Starting with Merton (1974), financial researchers have long understood the theoretical links between equity risk and credit risk. While "structural models" of credit risk such as Moody's-KMV have been available for some time, we have developed a new approach for the use of such models. In our approach, we derive the market-implied expected life of a firm based on the firm's stock price, balance sheet leverage and the equity risk forecast from our models. We first translate the equity risk forecast into a forecast of volatility of a firm's assets. An option framework similar to Merton (1974) and Leland (1994) is then used to derive an expectation of market implied expiration date of the option, which is a proxy for expected life of the firm. Two methods for improving estimates of default correlation are provided. We will also show empirical uses of the technique at both the firm level as a measure of credit risk and at the market level as a metric for systemic risk. Finally, we will also present evidence that the concept of corporate "sustainability" as broadly used by socially responsible investors appears to be supported, with purportedly sustainable firms having average expected lives which are longer than those of non-sustainable firms to a statistically significant degree.

Basic Contingent Claims Literature

The literature in this area begins with Merton (1974), where it is posed that the equity of a firm can be viewed as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt. For limited liability companies where there is no recourse to shareholders, this can alternatively be viewed as having lenders that are short a put option on the firm's assets. Rather than pay debts when due, shareholders can merely deliver the assets of the firm to the creditors as part of a bankruptcy process. Under this approach, it is assumed that default can occur only at the debt maturity, when substantial repayment is required.

The approach was extended by Black and Cox (1976) where they introduce a "first passage" model wherein default can occur before debt maturity, making the problem more similar to an American option. In this method, we assume that firm extinction will arise if the value of the firm's assets hits some boundary value that is expressed by the bond covenants or other collateral agreements. Two subsequent papers, by Leland (1994) and Leland and Toft (1996) further extend the method to account for the tax deductibility of interest payments, and the procedural costs of a bankruptcy. They estimate the extinction boundary value as the point where the salvage value of equity is maximized in a bankruptcy proceeding. A very useful computational advance is in Yaksick (1998), which provides a
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closed form solution to the expected optimal exercise of a perpetual American option.

Default Correlation

As corporate bankruptcies bond defaults are rare, generally singular events it is impossible to directly observe the pair-wise correlations of default among set of firms. When we talk about default correlation, we are either describing the variations in the frequency of events within large samples of firms, or are describing the time series variation in some measure of the likelihood of default, such as yield spread or bond rating. In some semantic sense, we might describe default correlation as the degree of dependence between firms such that the joint likelihood of both firms defaulting within a defined time interval is correctly specified.

Papers by Hull and White (2001) and Overbeck and Schmidt (2005) illustrate that you can estimate default correlation if you know the true (but generally unobservable) dependence between firms. The analytical framework of structural models provides the ability to estimate default correlation, given the correlation between the asset values of firms, as described in Zhou (2001). Frey, McNeil and Nyfeler (2005) extend the approach using a factor model that estimates correlations. Two papers by Giesecke (2003, 2006) try to improve the estimation of default correlation by including the correlation of changes to default boundaries, such as the tightening of bond covenants during periods of tight credit conditions.

We can also make the simplifying assumption that asset correlations are equal to equity return correlations. The book value of firm assets is a very incomplete measure of firm assets, so observing asset volatility and asset correlations across firms from financial statement data provides only very weak statistical estimates. Conversely, equity return volatility and correlation are readily observable for publicly traded firms. This approach is followed in Hull and White (2004), and apparently in the commercial service CreditMetrics.

The assumption that asset correlation equals equity return correlation is intuitive for situations where firm financial leverage is low and time horizons are short. However, Zeng and Zhang (2002) show that the asset correlations must be inferred from both the correlation of equity and debt components of the firm’s balance sheet. A paper by DeSerigny and Renault (2002) also presents negative empirical results regarding this assumption.

Bring on the Factor Models

Given that almost all large asset managers have access to a commercial equity factor risk model, those same firms can estimate firm asset value correlations. From such an equity risk model, one can estimate the numerically equivalent full co-variance matrix among any set of included stocks, as described in diBartolomeo (1998). Qi, Xie, Liu and Wu (2008) provide a complex analytical derivation of asset correlations given equity return correlations. If we have a multi-asset class risk model that includes both equity and fixed income securities, we can use the fundamental accounting identity to get a direct factor representation of asset volatility and equity, as Assets = Liabilities + Equity. Asset volatility is just equity volatility de-levered, adjusted for covariance with the market value of debt. When interest rates rise equity values usually drop. However, the market value of debt definitely declines,
so leverage is sometimes reduced. Current accounting standards recognize this situation by allowing firms to show a profit by buying back their debt below par value in the secondary market. Using the same algebra as above, we can convert the factor representation to pair-wise asset correlation values across firms.

