Comments on the Efficacy of Sign Constraints in Portfolio Construction

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Abstract
In active management, one of the pragmatic issues of portfolio construction is whether it is commonly appropriate to overweight securities that are expected to underperform benchmark indices, or to underweight securities that are expected to outperform benchmark indices in order to improve the diversification of portfolios. After more than twenty years of observations across thousands of portfolios, we assert that sign constraints on portfolio construction are at worst benign, and often helpful. This assertion is based on a series of arguments, both theoretical and practitioner-oriented.

Introduction
In active management, one of the pragmatic issues of portfolio construction is whether it is commonly appropriate to overweight securities that are expected to underperform benchmark indices, or to underweight securities that are expected to outperform benchmark indices in order to improve the diversification of portfolios. A comparable question exists with long/short portfolios that are market neutral and hence benchmarked against cash. Would we short stocks with positive expected performance to improve risk control? Would we go long stocks with poor return prospects to better diversify our portfolios?

To the extent that long-only benchmark relative portfolios can be replicated with a long/short dollar neutral portfolio and an index overlay done with stock index futures or ETFs, the active component of the two cases collapses to the long/short case. Active extension strategies (e.g. 130/30) can be similarly decomposed into the same class of problem.

In considering this matter, we must first begin with a recognition that the sole purpose of conventional active management is to add return for the investor. In this context we refer to managers who are measured relative to a benchmark index, or the market neutral manager being measured in absolute return space. If a long-only investor wished to maximally diversify their portfolio relative to a particular index, they could simply purchase a passive index fund at a considerable savings in fees and trading expenses. Similarly, no investor would rationally pursue a market neutral long/short strategy that was expected to produce returns below those of available risk-free assets. We exclude certain hedging strategies, such as portfolio insurance, from this assertion as being explicitly meant to truncate the potential for extreme negative outcomes, although the incremental expected return is negative.

If we accept this view, we have drawn a profound line in the sand of portfolio construction. While a sensible investor may choose to reduce concentration in
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high return assets to diversify their portfolio in a fashion consistent with their degree of risk aversion, once we introduce portfolio positions with negative incremental return expectations, we are no longer diversifying the portfolio, we are hedging.

Sign Constraints and Estimation Error

The question of whether or not to sign constraint portfolio positions often arises because mean-variance optimization algorithms operating in the classical Markowitz fashion will occasionally compute optimal weights that are of the opposite sign from the return expectation for that security. However, it should be noted that traditional optimization methods assume that input parameters such as returns, volatilities and correlations are precisely known quantities, when in practice such inputs are uncertain forecasts of the future. In addition, this framework presumes that investor portfolio allocations are concerned only with a single time period, which is equivalent to assuming that transaction costs are zero.

The finance literature suggests that inclusion of sign constraints actually helps investment performance by mitigating estimation error in optimization inputs. In essence, the sign constraint acts as an informed Bayesian prior. This view is presented in Frost and Savarino, (1988) and again in Michaud and Michaud (2008). Both studies use statistical re-sampling methods similar to Monte Carlo simulation to address estimation errors in optimization inputs.

A different path to the same conclusion follows from other papers on estimation error in optimal portfolios such as Chopra and Ziemba (1993) and Broadie (1993). These papers find that estimation errors in expected returns have a much stronger influence on portfolio optimality than estimation errors in asset volatilities, and that estimation errors in asset volatilities have a greater influence than estimation errors in correlations. Entering into a transaction where relative return expectations and the active portfolio weights are opposite is a deliberate error in the use of our return expectations.

Our basis for this perverse action is that it avoids an error in reflecting the correlations among assets in our portfolio. If errors on returns are more harmful than errors in correlations, this is an unproductive policy.

It's All Moot Anyway

Another way to consider the issue of whether or not to hedge active risks is to demonstrate whether such an activity can meet any tests for statistical significance in terms of adding value for investors. The most common test for the efficiency of portfolios is the Sharpe ratio, or the benchmark-relative analog, the Information Ratio. Hypothesis tests for these ratios have been present in the financial literature for a long time, starting with Jobson and Korkie (1981). Subsequent improvements in these methods were made by Memmel (2003) and again by Ledoit and Wolf (2008).

