



March 2011

# Northfield News

*A Newsletter for the Friends and Clients of Northfield Information Services*

## **Equity Risk, Credit Risk and the Returns to Corporate Sustainability** By Dan diBartolomeo

In recent years, Northfield has undertaken a substantial research effort in the area of corporate credit risk that has resulted in a unique new approach. Starting with Merton (1974), financial researchers have long understood the theoretical links between equity risk and credit risk. While “structural models” of credit risk such as Moody’s-KMV have been available for some time, we have developed a new approach to estimation and use of such models. In our approach, we derive the market-implied *expected life of a firm* based on the firm’s stock price, balance sheet leverage and the equity risk forecast from our models. We first translate the equity risk forecast into a forecast of volatility of a firm’s assets. An option framework similar to Merton (1974) and Leland (1994) is then used to derive an expectation of market implied expiration date of the option, which is a proxy for expected life of the firm. Obviously, the shorter expected life of the firm, the greater the credit risk.

Another output of the expected life method is a new and straightforward way to estimate the joint default probability across multiple bond issuers. Put another way, we have a mathematically tractable approach to estimating default correlations. We simply calculate the time series of percentage changes in the expected lives of the two or more firms we

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### Special Points of Interest:

- ▶ **Main Article: Equity Risk, Credit Risk and the Returns to Corporate Sustainability**
- ▶ **Tech Support Tip: Risk Acceptance Parameter**
- ▶ **Second Article: Estimating Unobservable Real Estate Returns**
- ▶ **Annual Conference Announcement- Colonial Williamsburg**



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## **Estimating Unobservable Real Estate Returns and What It Says About REIT Volatility** By Rick Gold

### Introduction

The shortcomings of private equity real estate ownership are well known. Appraisal smoothing, illiquidity, lumpiness (you cannot buy and/or sell 875 square feet of an office building to rebalance your portfolio), and placement issues, to mention a few, all contribute to the asset class’s structural problems. As a result, a growing number of institutional investors have been turning to Real Estate Investment Trusts (REITs) as both a substitute and as an additional diversifier over the past several decades. Initially created just over 50 years ago to allow small investors to buy commercial real estate, REITs would appear to be the perfect alternative for investors seeking to add commercial real estate to their portfolios. REITs offer partial ownership in companies that are traded on the world’s major exchanges and, while not a perfect substitute for direct ownership, they do not rely on appraisals for price discovery. Furthermore, like direct ownership, their tax structure allows all dividends to be distributed to the shareholder before taxes as long as 90% of a REIT’s net income is distributed making them extremely attractive to pension funds seeking current income.

Given their growing role in a mixed asset portfolio, Northfield decided to dig deeper into REIT behavior relative to the broader equity market during the recent financial crisis, and more specifically, whether that behavior offered investors any potential strategic opportu-

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### Inside This Issue:

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- ▶ **Staff Speaking Engagements**
- ▶ **Northfield on LinkedIn**
- ▶ **Newport Summer Seminar**

## Recent and Upcoming Events

### 2010 Newport Annual Summer Seminar

Tennis Hall of Fame • Newport, Rhode Island • June 3, 2011

Northfield's Annual Summer Seminar will be held at the International Tennis Hall of Fame in Newport, Rhode Island on June 3, 2011. The purpose of the seminar is to present recent research and technical advances to our clients and friends. Our meeting date has been selected to coincide with the US Professional Championships of Court Tennis. Following the day's presentations, there will be a Court Tennis demonstration by Northfield President Dan diBartolomeo, and then a Court Tennis match. Court Tennis, or "real tennis" is the medieval sport that is the progenitor of all modern racquet sports.

There is no charge for participation in any aspect of this event. The full seminar agenda and registration information will be posted on to [www.northinfo.com/events.cfm](http://www.northinfo.com/events.cfm) as it becomes available.

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### 2011 Northfield Annual Research Conference

Williamsburg Lodge • Williamsburg, Virginia • June 24th-26th, 2011

We are pleased to announce our 24th annual research conference at the Williamsburg Lodge, in Colonial Williamsburg Virginia. The conference will officially begin on Friday, June 24th and end on Sunday June 26th.

As is customary at Northfield events, a complete recreational and social calendar will accompany the working sessions. The recently renovated Williamsburg Lodge puts the excitement of Virginia's colonial capital at your doorstep with a generous helping of southern hospitality with impeccable service and a choice of comfortable accommodations.



Williamsburg Lodge

The full conference agenda and registration information will be posted to [www.northinfo.com/events.cfm](http://www.northinfo.com/events.cfm) as it becomes available.