Proposed Method of Measuring Sustainability

With asset volatility and correlations estimated we can use our preferred structural model to estimate the default probability of a firm. We can use the method from Zhou to convert asset correlations to default correlations, allowing us to produce joint default probabilities across firms. However there are some fairly restrictive assumptions. First, the firm must have debt today. Secondly, the firm must have positive book value today and third, the balance sheet leverage ratio must stay fixed in the future.

In order to reduce the impact of these assumptions, we propose to reverse the concept of structural models away from default and focus on the sustainability of firms. We will pursue this by estimating the market implied expected life of firms using contingent claims analysis. Firms with no debt can now be included since it is possible that they get some debt in the future and default on that. This approach also provides a quantitative measure of the fundamental and “social” concept of sustainability.

To estimate the expected life of a firm, we return to our basic option formulation of the problem. The option underlying is the firm’s assets with asset volatility determined from the factor model as previously described. We propose to solve the option pricing model numerically for the implied expiration date of the option that equates the option value to the stock price. The time to the implied expiration date is the market implied expected life of the firm. Given that we are now allowing the expiration date to be the free parameter of the problem, we must include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately. If you choose the basic Black-Scholes as your option model, then you can solve BS for the implied time to expiration using a Taylor series approximation. More complex option models allow for stochastic interest rates and stochastic volatility of the assets.

Using an actual option pricing model is problematic for firms with no current debt or a negative book value. For these firms, we fill in the missing data points using a simple “distance to run” measure. We simply assume that non-survival will be coincident with stock price to zero, since a firm with a positive stock price should be able to sell shares to raise cash to pay debt. For example, if you have a stock with 40% annual volatility you need a 2.5 standard deviation event to get a -100 return. We can then calculate probability of a 2.5 standard deviation event under your choice of distributional assumptions.

To calibrate the firms estimated under the short cut measure as compared to the full option pricing procedure, we convert both measures to the median of the distribution of future survival in years. In essence, we ask the number of years such that the probability of firm survival to this point in time is 50%. The individual firm expected lives have highly skewed distributions, so we upper bound these values at 300 years. The individual firm “median of life” scores under the two methods are both converted to Z-scores, and the score from the distance to run measure is
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mapped into the concurrent cross-sectional distribution of Z-score for the option method.

An Example Empirical Study

In our first study, we will use a simple Merton model (Black-Scholes European put). Our equity return volatility forecast input will be obtained at each moment in time from the Northfield US Fundamental Model, which has a forecast risk horizon of one year. It might be argued that our “near horizon” risk model (forecast horizon two weeks) is more suitable for this study, but at the current time less data history is available.

Using the procedures described here, we estimated the median life measure monthly for all firms in Northfield’s US equity universe from December 31, 1991 to March 31, 2010. We broke the universe of firms into three categories: all, financial firms and non-financial firms. It should be noted that there is considerable time series variation in the summary statistics. This variation arises from changes in stock price levels, interest rates, individual firm debt levels and in the Northfield risk forecasts (which encompass additional factors). Another important contributor is the time variation in the number of stocks publicly traded in the US and covered by the risk model, which ranged from 4660 at the start of the sample to a high of 8309 during 2000.

Given the recent financial crisis, it might be useful to begin with an examination of the summary information at the end of the study. Across the sample of 5068 firms, the median life expectancy was 23 years, with a mean of 22.18 and a capitalization-weighted mean of 25.71. For financial firms, a sample of 1132 firms had a median of 24 years, a mean of 21.69 and a capitalization weighted mean of 18.95. This result illustrates that the global financial crisis was very concentrated in large firms, as the capitalization weighted mean is substantially below other measures of central tendency. For the set of 3936 non-financial firms, the median life expectancy was 23 with a mean of 22.33 and a capitalization-weighted mean of 27.36.

There were also intuitive results at the individual firm level. Firms at the center of the crisis such as AIG, Citigroup and Goldman Sachs had very low life expectations of seven, six and six years respectively. Other “blue chip” firms such as Microsoft (32), IBM (30), Royal Dutch (39) and Exxon-Mobil (54) have life expectations of much greater length.

