Return Forecasting and Portfolio Construction: A Quantile Regression Approach

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Outline

- Introduction to Quantile Regression
- Tail Behavior in Investing: promising and challenging
- Alpha Distribution:
  - QRAD-location
  - QRAD-probability
- Portfolio Distribution: QRPD
- Summary

Quantile Regression: A Complementary Approach

Quantile regression, as introduced by Koenker and Bassett (1978), provides an alternative approach to conditional mean method.

- First regression in history is Median, half century earlier than OLS (1805).
- "Their souls seem as dull to the charm of varsety as that of a native of one of out flat English counties, whose retrospect of Switzerland was that, if the mountains could be thrown into its lakes, two nuisances would be got rid of at once." — Francis Galton

QR–Distributional View

- Sample Quantile: $F_y = Prob(Y \leq y)$,

$$Q(\tau) = inf\{y : F_y \geq \tau\}, \tau \in (0, 1)$$

- Conditional Quantile: $y = x^\top \beta + (x^\top \delta) u$

$$F_y^{-1}(\tau|x) = x^\top \beta + x^\top \delta F_u^{-1}(\tau)$$
$$Q_y(\tau|x) = x^\top (\beta + \delta F_u^{-1}(\tau)) = x^\top \beta(\tau)$$

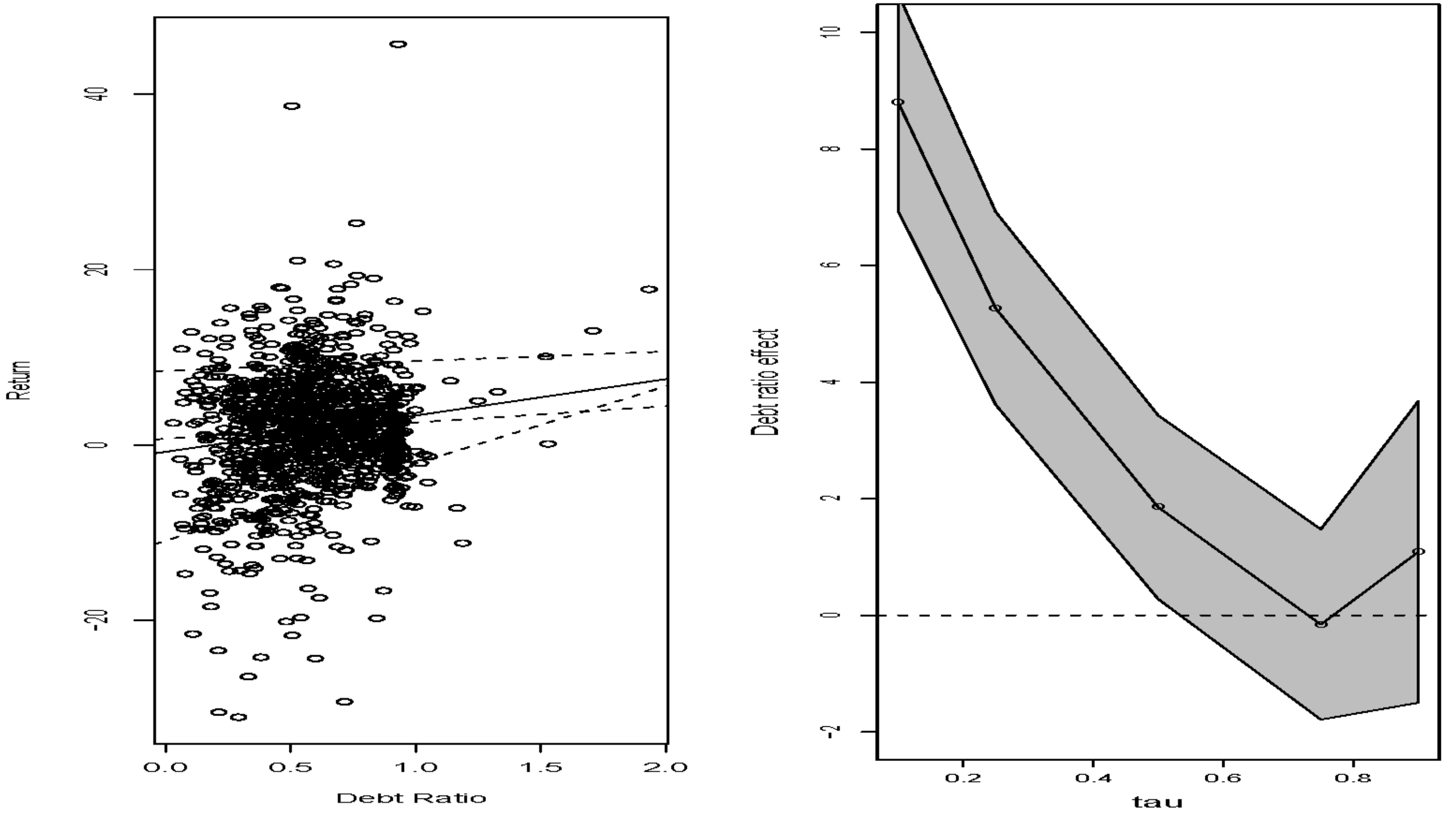
- Interpretation: $\beta_j(\tau) = \frac{\partial Q_y(\tau|x)}{\partial x_j}$, marginal effect of x_j on the τ^{th} quantile of response. With $\tau \in (0, 1)$, distributional view of causal effects.

