Risk, Uncertainty, Horizon and Investment Decisions

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"Doubt is uncomfortable, but certainty is absurd."

– Voltaire

Introduction

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  - Expected returns
  - Variances
  - Covariances

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  - Covariances
- We don’t actually know any of these numbers
  we *estimate* them
- This difference seems innocuous because
  - Short run it doesn’t matter (much)
  - **BUT, in the long run, it matters a lot**

Technical Notes

- This presentation only addresses the uncertainty in expected returns*
  - "Expectations risk"
  - I call conventional risk -- "variability risk"
- Lots of other work on uncertainty (see references)
  - Amazingly, almost all of it one-period
  - Black-Litterman (1990) and Michaud (1998)
  - Barberis (2000), Cvitanic, et al (2003)
- This presentation assumes rebalancing to fixed targets
  - But the qualitative results hold more generally

**Errors in variances and covariances matter, too. But not as much (Chopra and Ziemba (1993)).*

Outline

- Illustration of concept (thanks to Daniel Ellsberg)
- Asset class allocation
  - Impact of uncertainty on risk
  - Impact of uncertainty on optimal allocation
  - How much expectations risk?
- Active management
  - Portable alpha
  - How much expectations risk?
  - Optimizations with asset class and active risk

Urns and Uncertainty

- Consider two urns:
  - Urn 1 contains 20 red balls and 20 blue balls
  - Urn 2 contains 40 red and blue balls, the proportion is unknown
- You can pick a color to bet on
- The payoffs are: + $70 if we draw a ball of your color

  – $30 if we draw the other color

Which urn would you rather draw from?

Ellsberg (1961)

Urns and Uncertainty: Part 2

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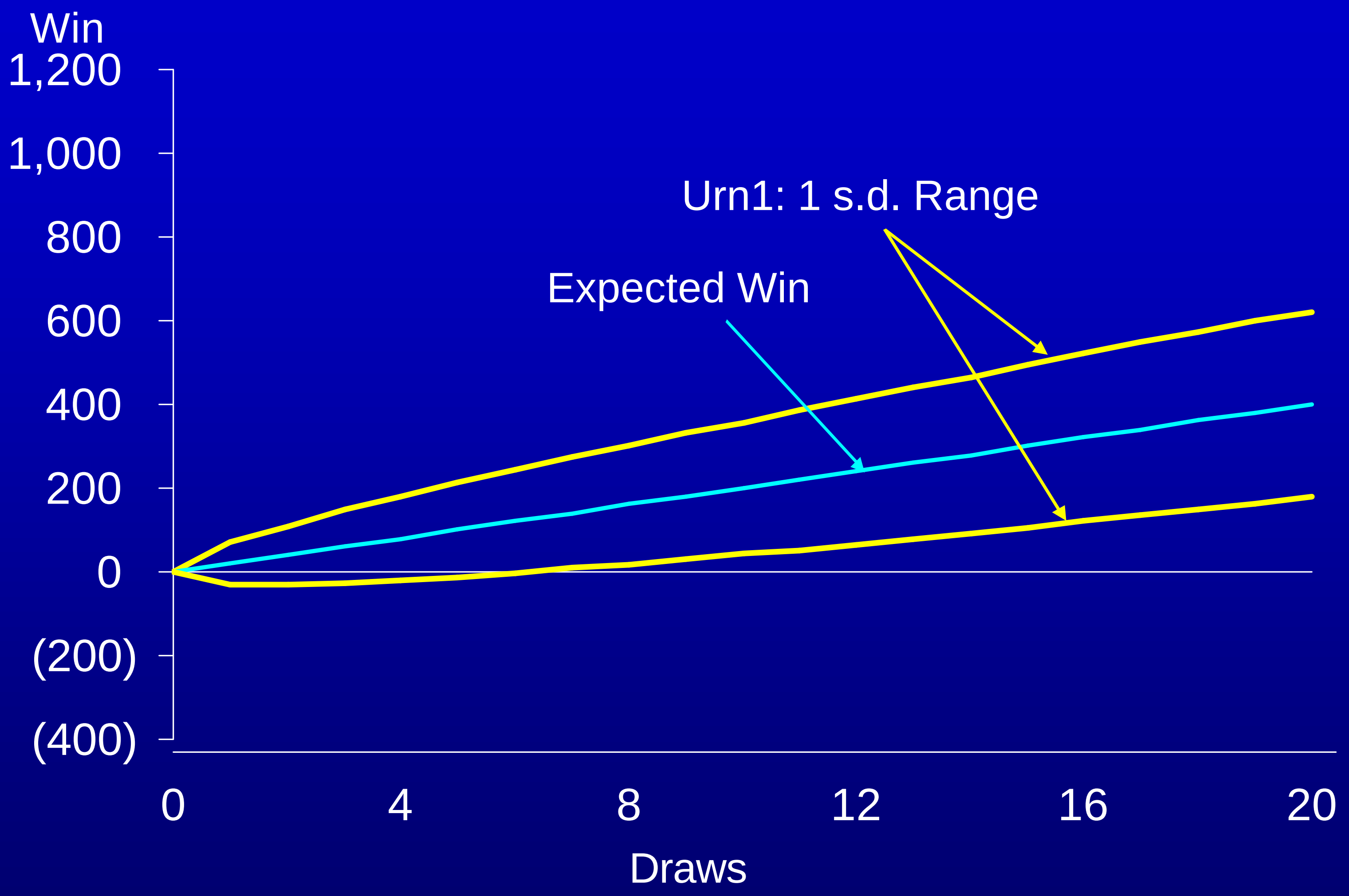
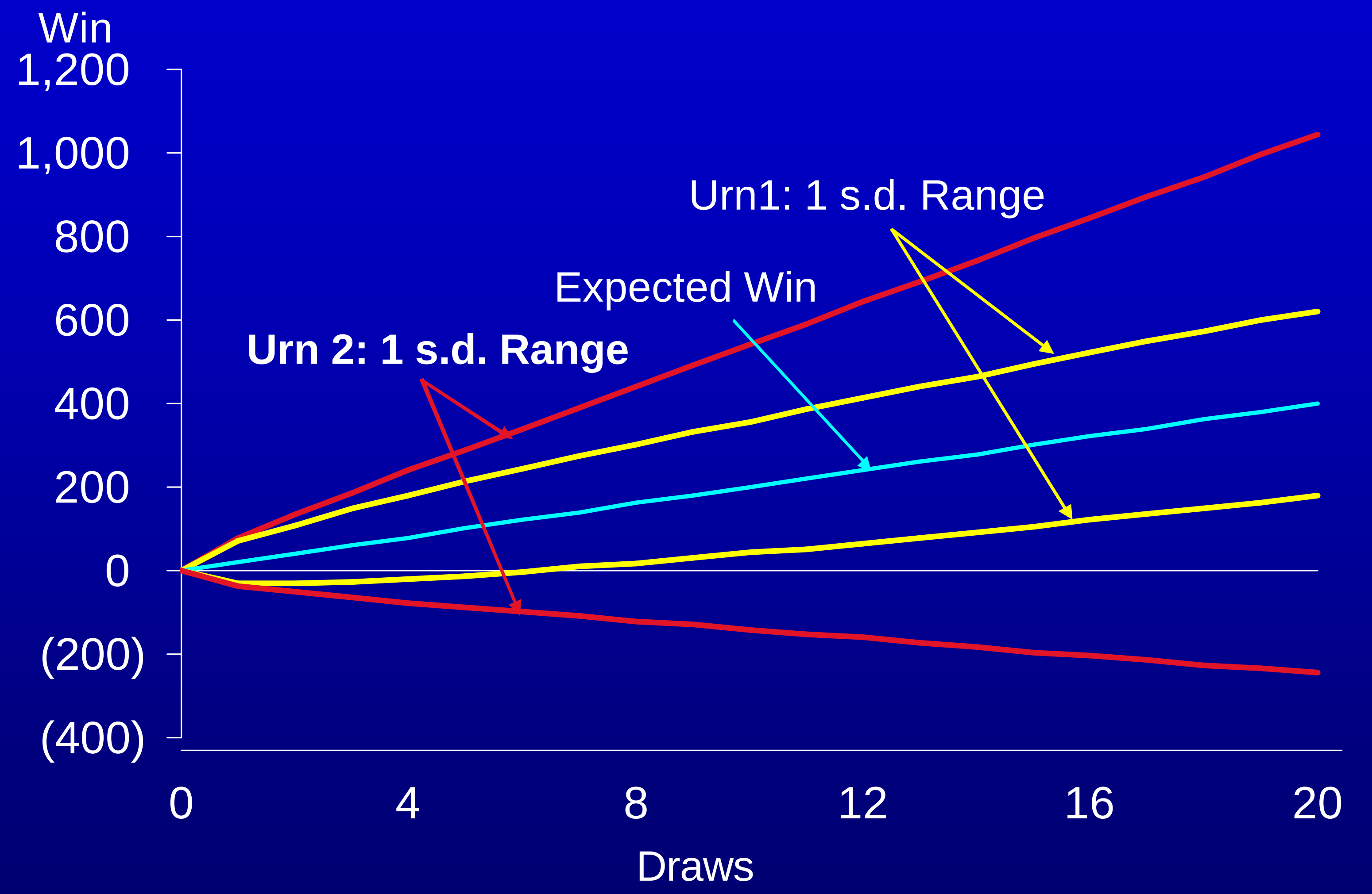
Urns and Uncertainty: Part 2

