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An Equilibrium Term Structure Approach to Asset/Liability Analysis

ABSTRACT

The purpose of this paper is to present an alternative approach to the traditional actuarial analysis of financial intermediaries such as pension funds and insurance companies. Rather than the usual actuarial study which establishes single value assumptions for parameters such as interest rates and inflation expectations, this approach allows for a rich set of possible future conditions which can be both state dependent and path dependent. The motivation for taking this unusual approach is the possibility that extreme economic conditions could lead to substantial underfunding of a pension plan or insurer, leading to economic strains on both the beneficiaries and the sponsoring corporation.

One can easily hypothesize about states of the US economy which would place extraordinary pressures on financial intermediaries with uncertain future liabilities. A recent example would be the period of strong inflation during 1970's which both magnified future pension liabilities and which was a period of very limited returns to conventional investment assets. However, it should be noted that even this period would be considered a very mild economic shock when compared to economic upheavals which have been observed at other times (e.g. the 1930's) or in other nations (e.g. the Brazilian hyper inflation).

METHODOLOGY

In a traditional actuarial study, the future benefit payouts of the plan are projected on the basis of a set of assumptions. Among these assumptions are the investment return expectation, inflation, real wage growth, mortality, and the rates of early retirements and terminations. Each of these items is forecast as a single value rather than a range of values. Once these assumed values are specified, the projected benefit payout for each employee is calculated, and results summed across all employees of the firm. This forms a time series of payouts which is discounted to present value and compared to the market value of plan assets. To the extent that plan assets are greater than or equal to the present value of liabilities, the plan is adequately funded. If the market value of assets is less than the present value of liabilities, the plan is underfunded and represents a potential hardship to both beneficiaries and the sponsoring company.

In this method, we will endeavor to project the **probability distribution** of possible "present values of future liabilities" for each year up to some selected horizon in the life of the plan (say the next thirty years). We will then make related forecasts of the **probability distribution** of possible asset values for each of the thirty years. To the extent that any portion of the range of possible asset values lies below the concurrent range of possible "present value of future liabilities", the potential for underfunding exists. As such we will not only be able to consider the issue of current funding, but we are able to project potential problems in each of the next thirty years. While some actuaries do include analysis of the sensitivity of their results to simple changes in assumptions, the path dependent interaction of assets and liabilities is not generally addressed.

The key benefit of this methodology is that funding status of financial intermediaries such as pension plans is no longer looked at as a single number but rather as a probability distribution at each moment in time. In addition, it gives force to the idea that while traditional methods contain assumptions that may actually be a proven correct on average in the future, we experience the future in the form of a single path of outcomes drawn from the distribution of events, only the mean of which has been traditionally considered.

Our task was then to construct a conceptual framework within which the potential future values of both liabilities and assets may be evaluated in a consistent fashion. We hypothesized that interest rates and inflation were the two key variables which drove valuations of liabilities and which also were instrumental in explaining the non-random aspects of asset returns. If we could devise a system of wide ranging future scenarios for interest rates and inflation in a way that was consistent with real world conditions, we would have the basis on which to proceed.

The idea of looking at pension funding as a set of path dependent probability distributions is not new. Many actuarial firms offer "Monte Carlo" type studies as one of the services which they sell. In addition, investment practitioners such as Michael Peskin, formerly of Morgan Stanley, have researched and advocates similar approaches. What is new in this approach is using the current term structure of interest rates as a mechanism to insure that the range of possible events within a Monte Carlo study are conformed to be consistent with an exogenously and unambiguously defined set of market expectations for key variables.

The basic model used in the study is a binomial tree of possible future interest rates. This sort of methodology is in widespread use for analysis of complex fixed income instruments such as callable bonds, mortgage-backed securities and other fixed income instruments where the future cash flows are not known with absolute certainty. For a detailed discussion, see Fabozzi and Dattatreya, *Journal of Portfolio Management*, Spring 1989. If we assume that the interest rate for a one year period as of today is I, and we have some estimate of the extent to which interest rates have varied historically called V (typically on the order of .15), we can say that a year from now there will be two possible states for the then one year interest rate. Interest rates could move up to $I * (1 + V)$. Interest rates could also move down to $I / (1 + V)$. In the subsequent year, interest rates could move either up or down from each of the two possible levels just defined. After two periods, there would be three possibilities for our interest rate assumption: interest rates have gone up twice from the beginning, gone down twice from the beginning or two chances be back where we started (up move followed by down, or down move followed by up). Proceeding in this fashion we would have thirty one possible rate levels in year thirty. This process is referred to as a random log-normal process. One should note that at each year of the level, the highest interest rate in the range is $I * (1 + V)^N$ where N is the number of the year. This exponential effect provides small probabilities of extremely high interest rates in later years. Similarly, the compounding of the division in a string of downward moves would take interest rates close to zero.