- Decomposition of conditional mean view:

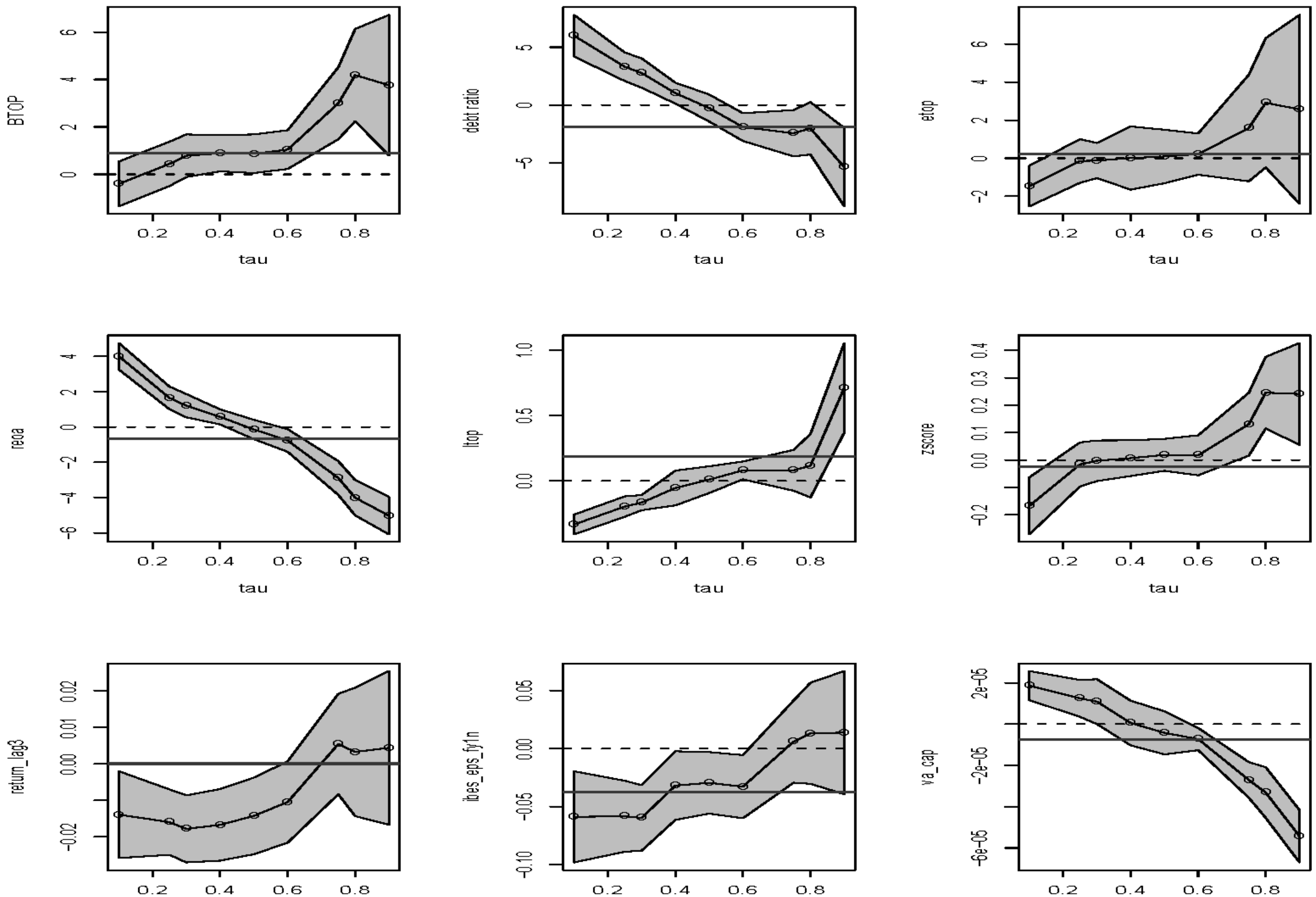
$$\beta = \int_0^1 \beta(t)\, dt = \int_0^1 (\beta + \delta F_u^{-1}(t))\, dt$$