- For a single trial, the two urns have the same a priori distribution
- But, most people still prefer to draw from urn 1
- The question Ellsberg didn’t ask: What about multiple draws?
  - **For multiple draws, there is a tangible difference**

Dispersion of Outcomes: Urn1



Dispersion of Outcomes: Urn1 versus Urn 2



Urn1: If we play long enough, you are sure to win

Urn 2: NOT

Urns and Uncertainty: Part 3

- This example captures the essence of expectations risk
  - You **think** the game is positive expectation **but you can’t be sure**
  - If the urn is unfavorable, it remains so
  - Expectations variance grows faster than linearly with time
- Now let’s apply it to investments

Simple Asset Allocation Question

- The simplest allocation question is two assets, one risky, one not
- Assume, for now, that the expected excess return and risk are constant
- For example:

	Expected Return	Excess Return	Variability Risk
Risk-Free	4%	--	0%
Stocks	8%	4%	16%

Dispersion Over Time: No Expectations Risk

- Define the excess stock return as:
  - $R_t = \mu + er_t$
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Dispersion Over Time: No Expectations Risk

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- Expected value of 2-year return = $\mu_1 + \mu_2 = 2\,\mu$
- Variance of the 2-year return = $s r_1^2 + s r_2^2 + 2\,?\,s r_1\, s r_2$
- **Assuming ? equals 0**
  - **2-year variance = 2 $s r^2$**
  - **Mean and variance both grow linearly with time**
  - **Optimal allocation does not depend on time horizon Samuelson (1969), Merton (1969))**

Dispersion Over Time: With Expectations Risk

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$$\overbrace{\mu e + em}^{\mu}$$

- **SO, from our perspective, $R_t = \mu e + em + er_t$**

  μe is our estimate of the mean. It’s error is em which has mean = 0 and standard deviation = s m

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- **Error in mean is fixed over time: (if urn 2 is "bad" it stays bad)**

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- **2-year expected return = 2 μe**

Dispersion Over Time: With Expectations Risk

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μ

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μe is our estimate of the mean. It's error is em which has mean = 0 and standard deviation = s m

- Error in mean is fixed over time: (if urn 2 is "bad" it stays bad)
- 2-year expected return = 2 μe
- **2-year variance = 2 s r ^{2} + 2 s m ^{2} + 2 ?m s m ^{2}**
  - **?m = 1**
  - **So, 2-year variance = 2 s r ^{2} + 4 s m ^{2}**
    - **In general: T s r ^{2} + T ^{2} s m ^{2}**

Relation of Variability and Expectations Risk Over Time

- Returning to our numerical example and adding expectations risk

	Expected Return	Excess Return	Variability Risk	Expectations Risk
Stocks	8%	4%	16%	**2%**

- Risk will evolve as:

	Variability Variance	Expectations Variance	Total S.D. (Annualized)	Expectations % of Total Variance
1-Year	0.0256	0.0004	16.1%	1.5%

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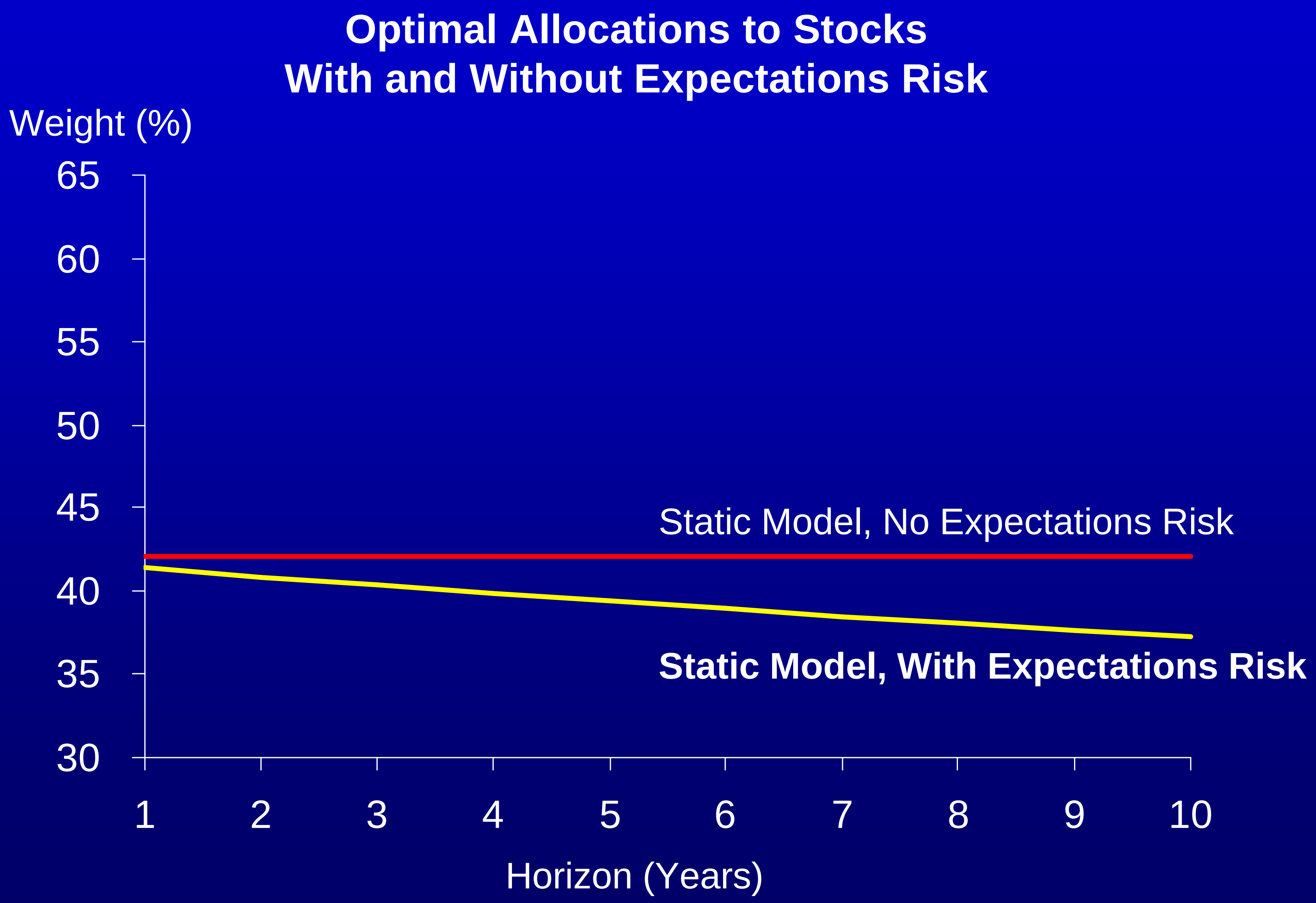
	Variability Variance	Expectations Variance	Total S.D. (Annualized)	Expectations % of Total Variance
1-Year	0.0256	0.0004	16.1%	1.5%
10-Years	**0.2560**	**0.0400**	**17.2%**	**13.5%**

