TAA With Pair-wise Strategies

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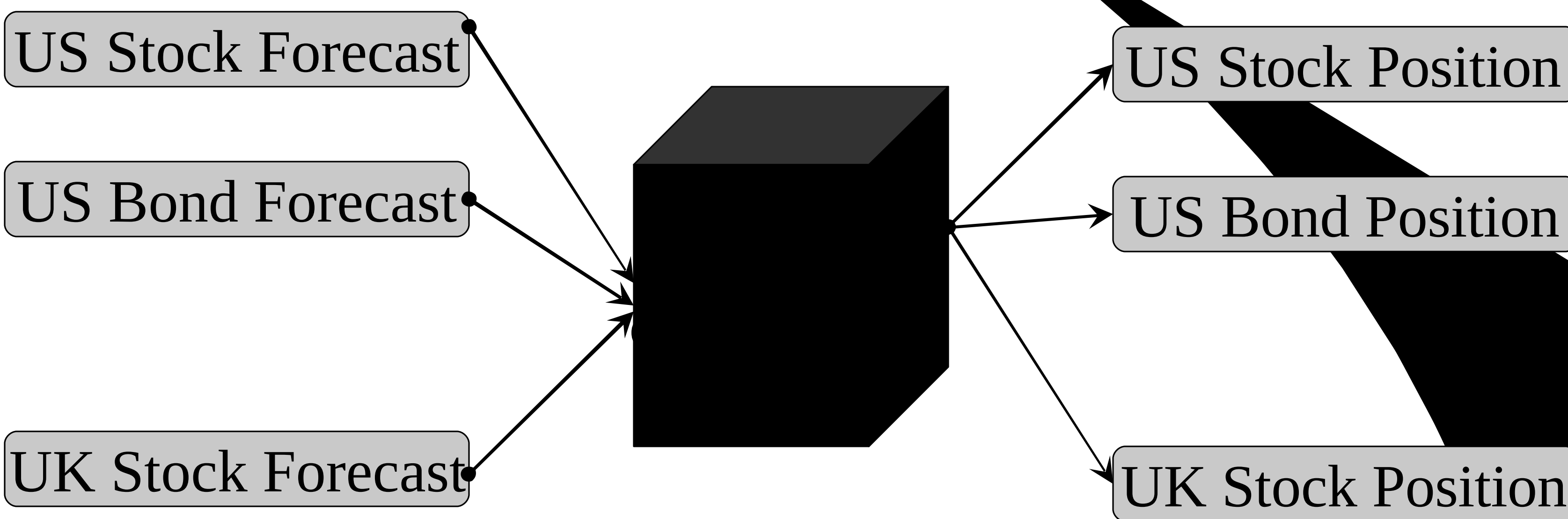
Outline

- Why pairs?
- Conventional approach
  - Tactical asset allocation
  - Active currency management
  - Sector strategies
- Pair-wise strategies
- TAA with pair-wise strategies

Conventional Approach



Conventional Approach

- Forecasting process
  - Individual time series model
  - Time series information coefficient (IC) $\mathrm{IC} = r\left(\mathbf{f}_t, \mathbf{r}_t\right)$
- Portfolio construction
  - MV optimization
  - Sample covariance matrix
- Performance
  - Information ratio (IR) $\mathrm{IR} = \dfrac{\mathrm{avg}(a)}{\mathrm{std}(a)}$

Problems With Conventional Approach

- Equity bias
  - Solution: de-mean the forecasts
- Three mysteries
  1) Steining doesn't help?
  2) Additional models don't help IR?
  3) Diagonal covariance is superior in back test?
- **Why? Answer: pair-wise analysis**

Pair-wise Strategies

- Risky asset versus risk-free asset or risky asset versus risky asset
- IC is only good for risky asset/cash pair; it is no good for pairs between two risky assets

$$w_1 = l^{-1} f_1 \qquad \longrightarrow \qquad a = w_1 r_1 = l^{-1} f_1 r_1$$
$$w_0 = -l^{-1} f_1$$

$$\longrightarrow \qquad \text{avg}(a) \propto \text{corr}(f_1, r_1)\, s(f_1)\, s(r_1)$$

Two Risky Assets

- Pair-wise IC matters! PIC – correlation coefficient between the forecast premium and return premium

$$w_1 = l^{-1}(f_1 - f_2) \qquad \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = l^{-1} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

$$w_2 = l^{-1}(f_2 - f_1)$$

➡ $$a = w_1 r_1 + w_2 r_2 = l^{-1}(f_1 - f_2)(r_1 - r_2)$$

➡ $$\mathrm{avg}(a) \propto \mathrm{corr}(f_1 - f_2, r_1 - r_2)\, s(f_1 - f_2)\, s(r_1 - r_2)$$