Note that at each node or intersection of the tree, not only is the one year interest rate expressed, but the rate for any maturity can be derived by averaging the many possible paths of interest rates arising from movements subsequent to that node. As such, the present value of any cash flow, irrespective of time of payment, can be discounted back to each node.

Once we have constructed a basic tree, our next step is to determine if our tree is consistent with real world conditions. To do this, we take the cash flows from each issue of popularly traded US Treasury bond and discount that bond's cash flows to present value. If our tree is consistent with the real world, the present value just calculated will be equal to the actual market price of the bond. If the market price and the calculated present value of the bond's cash flows are not equal, we can "bend" the branches of our tree until all popularly traded Treasury bonds price correctly. We now know that our binomial tree has the property of "**external consistency**". That is, its implicit spread of scenarios for future interest rates is consistent with the potential range implicit in the actual market prices of widely-held fixed income securities.

Now that our interest rate framework is in place, we move on to the relationship of inflation and interest rates. For this purpose, we estimate a regression equation where inflation is presumed to depend on interest rates, rather than the more typical way of looking at it, where interest rates are presumed to be a function of underlying inflationary trends. However, the correlation between changes in inflation and changes in interest rates is all that we need to define for this study. The direction of causality is unimportant. We need only to be able to define the most likely value (and potential dispersion) of inflation given a hypothesized level of interest rates. The average annual rate wage growth is assumed to be equal to whatever assumption was made in the traditional actuarial report. By making certain key assumptions at the same values assumed for the traditional actuarial approach, the mean result from this method are conformed to be equivalent to the result of traditional method, providing a characteristic of "**internal consistency**".

With regard to the asset side of the equation, the approach is to consider some particular mix of investment assets, calculate the return series which would have arisen from such asset mix over some historical time period and finally, estimate a regression equation which relates the non-random portion of asset returns to interest rates and inflation (potentially both contemporaneous and lagged).

Turning back to the liability aspect, we were now ready to conform our interest rate tree to the initial assumptions in the actuarial report. To do this, we took the projected cash flows listed in the actuarial report and discounted them to present value today using the discount rate specified in the traditional report. We then took those same projected cash flows and discounted them to present value today using the "tree" of rates which we had calibrated to the Treasury bond market. If these two present value calculations provide different present values for the liabilities, the interest rates in the tree are adjusted (trial and error) until the two calculations produce the same present value. In most cases, the interest rates in the tree will have to be increased in order to obtain the same result for both calculations. This incremental increase in the interest rate can be thought of as the additional yield we experience as compensation for the risk investing in instruments of less than Treasury credit quality. Our interest rate tree is now consistent with both the range of possible future interest rates implied by the current Treasury bond yield curve and with all the basic assumptions supplied by the actuaries.

THE ALGORITHM

Now that our underlying assumptions are complete, we need only conduct the actual computations to reach the results. To do this we proceed along the following series of steps:

1. Start at the root of the tree (today). Move forward randomly through the tree until you reach the other end at time equaling thirty years from now. This will give us a series of thirty one-year interest rate scenarios which we might experience.

2. Given the series of thirty interest rates along this time path, project the concurrent thirty annual inflation estimates. Recall that the inflation estimate has both a component related to interest rates and its own independent random movement.

3. Use the series of inflation estimates to adjust the series of projected cash flows, to reflect inflation experiences above or below the annual rate projected in the actuarial report.

4. We now discount the newly projected series cash flow liabilities using the series of thirty interest rates along our time path. Note that the projected "then" present value of subsequent liabilities is forecast for each year in the thirty year series. Save this series of thirty projected present values of subsequent liabilities.

5. Project the value of the assets for each of the thirty years given our assumptions of random returns with means and volatilities equal to historic averages, deducting the projected payouts along the way. Save this series of thirty yearly projected values for assets.