Impact of Uncertainty on Optimal Allocation



Mean Reverting Process

- More realistically, there is probably mean-reversion. Notably, the dividend yield effect (Campbell and Shiller (1988, 1998))*

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- **$R_t = \mu + B \times (d_t - D) + er_t$**
  - **B is a mean-reversion parameter, d_t is the dividend yield, D is the normal dividend yield**

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Mean Reverting Process

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- **But we observe:**
  - $$R_t = \underbrace{\mu e + em}_{\mu} + \underbrace{(Be + eb)}_{B} \; X \; (d_t - D) + er_t$$

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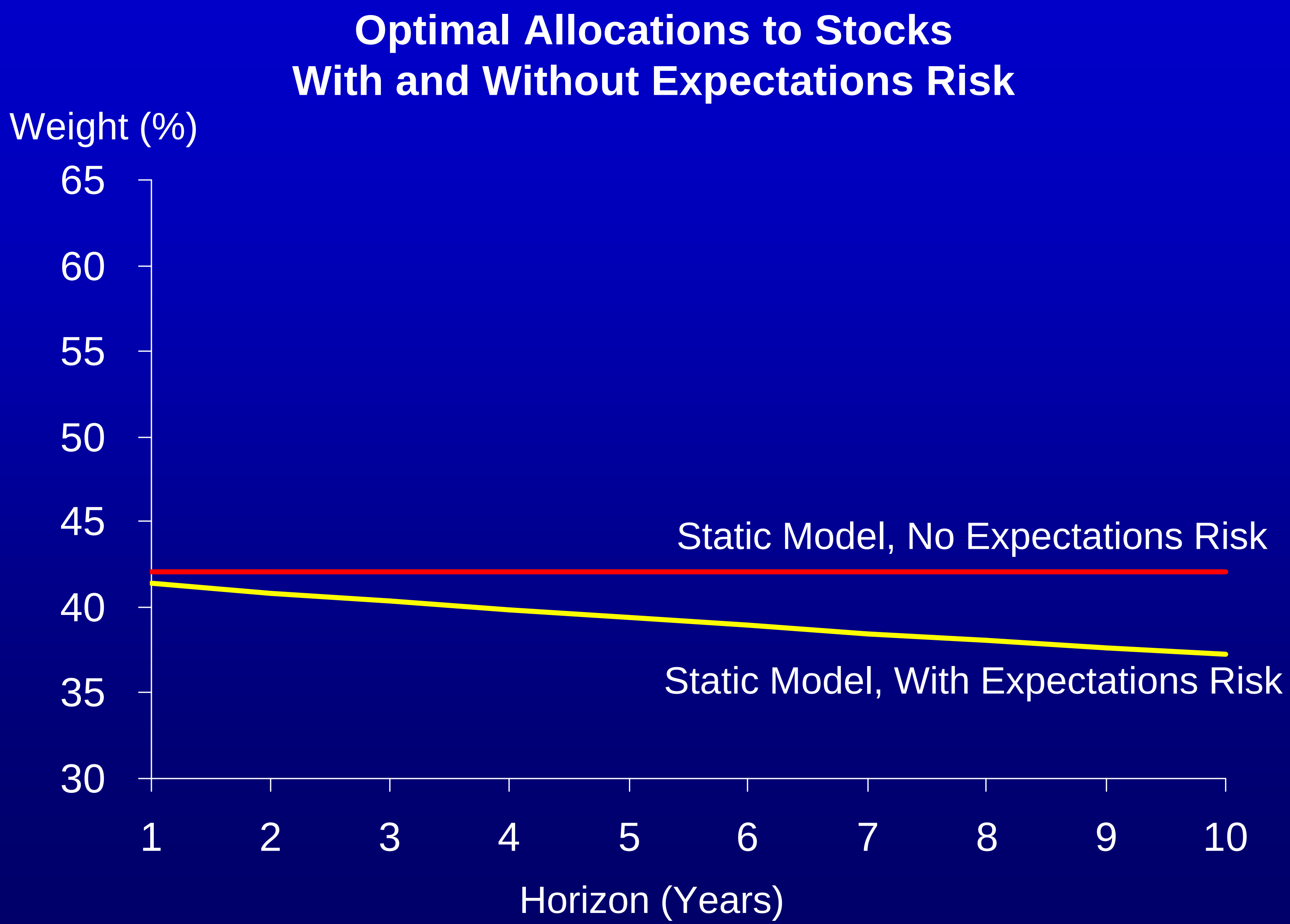
- **Mean reversion reduces long-term risk.**