Pair-wise IC (PIC)

- PIC is a combination of IC and cross IC

$$\text{PIC} = \frac{\operatorname{cov}(f_1 - f_2, r_1 - r_2)}{s(f_1 - f_2)\, s(r_1 - r_2)} = c_1 r(f_1, r_1) + c_2 r(f_2, r_2) - c_3 r(f_2, r_1) - c_4 r(f_1, r_2)$$

- Positive IC is good for PIC, but positive [illegible] is not

An Example

- Domestic TAA: stock/bond/cash
- IC: 0.2, 0.2; Cross IC: 0.2, 0.15
- PIC: 0.08!

	Stock Return	Bond Return	Stock Forecast	Bond Forecast
Stock Return	1.00	0.20	**0.20**	**0.15**
Bond Return	0.20	1.00	**0.20**	**0.20**
Stock Forecast	0.20	0.20	1.00	0.20
Bond Forecast	0.15	0.20	0.20	1.00

Pair-wise IR

- Pair-wise tracking error

$$\mathrm{std}(a) \propto s(f_1 - f_2)s(r_1 - r_2)$$

- Pair-wise IR is approximately pair-wise I

$$\mathrm{avg}(a) \propto \mathrm{PIC} \cdot s(f_1 - f_2)s(r_1 - r_2)$$

Mystery Solved

- Pair-wise IC is part of the reason for (1) and (2)
- Steining increased model IC, but it [illegible]lso increased cross IC. The combining effect on PIC [illegible] zero
- Additional models have good IC itself, b[illegible]ht have poor PIC when combined with other [illegible] existing models

Implications for Model Building

- When building separate models, be mindful of cross IC. Global or common factors often bring cross IC
- Focus on specific factors
- When possible (when N is small), build pa premium models
- Always analyze and assess forecasts through wise framework

TAA With Pair-wise Strategies

- We can prove MV optimization in the simplest form is equivalent to a combination pair-wise strategies
- Mathematics versus intuition
- We will demystify the black box

Pair-wise Trading With Stock/bond/cash

- Three pairs: (0 1), (0 2), (1 2)
- Three pair-wise bets

$$w_{1,0} = l^{-1}(f_1) \qquad w_{2,0} = l^{-1}(f_2) \qquad w_{1,2} = l^{-1}(f_1 - f_2)$$

$$w_{0,1} = -l^{-1}(f_1) \qquad w_{0,2} = -l^{-1}(f_2) \qquad w_{2,1} = l^{-1}(f_2 - f_1)$$

- Three pair-wise alphas

$$a_{ij} = l^{-1}(f_i - f_j)(r_i - r_j), \quad i, j = 0,1,2; i < j$$

Pair-wise Weights

- The only remaining decision is how to mix them
- Pair-wise weights

$$p_{01}, p_{02}, p_{12}$$

- Total alpha

$$\begin{aligned} a &= p_{01} a_{01} + p_{02} a_{02} + p_{12} a_{12} \\ &= l^{-1} \sum_{i<j} p_{ij} \left(f_i - f_j \right) \left(r_i - r_j \right) \end{aligned}$$

TAA With Pair-wise Strategies

- Construct TAA with pair-wise strategies
  - Select pair-wise weight
  - Scale active weights in pairs by p_{ij}
  - Aggregate weight in all relevant pairs
- The role of optimization
  - Optimization is one way to select pair-wise wei
  - Optimization gives a set of implied pair-wise wei

The Advantage of Pairs

- Each pair is a “security”
  - Expected return (alpha), risk (tracking error)
  - Correlation matrix among pairs
- TAA is a portfolio of pair-wise “securi[illegible]”
  - Given pair-wise weights, we can compute I[illegible]
  - We can find the optimal pair-wise weights
  - In practice, we can choose pair-wise weights to [illegible]
    - Treat pairs differently
    - Balance the risks of pairs
    - Trade pairs separately

MV Optimization and Pairs

- Two risky assets, one risk-free asset (stock/bond/cash) (2/1/0)
- No constraint, three pairs (0 1);(0 2); ([illegible])
- MV optimization
  - No correlation
  - With correlation

MV Optimization and Pairs

- We can write the alpha from MV optimization in terms of three pair-wise alphas.