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### Northfield Webinar: Optimization for Northfield Users

May 10, 2011 • 11:00 A.M., E.S.T.

Mike Knezevich, Northfield's Director of Technical Services, will be hosting a webinar presentation on May 10, 2011. The webinar will be a Northfield-centric description of optimization. Mike will build upon the tenets of modern portfolio theory as it is incorporated in Northfield's Optimizer and discuss the maximization of the utility function subject to different constraints and how the Optimizer mitigates constraint conflict during the optimization process. Although the content is theoretical in nature, this workshop is best suited for those familiar with the Northfield Optimizer.

There is no charge to participate. Visit <https://northinfoevents.webex.com> on April 18th when registration opens.

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### Northfield Webinar: Key Elements of Risk Control for Asset Managers

March 8, 2011

Northfield's Richard Pearce hosted a webinar presentation on March 8th, where he discussed the key elements of risk control for asset managers. The presentation discussed:

- Multiple reasons for using a risk model
- How risk models are built: model types, factor choices and completeness, time horizon
- Northfield risk models: analysis of reports and their part in portfolio construction

The event recording can be viewed at:

<https://northinfoevents.webex.com/northinfoevents/lsr.php?AT=pb&SP=EC&rID=3334332&rKey=d2d7a89232fe5ccd>

*(Equity Risk, continued from page 1)*

wish to consider and calculate traditional correlation measures. Unlike using time series of OAS or CDS spreads, this approach benefits from the greater liquidity of equity markets as compared to the very thin trading in corporate bond markets.

Our research in this area was recently published in the Journal of Investing (diBartolomeo, 2010). In this paper, numerous examples are given that may be of special interest to financial market participants. For example, as of December 31, 2004 the expected life measure for AIG was at the median of all publicly traded US financial institutions at approximately 30 years. AIG still had a AAA credit rating at this date, far better than the median credit rating of US financial firms. AIG lost their triple-A rating a few months later in 2005.

*Another interesting empirical finding is that the idea of “too big to fail” among financial institutions is not a recent development.* Over the pooled sample of all US financial firms (including ADRs) from January 1992 through March 31, 2010, the median life expectancy was 22.24 years with a capitalization weighted mean of 17.06 years. However, the revenue-weighted mean was only 7.86 years. This suggests that there has been a tremendous concentration of risk-taking among large financial institutions over the entire sample period beginning in 1992.

Our broadest empirical finding relates to the concept of corporate sustainability itself. It is often asserted that firms that behave in a fashion perceived as “socially responsible” will have greater sustainability. To test this assertion, we constructed two very broad portfolios. The first portfolio consisted of stocks that were in the MSCI DSI 400 index, (the oldest US stock index with inclusion based on social criteria), but were not members of the otherwise similar S&P 500. The second portfolio consisted of stocks in the S&P 500 that were not members of from the DSI 400.

The set of securities that were “DSI NOT S&P” had longer expected lives on average than those stocks in “S&P NOT DSI” for every date tested. In almost every case, a standard “difference in means” test evidenced a high degree of statistical significance. At least in terms of expectations, the SRI concept of sustainability is supported.

Our most recent line of research was to consider the influence on asset pricing of the bankruptcy risk of a firm, as measured by the lack of sustainability. To empirically research this question, we extended our existing data set through the end of 2010. We then formed quintile portfolios in decreasing rank order of sustainability and tracked the long term return performance and volatility for both

equal weighted and capitalization weighted portfolios. The results are summarized in **Table 1** for the period of January 1992 through December 2010.

	Mean Monthly Return	Cum. Return	Monthly Standard Dev.	Annual Compound Return	Leveraged S&P Risk Eqv. Return
Q5 Equal	1.33	713.77	9.15	10.90	7.45
Q1 Equal	1.03	790.86	3.64	11.50	12.83
Q5 Cap	0.77	251.60	6.62	4.98	4.76
Q1 Cap	0.79	414.32	3.78	7.77	8.26
S&P 500 <sup>2</sup>	0.75	347.74	4.32	6.78	6.78

As might be expected, an equal weighted portfolio of the low sustainability stocks (high risk) in Quintile 5 produced the highest monthly mean return of 1.33%. However, the volatility of such this portfolio was so high at over 9% per month that the annualized compound return was only 10.9%, which is inferior to the 11.50% annualized compound rate achieved by the equal weighted portfolio of high sustainability (low risk) stocks in Q1. It should also be noted that a market capitalization weighted portfolio of high sustainability stocks produce both higher returns and lower risk than the S&P 500. *These results ignore trading costs which are apt to be very substantial in the case of equal weighted portfolios.*

To further illustrate the return and risk characteristics of our portfolios, we calculated the return that would have been achieved by holding the subject portfolio and a cash position (positive or negative) such that the volatility of the portfolio was equivalent to the S&P 500. The yield on the cash position was assumed to be floating monthly at one percent over the three-month US Treasury Bill yield.