QR–Example 1

Heterogenous causal effects of debt ratio on returns



QR–Example 2


BTOP
debt ratio
etop
reoa
ltop
zscore
return_lag3
ibes_eps_fy1n
va_cap
tau

QR in Finance

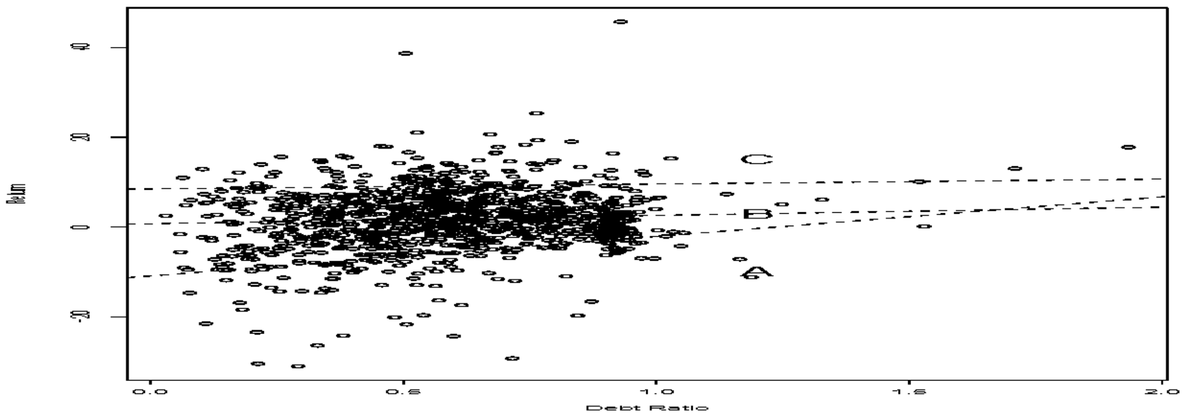
Recently, there has been a rising interest of QR in Finance. A primary example is the employment of QR in **VaR** models.

- Engle and Manganelli (1999): QR in nonlinear AR VaR models.
- Giacomini and Komunjer (2002): a Wald-type test for QR in VaR models.
- Chen and Chen (2003): empirical comparisons and found that VaR calculations with the quantile regression approach outperform those with the variance-covariance approach.
- Barnes and Hughes (2002): CAPM is retold through QR.
- Bassett and Chen (2001): QR for porfolio management styles.
- Koenker (2005): Copula and nonlinear QR

QR in Finance–Our Approach

- To the best of our knowledge, no practical use of quantile regression in the practical investment yet.
- Challenge: How to employ the distributional information for return forecasting and portfolio construction?
- Our approach:
  - General interpretation of QR in Financial markets
  - Return forecasting
  - Portfolio construction

Alpha Distribution: QRAD-location



According to the past return, the empirical conditional distribution is derived and then the $\beta(\tau)$ is assigned for the forecast of next period. For example, let $\tau = \{0.1, 0.5, 0.9\}$, we would obtain,

$$\hat{R}_{i,t+1} = \begin{cases} x_{i,t+1}^{\top}\hat{\beta}(0.9) & if \quad R_{i,t} \geq x_{i,t}^{\top}\hat{\beta}(0.9) \\ x_{i,t+1}^{\top}\hat{\beta}(0.1) & if \quad R_{i,t} \leq x_{i,t}^{\top}\hat{\beta}(0.1) \\ x_{i,t+1}^{\top}\hat{\beta}(0.5) & otherwise \end{cases} . \qquad (1)$$

Alpha Distribution: QRAD-location

The goodness-of-forecast may be decomposed as follows,

$$\sum_{i=1}^{N}|\hat{R}_{i,t+1}-R_{i,t+1}| = \sum_{A}|\hat{R}_{i,t+1}-R_{i,t+1}|+\sum_{B}|\hat{R}_{i,t+1}-R_{i,t+1}|+\sum_{C}|\hat{R}_{i,t+1}-R_{i,t+1}|,$$

where A, B and C denotes the area of $(0, 0.1]$, $(0.1, 0.9)$ and $[0.9, 1)$.