$$S = \begin{pmatrix} s_1^2 & z s_1 s_2 \\ z s_1 s_2 & s_2^2 \end{pmatrix} \qquad \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = S^{-1} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \qquad a = w_1 r_1 + w_2 r_2$$

$$a_t \propto \left(\frac{1}{s_1^2} - \frac{z}{s_1 s_2} \right) r_1 f_1 + \left(\frac{1}{s_2^2} - \frac{z}{s_1 s_2} \right) r_2 f_2 + \left(\frac{z}{s_1 s_2} \right) (f_2 - f_1)(r_2 - r_1)$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$p_{01} \qquad\qquad p_{02} \qquad\qquad p_{12}$$

MV Optimization and Pairs

- MV optimization implies a set of weights
- These weights are a function of the covariance matrix
  - No correlation
    - Only two pairs between the risky assets and the ris asset
    - No bet between the two risky assets
  - Correlation
    - All three pairs

Another Mystery Solved

- If the pair between the two risky assets is inferior to the two other pairs, then using a full covariance matrix rather than a diagonal one leads to inferior performance

TAA Performance

- Expected IR $a_t = l^{-1} \sum_{\substack{i,j=1 \\ i<j}}^{N} p_{ij} a_{ij}$
  - Expected return of each pair $\overline{a_{ij}}$
  - Covariance matrix of pairs $\mathrm{cov}(a_{ij}, a_{kl})$
- Given these results, we can obtain the ex[illegible]d alpha of TAA and expected tracking error [illegible] traditional portfolio theory
- We can also obtain the optimal pair-wise wei[illegible] that give the TAA with the highest IR

Summary

- Beware of MV optimization
  - What are the implied pair-wise weights?
  - Are they consistent with pair-wise IC (PIC)?
- The role of covariance matrix
  - It is implicitly assigning pair-wise weights
  - Sample estimate
    - Good for long-term strategic purpose
    - Good for single-period risk management
    - Not a good choice for multi-period tactical asset alloca

Summary

- Advantage of pair-wise framework
  - Apply to a variety of macro quantitative strategies
  - Simplify modeling process
  - Balance risk contribution of pairs
  - Easy use of trading concept
  - Calculate expected performance
  - Attain optimal information for given set of forecasts

Appendix

Mathematical Pr

Active MV Optimization

- Objective function $G(\vec{A}_t) = \vec{A}_t' \cdot \vec{f}_t - \frac{1}{2} l (\vec{A}_t' \cdot S \cdot \vec{A}_t)$
- Possible constraint

$\vec{A}_t' \cdot \vec{i} = 0$ No risk-free asset

- Solution $\vec{A}_t = l^{-1}\left(S^{-1} \cdot \vec{f}_t\right)$ No constraint

$\vec{A}_t = l^{-1}\left(P \cdot \vec{f}_t\right), P \cdot \vec{i} = 0$ No risk-free ass

- Alpha $a_t = l^{-1}\left(\vec{f}_t' \cdot P \cdot \vec{r}_t\right)$

MV Optimization – A Linear Combination of Pairs

$$\vec{A}_t = l^{-1}\left(P \cdot \vec{f}_t\right) \quad P \cdot \vec{i} = 0$$

$$\Rightarrow \quad P = \begin{pmatrix} \sum_{i \neq 1} p_{1i} & -p_{12} & \cdots & -p_{1N} \\ -p_{21} & \sum_{i \neq 2} p_{2i} & \cdots & -p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -p_{N1} & -p_{N2} & \cdots & \sum_{i \neq N} p_{Ni} \end{pmatrix} \quad \Rightarrow \quad P = \sum_{\substack{i,j=1 \\ i<j}}^{N} p_{ij} Q_{ij}$$

$$Q_{ij} = \begin{pmatrix} 0 & \cdots & 0 & \dots & 0 & \cdots & 0 \\ \vdots & \ddots & & & & & \vdots \\ 0 & & 1 & & -1 & & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & & -1 & & 1 & & 0 \\ \vdots & & & & & \ddots & \vdots \\ 0 & & & & & & 0 \end{pmatrix}$$

MV Optimization – A Linear Combination of Pairs

$$a_t = l^{-1}\left(\vec{f}_t' \cdot P \cdot \vec{r}_t\right)$$

$$\vec{f}_t' \cdot Q_{ij} \cdot \vec{r}_t = \left(f_i - f_j\right)\left(r_i - r_j\right)$$

$$\Rightarrow \quad a_t = l^{-1} \sum_{\substack{i,j=1 \\ i<j}}^{N} p_{ij}\left(f_i - f_j\right)\left(r_i - r_j\right)$$

MV Optimization – A Linear Combination of Pairs

- This proves that MV optimization with the constraint is equivalent to a linear [illegible]ombination of pair-wise strategies between risky ass[illegible]
- For unconstrained optimization, in addit[illegible] these pairs, it also includes pairs between a[illegible] assets and the risk-free asset