Leveraging up the equal weighted portfolio of high sustainability (low risk) stocks produced nearly a 13% annual compound rate of return, almost double the equivalent risk portfolio of the S&P 500. These results are consistent with an extensive financial literature regarding “low volatility” or “minimum variance” equity portfolio strategies as summarized in diBartolomeo (2007) and Scherer (2010). As the portfolio construction rules here are simply equal or capitalization weighting, credence is given to the idea that the results relate more to holding stocks that are individually low in risk, rather than forming portfolios that are jointly low in risk.

*(Equity Risk, Continued on Page 9)*

(Estimating, Continued from page 1)

nities. If nothing else, the research helped satisfy our own curiosity as to stability, the underlying relationship between REITs and the broader equity markets and whether this behavior fit our priors - especially with respect to the how they interacted leading up to the July 2007 market crash. Using *ex ante* estimates, our goal was to calculate the market characteristics of “true” unobservable real estate returns and what clues these “stripped” returns might offer investors in today’s market.

**Background**

We start off by making the simple assumption that REIT returns can be generalized by the following relationship:

$$REIT\ Returns_t = Unobservable\ Real\ Estate\ Returns_t + Stock\ Market\ Influence_t + \epsilon_t \quad (1)$$

Of course, the two independent variables are not orthogonal because of linkages in the real economy such as the credit markets. In the case of REITs, consumer spending, housing, consumption, and a host of other real estate-related activities are correlated. The error term is essentially white noise with a mean of zero but with potentially large positive and negative values in any single time period. This simple relationship is obviously not unique to REITs as many other equity sectors can be similarly bifurcated and analyzed.

Private equity real estate could also be defined by the same relationship. Because of its use of appraisals to set values, however, price discovery is limited and biased. In the long-run, private equity real estate’s self-immunization from the volatility of public market pricing through its use of appraisals may largely offset the non-real estate equity market factors that may influence REIT pricing. But that line of research will have to wait for another article. The question we hope to answer is once isolated: what are the characteristics of these unobservable real estate returns, are they stable over time, and do they have any practical applications?

Turning the previous equation around, we estimate “unobservable” or “stripped” real estate returns as REIT returns less the product of beta times the stock market influence which, for the purposes of this article, is defined as the S&P 500 Index such that:

$$R_t = \alpha + \beta * M_t + \epsilon_t \quad (2)$$

$$S_t = R_t - (\alpha + \beta * M_t) \quad (3)$$

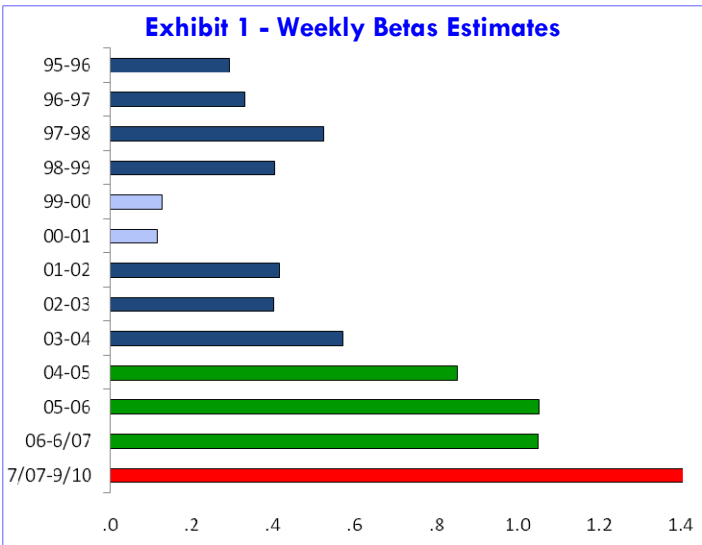
**Where:**

**S<sub>t</sub>** = Stripped or Unobservable Real Estate Returns

**R<sub>t</sub>** = REIT Returns

**M<sub>t</sub>** = Market Returns (S&P 500 Index)

There are several options available to infer the appropriate beta, from a simple time series regression to a direct derivation of correlation and standard deviations from one of Northfield’s risk models. We chose to estimate a series of weekly two-year regressions starting in January 1995 and ending June 2007. June 2007 was chosen as the termination date because it was the last complete month before the first casualty of the financial crisis emerged in July 2007. The intercepts were kept because, while they were found to be insignificant in eight of the twelve regressions, for those times that they were significant, dropping the intercept would bias the stripped return series. A total of twelve betas were estimated using weekly total return data. The first regression started in January 1995 and ran through December 1996. The second started in January 1996 and ran through December 1997. The one exception to this two year rule was the last regression which was truncated to 78 weeks (January 2006 through June 2007) ending just prior to the beginning of the financial meltdown in July 2007. We also estimated a regression from July 2007 through October 2010 to see how the relationship changed during the crisis itself. As can be seen in **Exhibit I**, the weekly betas are not constant over the fifteen years, rising steadily for the years immediately preceding the financial bubble in 2007.



As can be seen in **Exhibit I**, the betas remained within a fairly narrow range between 1995 and 2004 with the one exception being the period between 1999 and 2001. In 1999, REIT returns fell by approximately 6.5% while the S&P gained over 20%. However, over the next two years, S&P gave up all those returns as business activity peaked in mid-to-late 2000. REITs, on the other hand, paid investors handsomely as real estate markets continued to remain in balance because of a dearth of new supply entering the markets.

(Estimating, Continued on page 5)

(Estimating, Continued from page 4)

The post-2004 upward trend in the weekly betas can likely be traced to two factors. First, there was a post-recession decrease in volatility in the S&P 500 relative to the property sector after the 2002 recession. The second effect was the steady rise in correlation between the listed property sector and the larger equity market as the rising tide of the financial bubble lifted all boats (just before the financial tsunami crashed them on the rocks). The beta climbed even higher during the crash when the standard deviation on the NAREIT Index dramatically increased as REITs - facing declining rents and occupancy levels as well as balance sheets that were levered upwards of 50% or - more had become liabilities in the new world order.

**Stripped Returns Tell a Somewhat Different Story**

Once the beta had been estimated, it was possible to then go back and create the “stripped” or the unobservable real estate returns in **Equation 3**. In addition to the *ex post* stripped series, we also produced two *ex ante* series: a two year forward series for the twenty four months immediately following each observation period and, for all 11 regression results, *ex ante* estimates of stripped returns, from July 2007 to September 2010, the latest time period for which we had data when this study was conducted.

**Exhibit II-Unobservable Return Correlation Comparisons**

Weekly	Stripped Correlation Between			
	July 2007-Sept 2010		Two Year Forward	
	NAREIT	S&P	NAREIT	S&P
95-96	.99	.72	.95	.58
96-97	.99	.71	.93	.47
97-98	.97	.64	.70	-.55
98-99	.99	.68	.78	.46
99-00	1.00	.77	.99	.45
00-01	1.00	.77	.84	-.01
01-02	.98	.68	.93	.16
02-03	.99	.69	.97	.32
03-04	.97	.62	.94	.36
04-05	.90	.46	.87	.15
05-06	.82	.31	.79	.26
06-6/07	.83	.32	.80	.29
Avg	.95	.61	.87	.24

The estimated stripped series shows an extremely high correlation with the historic NAREIT series until 2004 when it begins to erode slightly, but it does maintain a consistently and significantly lower correlation with the S&P Index over each of the two year regression periods. In fact, during the estimation period just before the crash, the correlation between the two series falls to .32, almost half its historic average.

Even more dramatic are the differences in the *ex ante* correlations between the stripped series and the S&P 500. While the NAREIT Index and the S&P 500 had a correlation of .80 between July 2007 and September 2010, the highest correlation any of the stripped regression series could muster was .58. In contrast, the 97-98 regression produced a correlation of -.55. The average correlation was .24, hinting that this transitory beta may offer hedging strategies for investors - especially those whose real estate portfolios include both private and public real estate equity positions.

Standard deviations were also calculated for various periods. **Exhibit III** shows the estimated standard deviations of the stripped returns for each of the two year weekly regressions and their two year forward forecast. Again, a similar pattern emerges. Here we see that the standard deviation of the forecasted stripped series is smaller than the standard deviation of the actual *ex ante* NAREIT series with one exception. In 1999 and 2000, the unobservable return’s standard deviation was estimated to be greater than NAREIT’s by 30 bps. We attribute this to the high tech boom in Silicon Valley which drove down the correlation between NAREIT and the equity markets in 1999 and 2000 just as REITs were going through a choppy period. **Exhibit III** also shows the “estimated” stripped standard deviations from July 2007 through September 2010 for each of the 11 estimated regressions. Again, similar reductions in risk were achieved and, like the rolling two year forecasts, the smallest gap occurred in the 1999-2000 period.