**The argument for uncertainty is made by Goyal and Welch (2003)*

Impact of Uncertainty and Mean-Reversion on Optimal Allocation



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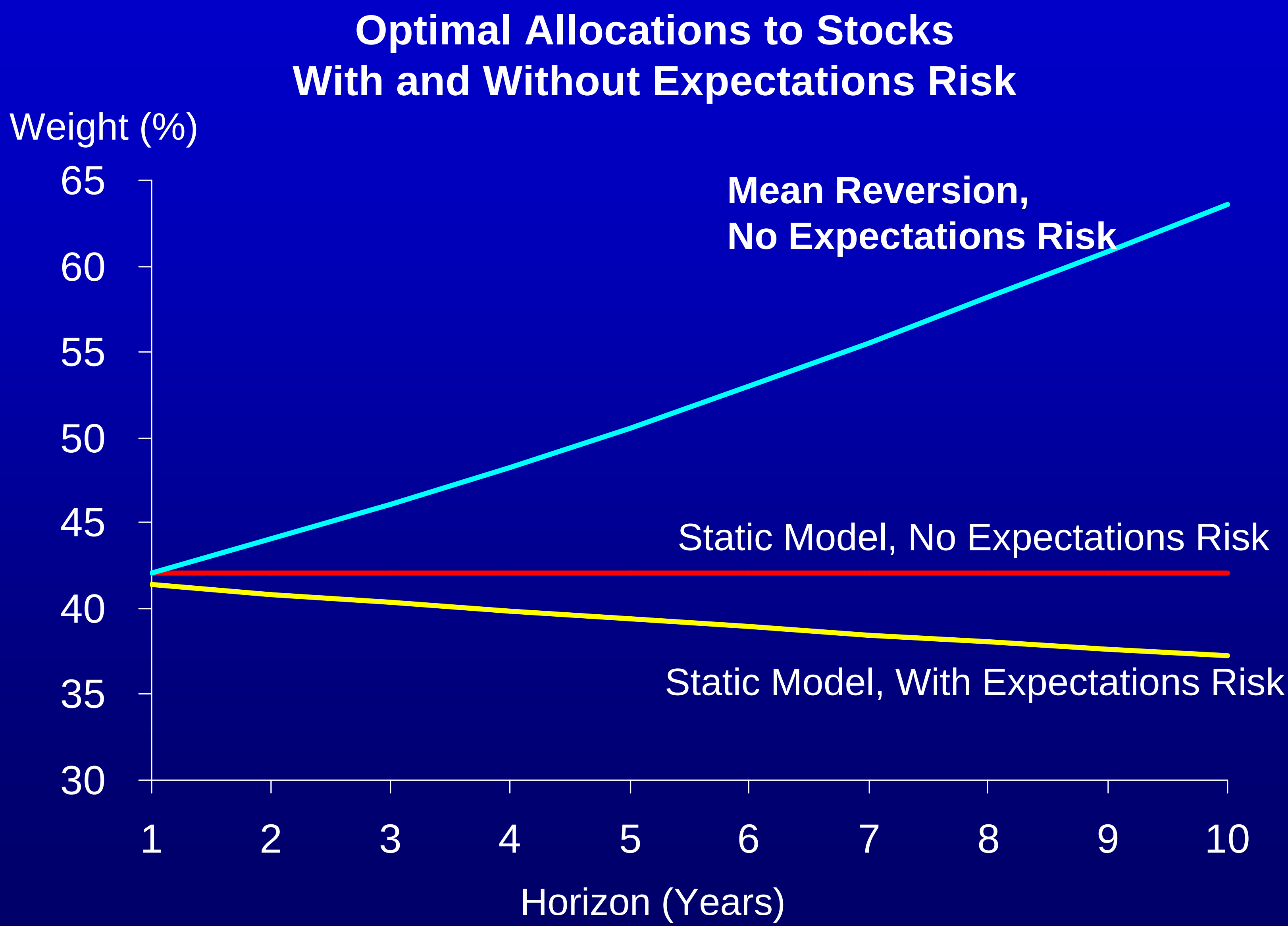

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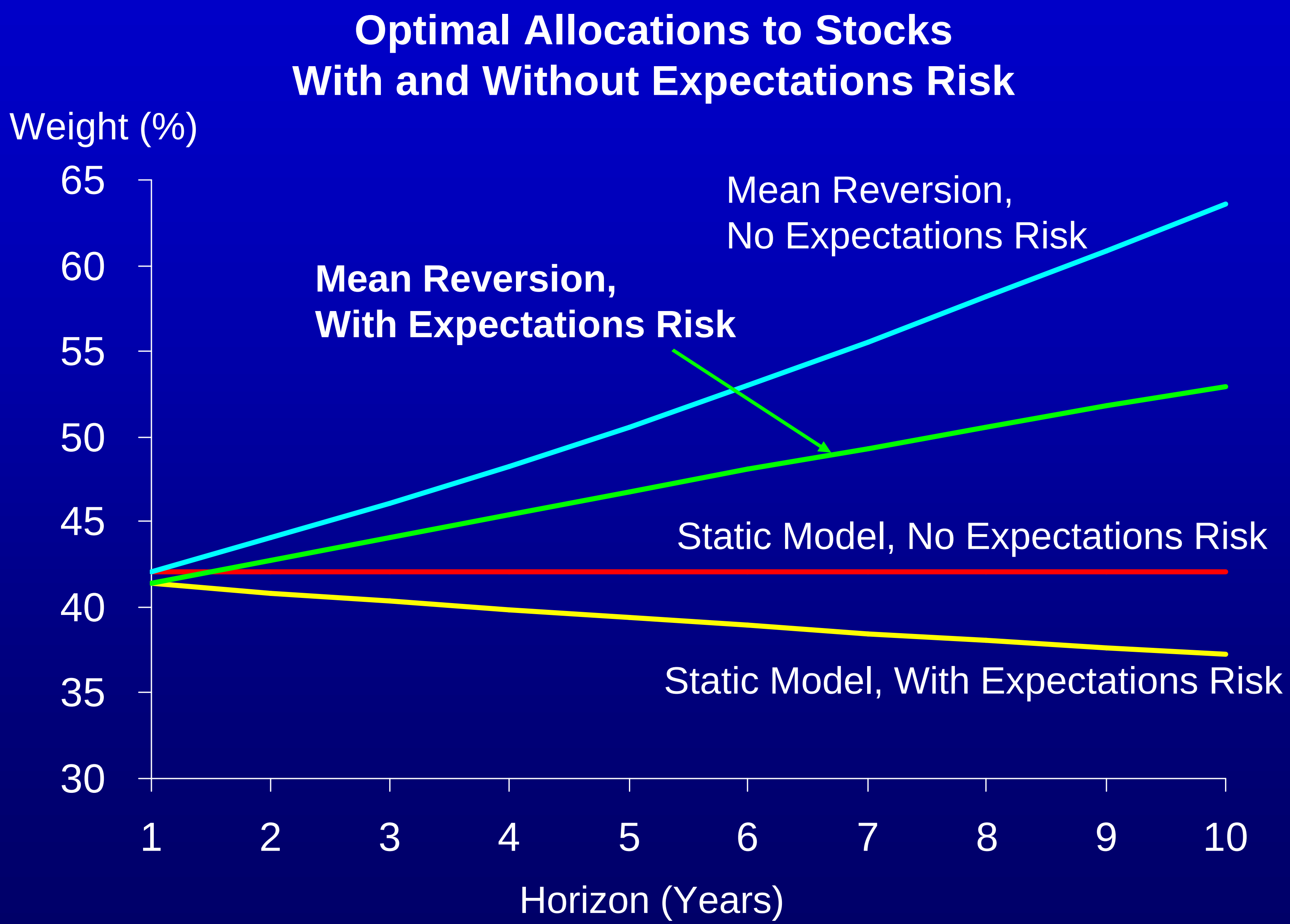
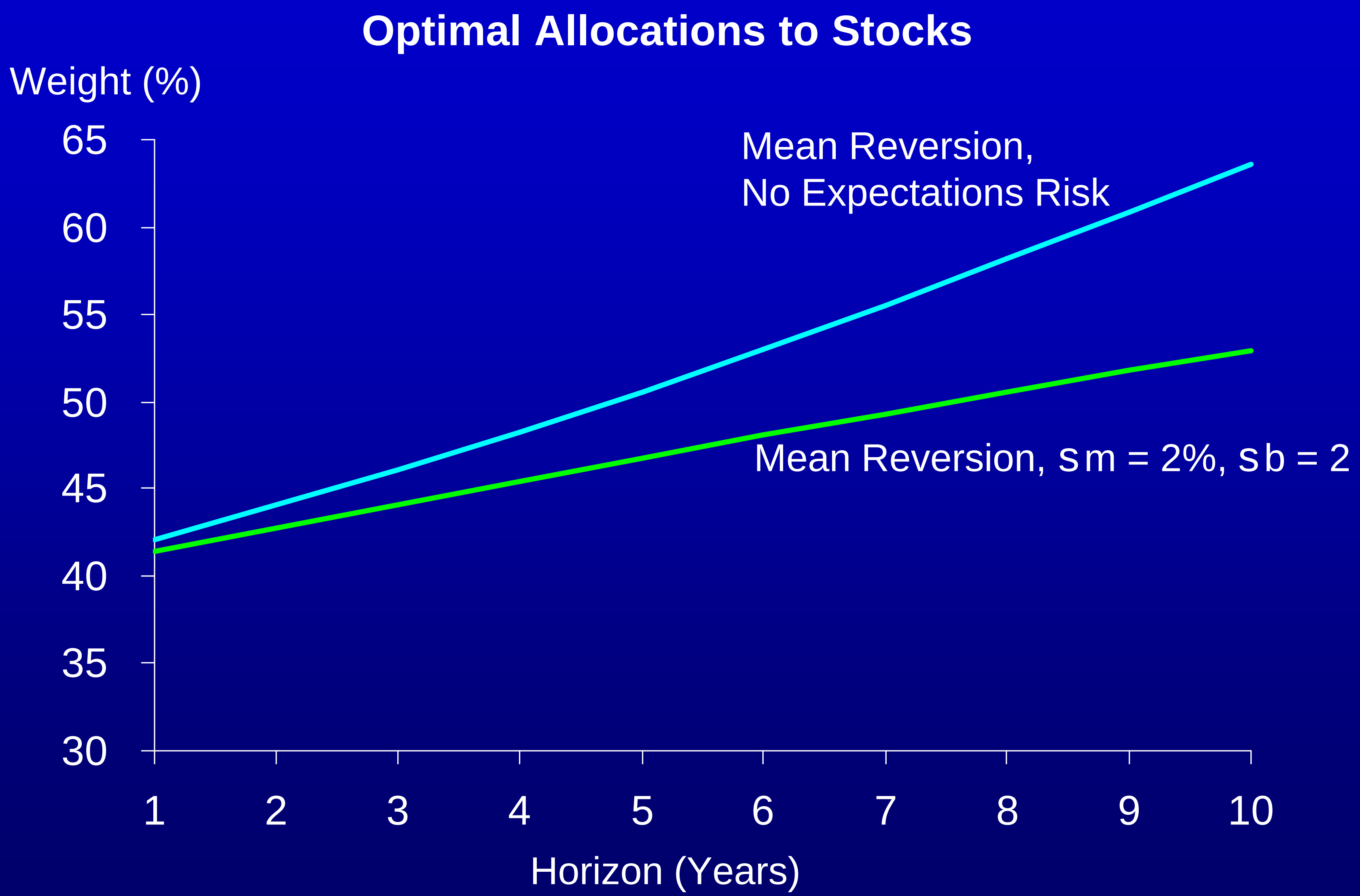