Proposition 1 *Let $\hat{R}^l(\tau|x)$ be the composite quantile return from the QRAD Location method, also $\hat{R}^m(0.5|x)$ and $\hat{R}^e(.|x)$ be the median and mean return, respectively, then*

$$\sum_{i=1}^{N}|\hat{R}^l_{i,t+1} - R_{i,t+1}| \leq \sum_{i=1}^{N}|\hat{R}^m_{i,t+1} - R_{i,t+1}| \leq \sum_{i=1}^{N}|\hat{R}^e_{i,t+1} - R_{i,t+1}|,$$

where the letter l, m and e denotes location, median and mean, respectively.

Alpha Distribution: QRAD-probability

To overcome the disadvantage of the QRAD Location method where forecasts depend heavily on previous conditional location, we propose an alternative approach, which is to assign probabilities to the forecasts,

$$\hat{R}^{p}_{i,t+1} = p_1 \hat{R}_{1,i,t+1} + p_2 \hat{R}_{2,i,t+1} + ... + p_k \hat{R}_{k,i,t+1},$$

where p_k is the probability of the occurence of $\hat{R}_{k,i,t+1}$. For example, for $\tau = \{0.1, 0.5, 0.9\}$, we would have that

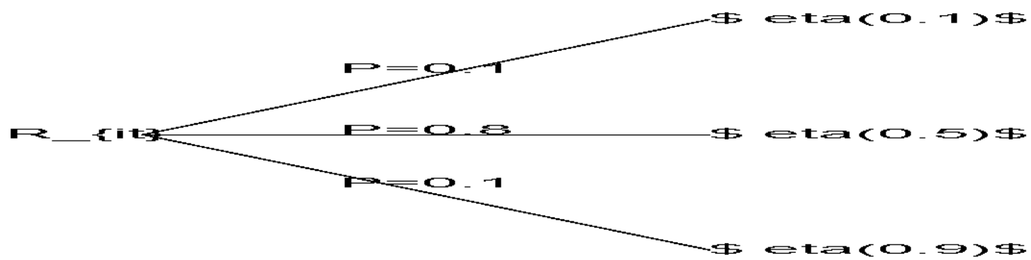
$$(p_1, p_2, p_3) = (0.1, 0.8, 0.1),$$

and,

$$(\hat{R}_{1,i,t+1}, \hat{R}_{2,i,t+1}, \hat{R}_{3,i,t+1}) = x_{i,t}^{\top} \big(\hat{\beta}(0.1), \hat{\beta}(0.5), \hat{\beta}(0.9) \big).$$

Clearly, the above formulation is the familiar expected value.

Alpha Distribution: QRAD-probability



Proposition 2 *Let $\hat{R}^p(\tau|x)$ be the composite quantile return from the QRAD Probability method,*

$$\sum_{i=1}^{N} |\hat{R}^p_{i,t+1} - R_{i,t+1}| \leq \sum_{i=1}^{N} |\hat{R}^m_{i,t+1} - R_{i,t+1}| \leq \sum_{i=1}^{N} |\hat{R}^e_{i,t+1} - R_{i,t+1}|.$$

- Under mild conditions, QRAD-location and QRAD-probability yield better goodness-of-forecast than the traditional methods.
- As $n \to \infty$, QRAD-location $\Longrightarrow$ QRAD-probability.

Portfolio Distribution: QRPD

Now, let's focus on *portfolio* distribution.

- At each τ, let $W_\tau = \left(w_{1,\tau}, \ldots, w_{N,\tau}\right)^\top$ be the optimal weights of the portfolio.
- We then have an empirical distribution of the portfolio.
- The final portfolio is constructed as

$$W = p_1 W_{\tau_1} + \ldots + p_k W_{\tau_k},$$

where p_k is the probability of occurence of W_{τ_k}.

For example, as $\tau = \{0.1, 0.5, 0.9\}$, *we would have three sets of weights,* $W_{0.1}$, $W_{0.5}$ *and* $W_{0.9}$, *corresponding to the three sets of forecasting returns. Thus, the weights of the proposed portfolio are*

$$W = 0.1W_{0.1} + 0.8W_{0.5} + 0.1W_{0.9}.$$

Conclusion

- Conditional mean method is still very attractive
- QR provides a full-picture distributional view
- Investigate the tail behavior in financial market
- Empirical study in the next step