**Exhibit III-Ex Ante Risk Calculation Comparisons**

July 2007-Oct 2010			Rolling Two Year Forecasts		
Standard Deviations			Standard Deviations		
Weekly Data	Actual NAREIT	Fitted Stripped	Weekly Data	Actual NAREIT	Stripped Forecast
95-96	6.28	5.48	95-96/97-98	2.12	1.81
96-97	6.28	5.39	96-97/98-99	2.23	1.98
97-98	6.28	4.93	97-98/99-00	1.77	2.07
98-99	6.28	5.21	98-99/00-01	1.96	1.77
99-00	6.28	5.92	99-00/01-02	2.09	1.89
00-01	6.28	5.96	00-01/02-03	1.88	1.58
01-02	6.28	5.18	01-02/03-04	2.05	1.79
02-03	6.28	5.21	02-03/04-05	2.24	1.99
03-04	6.28	4.83	03-04/05-06	2.20	1.79
04-05	6.28	4.29	04-05/06-07	2.45	1.95
05-06	6.28	4.01	05-06/07-08	5.91	3.72
06-6/07	6.28	4.01	06-7/07 TO 7/07-12/08	6.65	3.72
Avg	6.28	5.03	Avg	2.79	2.17

(Estimating, Continued on page 6)

Estimating, Continued from Page 5

## Applications

In order to get a second perspective, we ran a decomposition report using Northfield's Fundamental Model over six different time periods. The report compared a portfolio of market cap-weighted U.S. REITs to a portfolio of every stock in the Fundamental Model minus those same REITs. We then calculated the correlation between the portfolio of REITs and the universe of stocks during those time periods to see whether they exhibited the same pattern as the stripped series. While the correlations are lower, most likely due to the presence of a large number of smaller firms that are less closely tied to the economy than the S&P 500 and REITs, the general pattern holds, as can be seen in **Exhibit IV**. For this reason, we believe that using the stripped series may have some practical applications for investors.

### Exhibit IV-Fundamental Model Results

Date	R-Squared	Corr(R,M)
July 31, 1997	.41	.64
July 31, 1998	.33	.57
July 31, 2000	.30	.54
July 31, 2003	.55	.74
July 31, 2008	.67	.82
September 30, 2010	.87	.93

For example, investors with positions in both REITs and private equity real estate may opt to protect their brick and mortar position if they believe a market downturn is approaching since there is no practical way to sell physical properties in the short-run. That might not be the case if an efficient private equities derivatives market existed which unfortunately is years away from being a reality. The amount of the short depends on the "risk" to the long portfolio (private equity's cash flow, refinancing, etc.) in conjunction with an investor's forecasts of stripped returns and correlation with the broader market. This needs to be in addition to whatever rebalancing is done to an existing REIT portfolio. REIT-only investors can gain better exposure to pure real estate returns by shorting the S&P against their REIT position, effectively getting rid of non-real estate effects. Of course, the opposite is true in a bull market.

While the concept of a hedged REIT portfolio is also nothing new (See Giliberto 1993 or Liang and Webb 1996), there has been little to show on the practical side of the ledger since isolating the non-real estate component of REIT returns or, conversely, netting out the real estate component of non-real estate companies has always been a futile real world exercise (what percentage of GM's value

was in its real estate holdings when it declared bankruptcy or is Sears a retail company or a real estate company that happens to sell clothing and power tools?).

The actual hedge for a REIT-only investor is as follows:

$$\text{Hedge} = \hat{P} * \hat{f}$$

$\hat{P}$  = Expected correlation between the REIT's and the Market

$\hat{f}$  = Expected ratio of  $\sigma(\text{REITS}) / \sigma(\text{MARKET})$

Therefore, investors expecting a correlation of .75 between REITs and the S&P 500 over their investment horizon and a standard deviation of 4 and 6 for REITs and the S&P 500 respectively over the same time period would short their REIT portfolio by 50% and would adjust that position as their expectations changed.

## Conclusions

There is still much work to do. We have done some preliminary research with Canadian REITs that also reinforces our results for American REITs and over time we hope to extend the work to Europe and Asia. Initially, we had also hoped to introduce a financing variable in our specification given the importance of leverage for REITs but with no success. We will continue to explore this issue at a later date.