Impact of Varying Levels of Uncertainty



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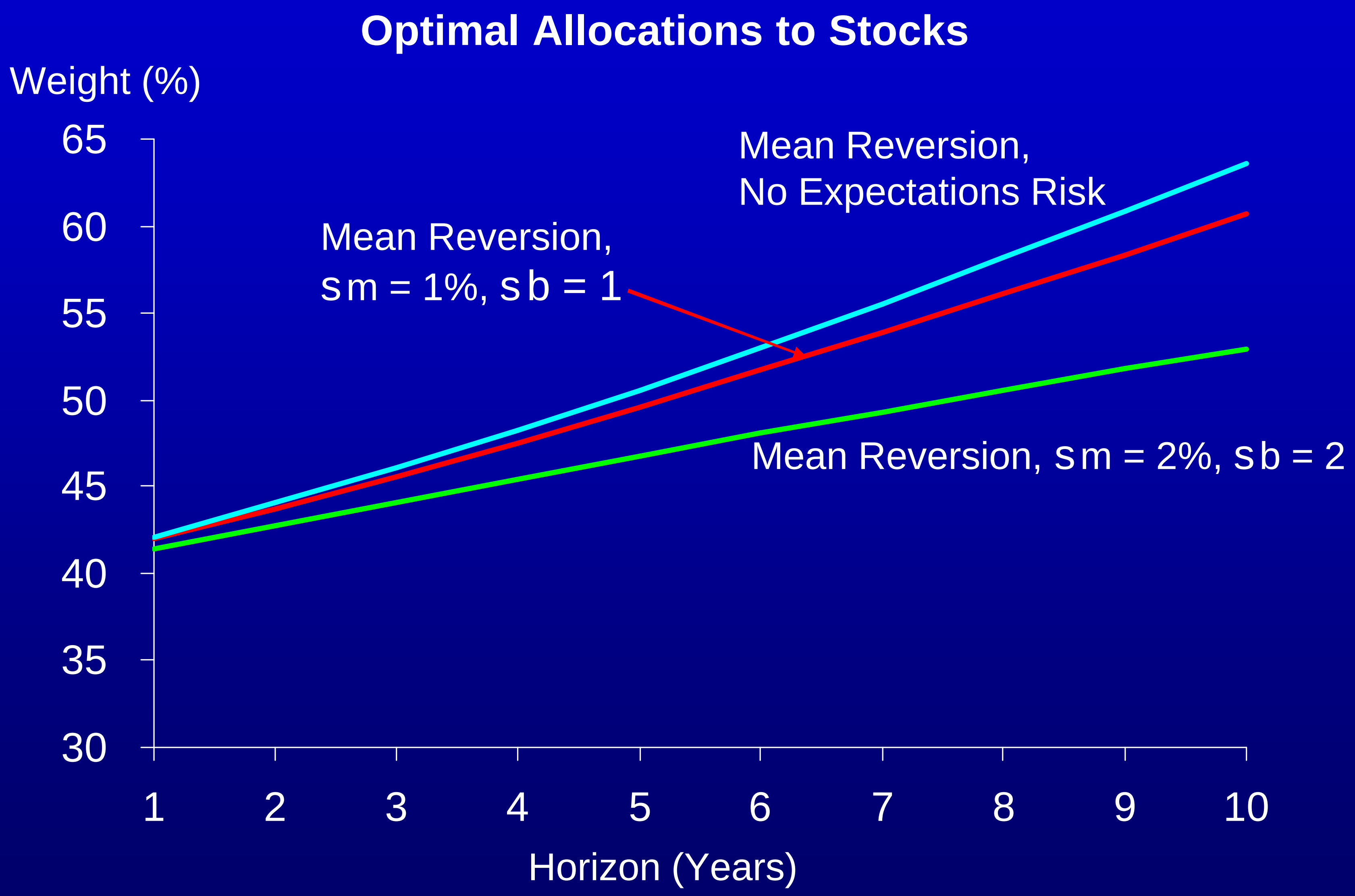