6. For each of the thirty years, subtract the projected "then" present value of subsequent liabilities from the concurrent projection for asset values. This result is the level of funding surplus or deficit in the plan. Save this information.

7. Go back and start again at step #1. Recall that since many of the steps in the analysis assume random movements and returns, each trip through the tree represents a different sequence of events. Repeat the entire procedure a large number of times (as many as necessary) to reach the desired level of confidence interval on the estimates of the parameters of the resultant probability distributions.

Assuming that we repeated this procedure a thousand times, we would have one thousand series of projected funding surpluses (deficits), each series containing thirty separate projections for each of the next thirty years. If we look at each year, we can assess the **probability distribution** of funding surpluses (deficits) at that future moment in time.

A REAL LIFE EXAMPLE

A study of this kind should serve as an addition to rather than a substitute for a traditional actuarial study. It should be viewed both as a research tool and as a pedagogical exercise meant to lead us into a dialogue and further research on the issues at hand. In carrying out the example problem presented here, we have made many assumptions, and the precision of our results is limited in many ways. Among these limitations are:

1. All of the actuarial information which is used herein was taken directly from a document supplied to us by the actuaries of a Fortune 500 pension fund. The base case estimated cash payouts of the fund were supplied to us. The actuarial discount rate was 8% and the assumed wage growth was 5.5% per annum.

2. The asset and liability valuations in this report are based purely on the projected asset returns, and discounting of liabilities. For example, no attention has been paid to accounting and actuarial policies which would smooth investment earnings. Similarly, all assets are assumed to be valued at market at all times. It should be noted, however, that due to the fact that the method preserves the path dependence of results, it would be easy to incorporate such accounting and actuarial policies as needed to extend the results into periodic projections of sponsor contributions.

3. The basis for expected returns to plan assets are derived from historical returns for the asset classes in which the assets are invested. We have made no investigation of the particular returns historically achieved by this specific plan. No adjustments have been made to reflect any beliefs as to whether the actual managers employed by the plan will achieve returns inferior or superior to market averages. S&P 500 returns were used to represent equities, and the Saloman Brothers Corporate Bond index to represent fixed income. Cash assets were proxied by three month Treasury bills.

4. Historical data for the period February of 1962 through November of 1994 is used for return and volatility assumptions for asset classes, and inflation. The volatility of interest rates was implied from the prices of options on futures contracts for T-Bills, T-Notes and T-Bonds as of 9/30/94. The assumed maximum level of the one year interest rate in the future was 25%.

5. The Treasury yield curve at 9/30/94 was:

1 Year Maturity	5.89%
2 Year Maturity	6.52%
3 Year Maturity	6.83%
5 Year Maturity	7.28%
7 Year Maturity	7.40%
10 Year Maturity	7.57%
30 Year Maturity	7.80%

EXAMPLE RESULTS

Using the pension plan's current asset mix as of 9/30/94, we ran the Monte Carlo process through 1000 trials. The market value of the included assets as of September 30, 1994 is approximately $325 million. By traditional actuarial calculation the present value of liabilities at the 8% actuarial discount rate is $241 million, leaving a 1994 surplus of approximately $80 million.

For 1995, the mean value of the surplus is projected at $86 million across the 1000 trials. The minimum surplus value for 1995 is negative $26 million, while the maximum value is $275 million. On average the projected surplus values are entirely positive, rising steadily to an average value of $1.659 billion in the year 2024. However, the minimum surplus or "worst-case scenario" starts to take on negative values for surplus (deficit) in the year 1995, with a potential worst-case deficit of $23 billion dollars by the year 2024. It should be noted that this worst-case scenario is more than five standard deviations away from the mean, suggesting the likelihood of such an event occurring is less than one in a thousand. The likelihood of there being any deficit at all by 2024 is less than one is six.

The assets of the plan (net of payouts) are projected to grow to about $1.7 billion by the year 2024. The minimum value of assets could possibly turn negative by the year 2011, indicating that the plan would be out of money and could meet payout requirements only through new contributions. Again, the likelihood of such a dire outcome is very small, around one in forty for the year 2014.

CONCLUSIONS

Current practices in asset/liability analysis of financial intermediaries are far too dependent on point estimates of key assumptions. The method presented here represents future outcomes as probability distributions, allows for both state and path dependencies, and for conformity with internal and external consistency criteria.