By bifurcating REIT total returns into unobservable/stripped real estate and equity market-dependent returns, we have undertaken to strip away the non-real estate factors that often dominate or hide true real estate returns. This simple model shows that once stripped to their core, listed property returns do indeed provide lower *ex ante* correlations and volatility than their non-stripped counterparts, adding an additional argument for a hybrid REIT vehicle in a mixed asset portfolio. Moreover, for many pension funds who hold both public and private real estate assets, a stripped hedge can help them during a real estate downturn. While the pure REIT hedge is rather straight forward, the hedge for a mixed private/public real estate portfolio is much more complex and involves understanding property-level risk in a securities market framework. Interestingly, Northfield has another model that does just that.

## References:

- Chatrath, Arjun. "Can We Hedge REIT Returns?," Real Estate Finance, 1999, v15(4, Winter), 78-85.
- Giliberto, S. Michael. "Measuring Real Estate Returns: The Hedged REIT Index," Journal of Portfolio Management, 1993, v19(3), 94-98.
- Liang, Youguo and James R. Webb. "The Hedged REIT Index And Mixed-Asset Portfolios," Journal of Real Estate Portfolio Management,

## Tech Support Tip: Risk Acceptance Parameter (RAP)

By Mike Knezevich

The Risk Acceptance Parameter (RAP) is an important input in the Northfield optimization process determining the magnitude of risk in the utility function. Higher RAP numbers correspond to an investor's larger appetite for risk whereas lower RAP numbers correspond to investors with a smaller risk appetite. The relationship between RAP and risk is intuitive given the position of RAP in the denominator of the risk cost portion of the utility function.

$$\text{Max } U = \alpha - (\sigma^2 / \text{RAP})^*$$

Before we can adequately discuss RAP we must begin by reviewing some basic tenets of the MPT.

### Efficient Frontier:

The efficient frontier is the set of portfolios along the investable set with the characteristics of having the:

- Greatest return for any given level of risk
- Lowest risk for any given level of return

The frontier is a concave curve with the most efficient portfolios along the curve. (Figure 1)

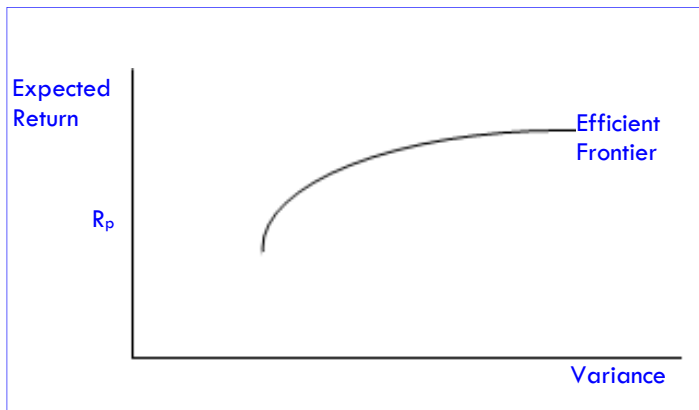


Figure 1

### Indifference Curves:

Indifference curves are a set of parallel upward sloping lines where the investor is indifferent between any point along the line. The upward slope characterizes investors demand for greater returns to undertake risk whilst still be indifferent between the investment choices along the line. Higher indifference curves are preferred to lower curves as this provides the highest attainable return for the level of risk (see dotted line, Figure 2).

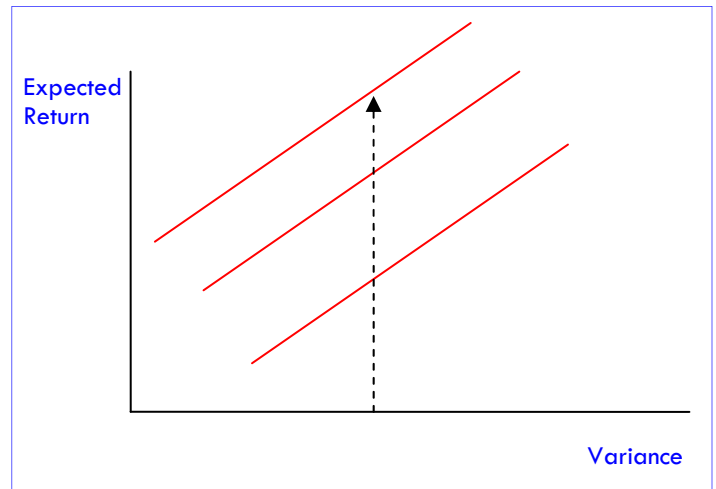


Figure 2

Superimposing the indifference curves with the efficient frontier we determine the highest point of tangency. At this point the investor is indifferent between a risk-free asset and a risky asset. In other words, the risk premium received by holding the risky asset compensates by at least enough to keep utility constant versus the risk free asset:

$$U = R_f = R_p - R_{\text{Risk Premium}}$$

This point represents the investor's optimal portfolio and the slope of the highest attainable indifference curve (as represented in blue, Figure 3)