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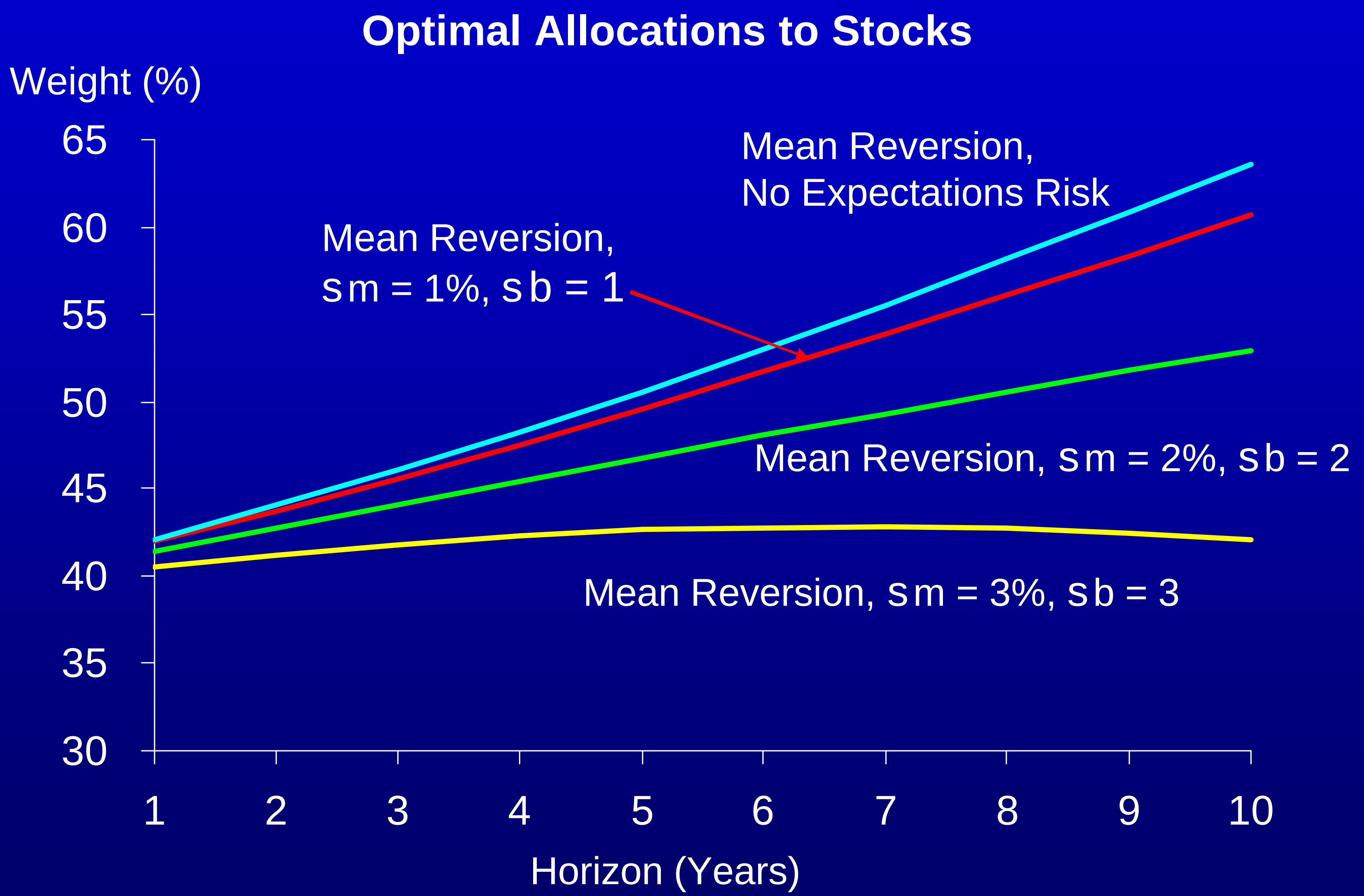

How Large is Expectations Risk for Stocks?

- There are many ways to think about the expected return of the market
  - Empirical
    - e.g. Ibbotson and Chen (2002), Seigel (2002)
  - Theoretical
    - e.g. Mehra and Prescott (1985)
  - Conceptual but practical
    - e.g. Arnott and Bernstein (2002), Fama and French (2002)
  - Combined
    - TIAA-CREF (2001)
- There is expectations uncertainty about the Equity Risk Premium
- For stocks, sm = 2% seems about right to me

What We Have Learned So Far?

- Myopic Optimization is not optimal in long run
- Expectations risk raises total risk (lowers allocation) as horizon lengthens
- Mean reversion lowers risk (raises allocation) as horizon lengthens
- See Barberis (2000)

Active Strategies and Expectations Risk

- So far, we have considered an asset class
- Investors also take active risks
- What impact does expectations risk have on allocations to those strategies?

Active and Asset Class Risks: Portable Alpha

- Portable Alpha
  - Separates asset class and active risks
  - Allows leverage
  - Removes constraints
  - See references
- Gives investors freedom
- Kritzman and Thomas (2004), Litterman (2004) run common expected return assumptions through an optimizer and find that investors should be taking **much** more **active risk**.
- Litterman dubbed this the "Active Risk Puzzle".
- Part of the solution to puzzle: expectations risk

How Large Is Expectations Risk of Active Management?

- It depends on the manager and on you
- Judgment arrived at from a combination of empirical and conceptual analyses
- Remember the background information*
  - The average active manager has underperformed
  - Portable alpha often has large fees and costs
- Allow for significant possibility of illusory alpha

**See: Base-Rate Neglect: Kahneman and Tversky (1973) and many others,*
See: Samuelson (1990) for a skeptical perspective on active management.

How Large Is Expectations Risk of Active Management?