Across the entire time period of 220 months, the average sample size was 6587 firms, for a total of more than 1.3 million observations. The time-series average of the monthly medians, 21.63 years, with the times series average of the simple mean of 24.42 years and the capitalization weighted mean of 22.67. Particularly low values occur at several points in time, such as the recession period of 1992. At January 1992, the median was reduced to just 10 years with a mean of 13.20 and a capitalization weighted mean of 11.05. Conversely, the peak values were observed in January 2005 with a median of 30 years, a mean of 41.09 and a capitalization weighted mean of 32.36.

For the financial firm sub-sample, the average sample size was 1630 firms. The time series average of the monthly medians was 31.03, the time series average of the monthly means was 31.51 with the average capitalization
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weighted mean at 24.09 years. For non-financial firms, the average sample size was 4955. The time series average of monthly medians was 20.03 years, the times series average of the monthly means at 22.13, with a capitalization-weighted mean of 22.23 years. Note that for the full time series, financial firms were expected to survive about 50% longer than non-financials. At last date in March of 2010, financials have slightly lower expected lives. Our time series information also gives us another view of default correlations. Once the time series of expected lives have been calculated, we can estimate default correlation as the correlation of percentage changes in expected lives across firms. As expected lives shorten, changes of a given magnitude become larger percentage changes.

Since correlation is a bounded function (-1 to +1) larger events drive the correlation values toward the extreme values. For example, two bonds whose issuers each have one day of expected life each will have a very high default correlation. We believe this approach will prove more satisfactory than trying to correlate movements in option-adjust yield spreads (OAS) since changes in bond prices are substantially impacted by liquidity effects.

Quantifying “Sustainability” of Firms and Nations

We can also use our model to test the empirical existence of the property of firm “sustainability”. In this example, we will examine the FTSE/KLD DSI 400 stock index of US large cap firms considered socially responsible. A history of more than 20 years of the index constituents was reviewed. Typically about 200 firms are in common with the concurrent S&P 500.

Let’s look in some detail at just two illustrative points in time. First, let us consider July 31, 1995. For the DSI 400, the median expected life is 17 years, a mean of 17.91 with a standard deviation 9.93. For the S&P 500 the median is 14, the mean is 15.40, and the standard deviation 9.28. This difference in means is statistically significant at the 95% level. Similar information is observed at March 31, 2010. For the DSI 400, the median is 30 years, the mean 26.39, with a standard deviation of 11.45. For the S&P 500, we have a median of 30, a mean of 24.93 and a standard deviation of 10.92. This difference in Means is statistically significant at 90% but not 95%.

However, we must recall that the two sets of index constituents typically had about 200 common members. If we remove all firms that are present in both indices to just examine the disjoint sets (DSI NOT S&P and S&P NOT DSI) we have obtained statistically significant differences in all periods tested, supporting the SRI concept of firm sustainability. This line of research will be the subject of a full forthcoming paper.

Obviously, if the market thinks public companies are not going to be around very long, the economy is in a bad way. Under the structural model, low equity valuations and high leverage equate to short life expectancy, while higher leverage can be sustained with higher growth rates that contribute to higher equity valuations.

We propose to use the “revenue weighted” expected life across the universe of firms as a measure of systemic stress on an economy. By revenue weighting we capture the stress in the real economy, since failure of a high revenue firm typically leads to job losses and substantial effects of supplier firms. We also avoid a bias introduced
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cap weighting since failing firms have small equity market capitalization and effectively fall out of the sample.

For the full sample, the maximum value of the revenue weighted mean was over 36 years, while the minimum of about 6.5 years was reached during the 1997 currency crisis. For the financial sub-sample, the lowest value was 4.55 years also during the same events, while the 2008 financial crisis only got values down into the nine to ten range. We find the time series properties of the revenue weighted mean to be a very intuitive representation of systemic stress in the global economy.

Our next research project based on this method will be to investigate the ability of the expected life data to predict changes in firm level credit ratings. To remove any possible biases in the credit rating changes, we have hand collected (copied from Barron’s week by week) every credit rating down grade and upgrade since 1991. Our effort will be to relate changes in expected life and expected life values that are outliers within their rating category to subsequent changes in bond ratings. Eventually, we will use this mechanism to refine the credit risk expectations for both bond issuers and financial counterparties in our Everything, Everywhere model of risk.

Conclusions

Our research indicates that combining factor models and structural models of credit risk allows for consistent estimation of equity risk, credit risk and default correlation. Structural models based on contingent claims methods are a direct and informative approach to assessing the expected survival of firms. Empirical studies comparing constituents of SRI and conventional US stock indices reveal a positive and significant difference in expected lives, confirming the existence of “sustainability.” Finally, we believe this technique will have usefulness as a measure of systemic risks in developed economies.