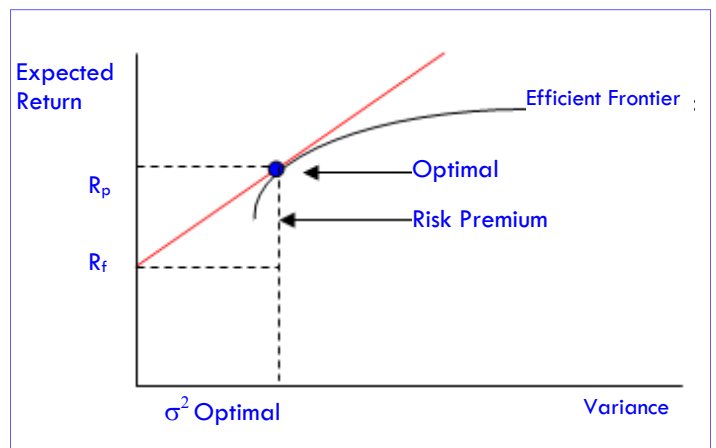


Figure 3

We calculate the slope, referred to as  $\lambda$ , by taking the partial derivative of the utility function and setting this value to 0 (see Rudd, p79):

$$\lambda = \alpha / (2 * \sigma^2)$$

(Tech Support Tip, continued on page 8)

\* For the purposes of this discussion, a simple utility function just comparing risk and return is used

(Tech Support Tip, Continued from page 7)

The slope is the risk aversion parameter ( $\lambda$ ) determining the units of return necessary to undertake each additional unit of risk. Mathematically RAP is equal to the inverse of the slope,  $RAP = 1/\lambda$ .

**RAP:**

We demonstrate the affect of RAP by comparing two investors with the same expected return and risk numbers, yet different levels of risk acceptance. The investors' differing preferences for risk results in two different optimal portfolios along the same efficient frontier. An investor with a higher RAP number (as represented in blue, Figure 4), will have a flatter indifference curve which will be tangent to a point higher on the efficient frontier. Whereas an investor with a lower RAP number (as represented in red, Figure 4) will have a steeper indifference curve which will be tangent to a portfolio with less risk..

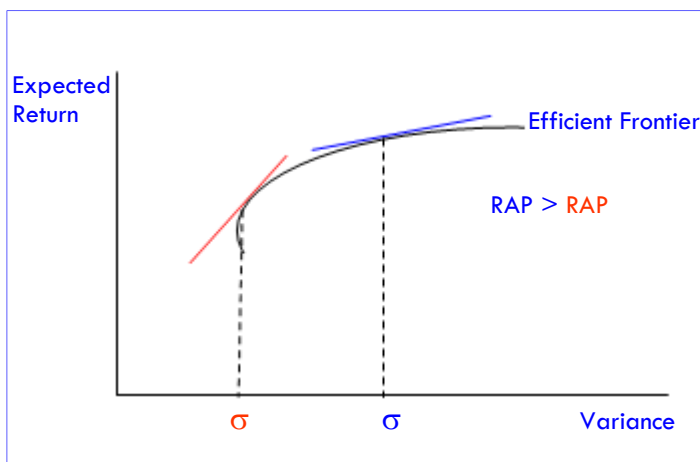


Figure 4

Since investors typically don't think in terms of their risk tolerance as the slope of their highest attainable indifference curve some insight into how Northfield parameterizes RAP and rules of thumb should help determine an appropriate number.

**Default RAP values:**

There is a historically rational basis to the Northfield default RAP of 100. Lets think of the collective US market as our typical investor. What long term risk premium has induced this investor to be invested at the long term level of risk? Begin by inverting lambda to calculate RAP:

$$RAP = 1/\lambda = (2 * \sigma^2) / \alpha$$

Then plugging in long term historical averages we calculate the default RAP:

$$RAP = (2 * (20)^2) / 6 = 133.34$$

Users should be cognizant of which values best suit their investment objective as historical values may not be the best predictor of future results. Alternative or expected values may better represent some investors' objectives (see Damaradon). Historical values are provided in absolute return space, thus the resulting RAP default is more appropriate for users concerned with absolute returns. For those managing against a benchmark, RAP may typically be much smaller.