- Suppose we estimate that a manager has a 3% alpha and 6% variability risk
- If we assume no expectations risk, we are implicitly assuming we are sure they will outperform in the long run
- In the long-run, variability risk becomes relatively unimportant,
  - Alpha / Expectations risk determines the odds of long-term outperformance

Odds of Outperforming

Expected Alpha = 3%

Variability Risk = 6%

Years	No Exp. Risk s m = 0%	Exp. Risk s m = 4.5%	Exp. Risk s m = 9%
1	69%	66%	61%
2	76	69	62
5	87	72	63
10	94	73	63
20	99	74	63

Incorporating Expectations Risk into Optimization

- Bringing this all together, we can determine optimal portfolios that include both passive asset class risks and portable alpha
- Because of expectations risk and mean reversion*, the optimal allocations will depend on the horizon
- Consider the following assumptions for 1 and 10-year time horizons:

	Excess Return	Variability Risk	Information Ratio**	Expectations Risk
Stocks	4%	16%	0.25	0 or 2%
Active Strategy	3%	6%	0.50	0 or 4.5%

*Notes: *Only stocks are ascribed mean reversion.*
***Information Ratios ignore expectations risk.*

Optimization: Impact of Expectation Risk and Horizon

	Case 1 1-Year No Expect. Risk
Portfolio Weights	
Stocks	42%
Stocks	225%

Optimization: Impact of Expectation Risk and Horizon

	Case 1 1-Year No Expect. Risk	Case 2 1-Year With Expect. Risk
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Optimization: Impact of Expectation Risk and Horizon

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Optimization: Impact of Expectation Risk and Horizon

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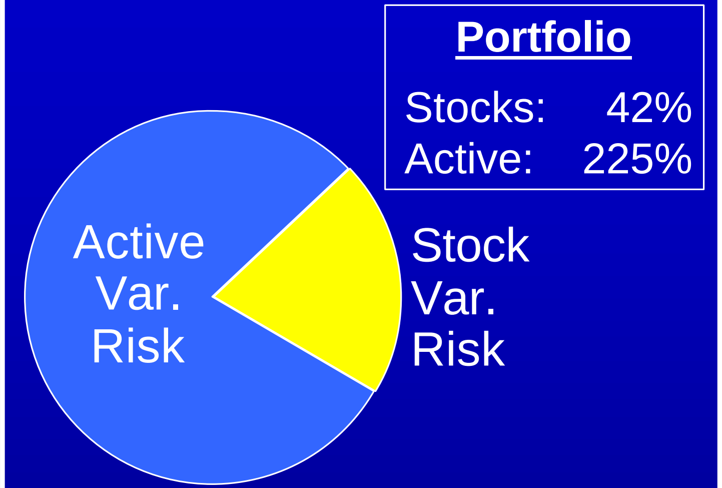
Portfolio Weights				
Stocks	42%	41%	64%	53%
Active	225%	160%	225%	44%

Risk Allocation: Without Expectations Risk

Case 1



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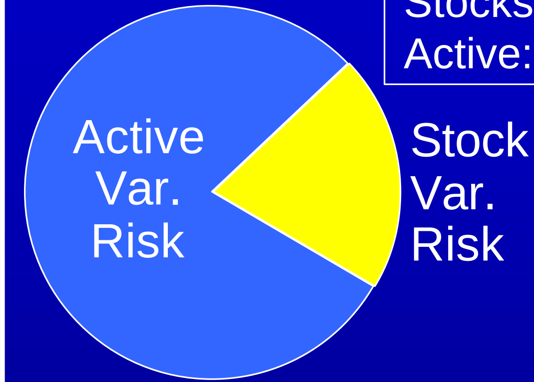
Risk Allocation: Without and With Expectations Risk

Case 1

Portfolio

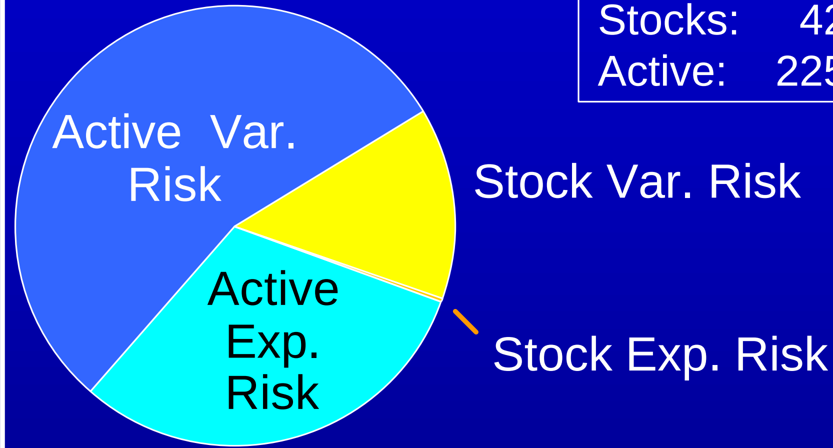
Stocks:	42%
Active:	225%



Case 1: Acknowledging Expectations Risk

Portfolio

Stocks:	42%
Active:	225%



Risk Allocation: Without and With Expectations Risk

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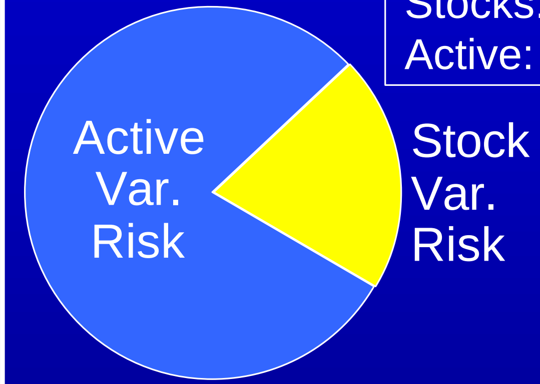

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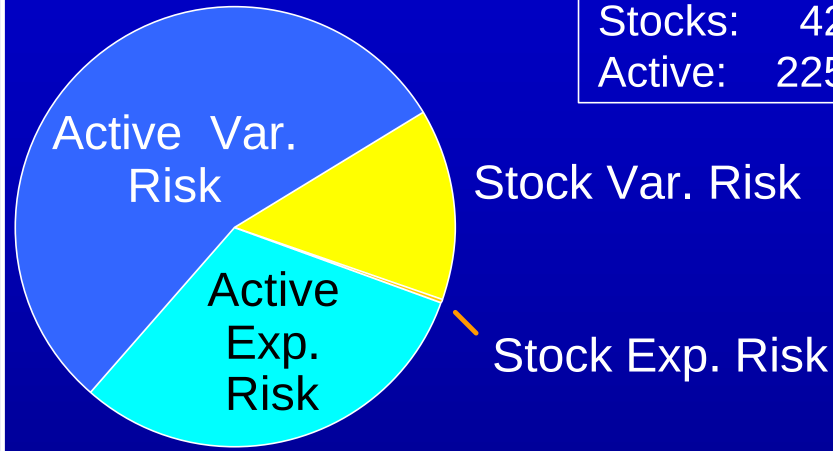
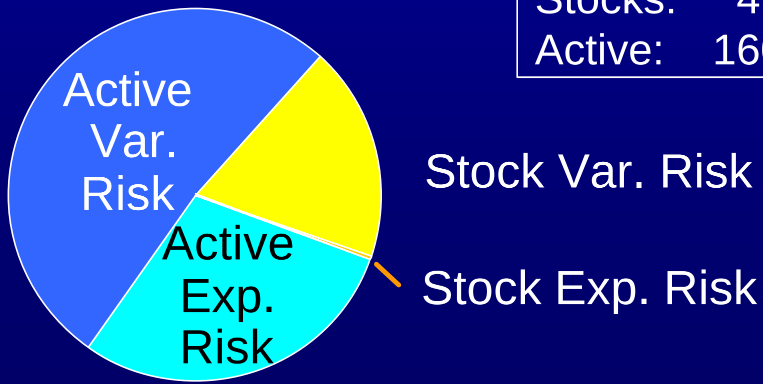


