Estimation Error in Asset Allocation

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The purpose of any asset allocation technique is to determine, with greatest accuracy, that mix of asset classes which will produce the most satisfactory result for the owner of funds during some FUTURE investment period. Use of mean-variance optimization is a widely adopted technique for obtaining the "most satisfying portfolio". Inputs to MV optimization are expected mean future returns for each asset, expected volatility of returns around the future expected means and the expected matrix of correlations of all returns.

The traditional approach of using historic returns for expected returns is grossly inadequate and technically inadmissable (See Stein). For reasonable time horizons, current market conditions (yields) represent a far superior forecast for fixed income securities. At worst, other asset classes can then be initially forecast based on historic premia relative to the fixed income class.

Once we admit that estimation error exists, it follows that the efficient frontier is not made up of discrete points. Each point is surrounded by a small region of other points which are not statistically different from the center point. As such, the efficient frontier becomes a region of overlapping circles (See Michaud for a good general discussion).

We must also consider that estimation error can have two sources. The first source is that the historic sample mean is not representative of the population mean. The second source is that for a given time horizon, our forecasts may differ from our own estimates of population parameters. If we simply wish away estimation error, we are systematically understating the uncertainty in the problem. As such, any of the following three approaches will bias the MV optimization toward a better result:

a) shrink the expected returns for each asset class toward an overall mean. This approach is technically use of Bayes-Stein or James-Stein estimators and has been well explored in the literature. These methods are typically used to deal with the errors arising between finite period forecasts and population parameters. (See Jorion)

b) calculate the entire efficient frontier but select a more conservative risk/reward tradeoff than you otherwise would. This is an easy but rather rough cut at the process, as most investors have enough trouble simply deciding how risk averse they should be, let alone how much to increase that risk averseness to make up for estimation errors.

c) explicitly quantify the magnitude of the potential estimation errors and add this uncertainty to the expected variances for each asset class. This approach is the most work but the most conceptually correct.

Statistical procedures exist for explicitly estimating the potential for difference between sample parameters (historic) and population parameters. In particular, a form of Monte-Carlo simulation known as "bootstrapping" can be used. In this technique, the historical return series is reconstructed by selecting observations from the historic series at random. For example:

We could extend a ten year historical series by doing 120 random selections of individual months (all series together so as to maintain historic correlations) and then calculate the mean and standard deviation of the simulated series. Repeat this process 1000 times and plot the mean and standard deviation of each simulated series. The result will be a scatter plot of 1000 surrounding the point representing the original historic series. By use of the Pythagorean theorem use can calculate the Euclidean distance between the center and any point of the scatter plot. Calculate the mean and standard deviation of the Euclidean distances. The variance of the Euclidean distance can then be added to the expected variance of each series to get a better value for the uncertainty of future returns for each asset.

We can also make standard assumptions about significance such as only those points lying outside two standard deviations of Euclidean distance are really different than the original point. This idea has particular appeal for rebalancing of portfolios as a means to determine whether or not a new asset mix is sufficiently different from the old to make it worthwhile to incur the transaction costs. In short a method of this type avoids trading on "noise". (For a full discussion of this topic see Bey, Burgess and Cook)

A more simple test of the "differentness" of portfolios can be constructed as follows:

Given two asset mixes, simulate the historic returns to each mix over the longest time period available. Combine the two series into one and calculate the joint median. Construct a 2 X 2 contingency table a below:

Monthly Returns

	Returns<Joint Median	Returns > Joint Median
Asset Mix 1		
Asset Mix 2		

One may now perform traditional ChiSquare tests and determine whether the median of the return time series of asset mix one is significantly different from the median return of asset mix 2.

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