**Range:**

Northfield recommends a range for RAP between >0 and 200. RAP's closer to 0 provide a greater negative impact of risk in the utility function and should be minimally slightly greater than 0 to avoid undefined values. RAP settings <1 can exponentially increase the cost of risk as the value tends towards 0 and should be used with a cautious understanding of the results. RAP values 0 or below are non-sensible as increasing risk would provide positive utility and therefore not permitted within the Optimizer.

Although users may set RAP greater than 200, this is the theoretical maximum for growth optimal investing and is based on the long term relationship of the geometric mean being equal to arithmetic mean minus half the variance in percentage terms (see diBartolomeo):

$$-100 + \Pi (100 + r_i)^{1/n} \approx (1/n) * \Sigma r_i - \sigma^2/200$$

At RAP =200 risk has no bearing on utility, but is purely an exercise in return maximization.

**Rules of thumb:**

With a better feel for the logic behind Northfield default and range for RAP's, we can now address sensible user numbers. The following are only rules of thumb providing a starting point, if optimal portfolio results are counter intuitive or the user's expectation have changed, RAP numbers should be re-evaluated:

- In *absolute terms*, the default RAP provides a good starting point. A more specific RAP is approximately 2 times investor's net worth as a percentage of total assets:

$$RAP = 2x(A-L)/A$$

- In *benchmark relative terms*, RAP is approximately 6 times the desired tracking error (for a comprehensive description see diBartolomeo):

$$RAP = 6xE(TE)$$

(Tech Support Tip, Continued on page 9)

(Tech Support Tip, Continued from Page 8)

### Systematic vs. Non-Systematic:

Preference to which risk the optimal portfolio is exposed to can be influenced by setting different RAP numbers for the systematic versus non systematic components. We see the impact that each RAP setting will have in the extended utility function:

$$\text{Max } U = \alpha - (\sigma_s^2 / \text{RAP}_s) - (\sigma_u^2 / \text{RAP}_u)$$

Keep in mind that the ratio of RAP may not be attainable dependent on the investment universe and other constraints.

### Conclusion

The reader should now have a better understanding of:

- How RAP is mathematically derived
- RAP's geometric significance in determining an optimal point
- Logic used in determining Northfield's RAP parameters
- Starting points to identify a user's RAP

### References

Damodaran, Aswath. "Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2010 Edition", Updated: February 2010, Stern School of Business.

diBartolomeo, Dan & Jennifer Gerber. Tech Tip: Choosing RAP, Northfield Newsletter July 2004, <http://www.northinfo.com/documents/16.pdf>.

Rudd, Andrew and Henry Clasing. Modern Portfolio Theory. @Andrew Rudd 1988, Orinda.

### Northfield Now on LinkedIn

Northfield has now been established on the LinkedIn business oriented social networking site. The group currently has over 150 members. To become a member of the group, visit <http://www.linkedin.com/groupinvitation?groupID=2228261>

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### References

diBartolomeo, Dan. "Equity Risk, Default Risk, Default Correlation and Corporate Sustainability", Journal of Investing, December 2010.

diBartolomeo, Dan. "The Equity Risk Premium, CAPM, and Minimum Variance Portfolios", *Northfield News*, August 2007.

Scherer, Bernd. "A New Look at Minimum Variance Investing", *ED-HEC Working Paper*, July 2010.

### Endnotes

1. We thank Steve Dyer for compiling this data.
2. Return information for the S&P 500 may differ slightly from published information due to timing differences in the application of corporate actions and index membership changes. The procedures used are consistent across all securities within the study universe.

### Northfield Speaking Engagements

Northfield President Dan diBartolomeo presented "Equity Risk, Default Risk and the Returns to Corporate Sustainability," at the Feb. 16th QWAFEFW meeting in Princeton, NJ.

Dan discussed "Parameterization of Trading Algorithms in Volatile Markets," at the Feb 24th High Frequency Trading Conference in London.

On March 14th, Dan presented "Risk Analytics for Multi-strategy Hedge Funds" at the Credit Suisse Hedge Fund Conference in Palm Beach, FL.

Dan will be presenting "Performance Attribution: Lies, Damned Lies and Statistics," at the April 21st London CFA Society Meeting.

On May 16th, Dan will be at the London Quant Group Spring Seminar where he will be discussing "The Central Paradox of Active Management."

Northfield's Nick Wade conducted a one-day "Quant Workshop" for the CFA society in Singapore on Feb 18th. Topics included: advances in asset allocation, risk model methodologies, and issues with existing risk measures & new advances.

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