Case 2: Recognizing Expectations Risk

Portfolio	
Stocks:	41%
Active:	160%

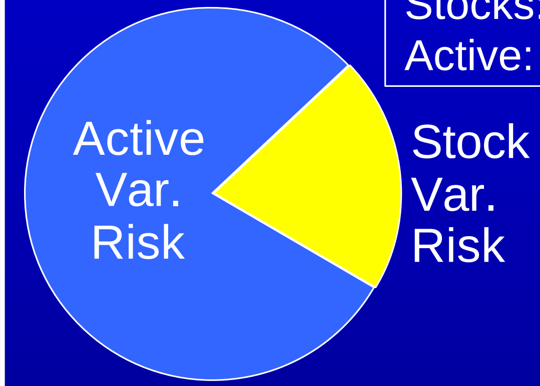


Risk Allocation: Without Expectations Risk

Case 3

Portfolio

Stocks:	64%
Active:	225%



Risk Allocation: Without and With Expectations Risk

Case 3

Portfolio

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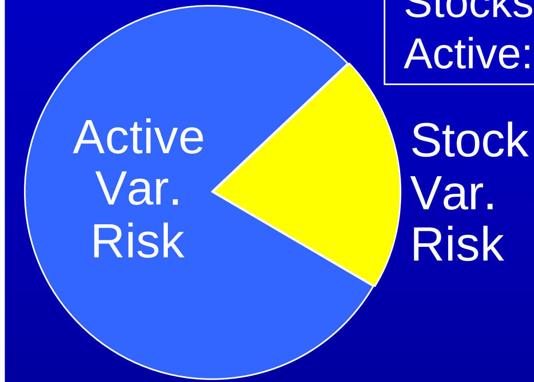

Case 3: Acknowledge Expectations Risk

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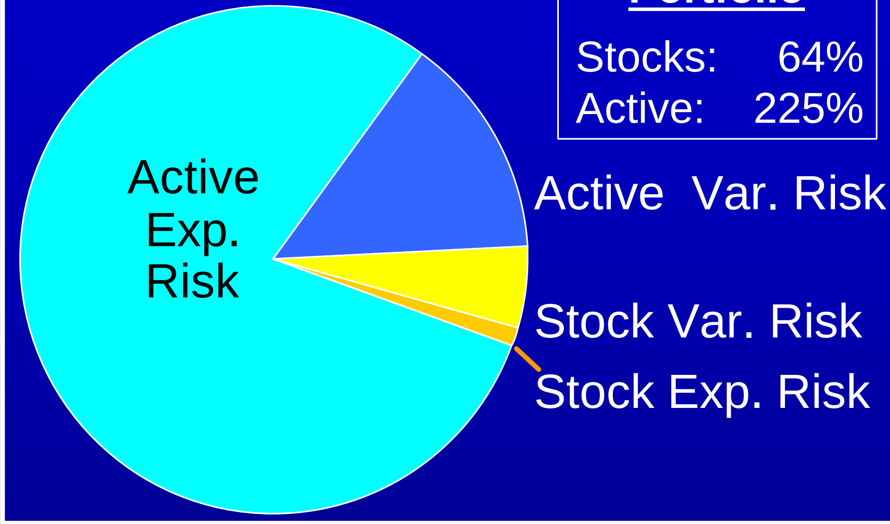

Risk Allocation: Without and With Expectations Risk

Case 3

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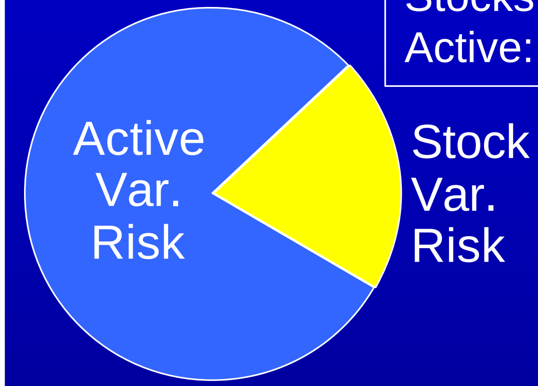

Case 3: Acknowledging Expectations Risk

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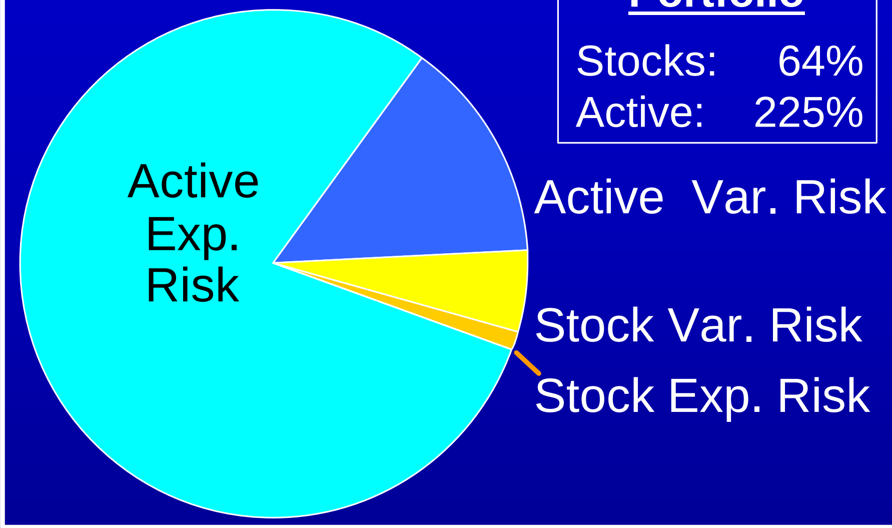
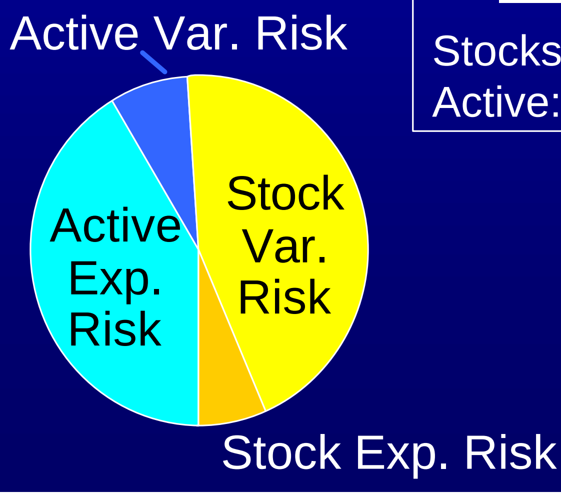


Case 4

Portfolio

Stocks:	**53%**
Active:	**44%**



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Portfolio Weights				
Stocks	42%	41%	64%	53%
Active	225	160	225	44
Portfolio Characteristics (Annualized)				
Excess Return	8.44%	6.44%	9.06%	3.44%
Variability Risk	**15.09**	11.66	**15.55**	7.48
Expectations Risk	10.16	7.23	32.36	2.25
Total Risk	18.19	**13.71**	35.90	**7.81**

Summary

- Expectations risk is:
  - Subtle
  - Of modest impact for one year
  - Crucial at long horizons
  - Especially for active management
  - Potentially outweighing conventional risk

"To know that we know what we know

and

to know that we don't know what we don't know,

that is true knowledge."

– Copernicus