In particular, the Ledoit and Wolf approach takes into account the existence of fat tails in the distribution of returns. They assert that the normal distribution assumption used in Jobson and Korkie leads to a large number of false positive results and therefore making the particular differences between portfolios seem more significant than they actually are.

To the extent that portfolio formation is carried out under uncertainty of the input parameters, the above tests would show that for any portfolio, even those considered Markowitz optimal, there are a large number of other portfolios that are statistically indistinguishable in their probable economic value to investors. Numerous studies have highlighted this issue, including Jorion (1992). Given the alpha and risk estimates of typical institutional portfolios, the likelihood that any specific hedging transaction could pass these significance tests is quite small.

The Impact of Transaction Costs

If securities could be traded with zero transaction costs, then it would be appropriate for managers to hold very broadly diversified active portfolios. Any expected incremental return above the risk free asset could be pursued
by holding a long position, and any expected negative incremental return would be exploited by a short sale. Once transactions are non-zero, we should only undertake positions when the certainty-equivalent incremental return (alpha) is greater than the associated expense. To the extent that alpha expectations are highly uncertain, only large-magnitude return expectations can overcome the transaction cost drag. In essence, active managers select securities largely from the tails of the expected return cross-section.

The same consideration of transaction costs should be applied to positions undertaken for the purpose of hedging. To undertake a hedging position against our own active strategy, we must consider the potential uncertainty in the required negative correlation, the expected cost of the hedge (negative alpha) and the associated transaction costs. Given the high degree of asset specific risk at the individual stock level, the uncertainty in such negative correlation relationships is high. In addition, to the extent we have explicit constraints (i.e. a 1 * 1 market neutral) or in implicit constraints (long-only) on the absolute amount of active positions, inclusion of the hedging asset also reduces the participation of high alpha assets in the portfolio, increasing the net cost.

It might be argued that for some very risk-averse investors, it might still make sense to hedge active risks, even after consideration of all the related negative aspects. However, investors that have such a low tolerance for risk would be unlikely to pursue actively managed equity portfolios in the first place.

Concerns about Hedging Active Management
The economic efficacy of diversification lies largely with reducing uncertainty over return expectations. To quote Antonio in Shakespeare’s Merchant of Venice, “My ventures are not to one bottom trusted, nor to one place; nor is my whole estate upon the fortune of the present year.” Conversely, the economic of hedging lies explicitly upon the necessary (but realistically uncertain) expectation that the principal asset and the hedging position will be negatively correlated in their payoffs.

Hedging market risks over which an investor or manager has no control is a long established and very sensible action for participants in both financial and tangible goods markets. However, the more subtle concept of an active manager hedging the risks of being wrong in their pursuit of their own active strategy (for which they are being paid a substantial fee) is a very different matter.

As long as investors have inexpensive access to risk-free assets, hedging active risks by an active manager represents a clear ethical conflict and potential regulatory risk. There is an excellent discussion of these issues in Scherer (2008).

Conclusion
Active management is all about constructing portfolios that reflect what the manager believes the future returns of their portfolio assets will be. Use of sign constraints on portfolio weights is well supported in the finance literature in terms of acting as an informative Bayesian prior for the portfolio formation process.

A broad range of literature also suggests that the inclusion of sign constraints in the portfolio construction process is very unlikely to ever do statistically significant harm to a portfolio, even on an ex-ante basis. This is doubly true if we have the real world situation of risk aversion levels consistent with active equity investing and non-zero transaction costs.

Finally, the inclusion of sign constraints avoids any implicit hedging of active management strategies by agent managers, and requires that risk control activities be limited to diversification. Such limitation removes both ethical and regulatory questions regarding the appropriateness of such hedging. In the absence of strong statistical evidence showing the
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benefit of such hedging in a specific case, avoidance of such activities seems advisable.

References


