A New Approach to Real Estate Risk
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Abstract
Traditionally relegated to the back of the bus by institutional investors, private equity real estate has recently been afforded larger allocations in recent years on the brute strength of its performance, rather than any theoretical justification arising out of new methodology or data. While real estate is a known diversifier, the true extent to which it increases a portfolio’s risk adjusted return is difficult to quantify. The purpose of this paper is to present a model that bridges the methodology divide between real estate risk assessment methods, and those used in securities markets. Using this approach, it is possible to assess the risk of specific properties and measure the expected contribution of such properties to the enterprise-wide risk of typical institutional portfolios.
Introduction
Traditionally relegated to the back of the bus by institutional investors, private equity real estate has recently been afforded larger allocations in recent years on the brute strength of its performance, rather than any theoretical justification arising out of new methodology or data. While real estate is a known diversifier, the true extent to which it increases a portfolio’s risk adjusted return is difficult to quantify. Data constraints and definitional issues make the answer to this question rather fuzzy. Published real estate returns are not only notoriously smooth, but also suffer from return persistence as well as sample size issues. In addition, real estate is a notoriously local asset, and published index returns provide little insight into the impact of ownership of specific properties on a broad investment portfolio that is often dominated by stocks and bonds.

The long time horizons in real estate have encouraged property investors to concentrate their research on forecasting absolute returns, while institutional security analysts often focus on risk or uncertainty of returns, as this is the dominant influence on performance over shorter horizons. This dichotomy arises as the cumulative return on an investment rises roughly linearly with time (ignoring compounding), while the volatility (standard deviation of returns) grows at the square root of time (assuming a random walk process).

The purpose of this paper is to present a model that bridges the methodology divide between real estate assessment methods, and those used in securities markets. Using this approach, it is possible to assess the risk of specific properties and measure the expected contribution of such properties to the enterprise-wide risk of typical institutional portfolios.

The model is economical in its structure; however, instead of forecasting the level of expected returns for future periods, it directly forecasts the variance or risk of returns at both the property and portfolio levels. The expected level of correlation among properties in the portfolio and to other asset classes is also a direct output.

With risk and correlation estimated, the investor can then provide expected returns and form an efficient frontier, as well as measure the incremental risk contribution of a particular asset and the components of risk at the property level. For example, the model allows an investor to consider whether buying a shopping mall in either San Jose or Syracuse would be more diversifying, given that their stock portfolio is typically concentrated in high tech companies. Similarly we can consider the decision to use fixed rate or variable rate financing, given the nature of prepayment options in the investor’s bond holdings. By assessing risk directly, we sidestep most of the problems with reliance on traditional property appraisals. We also present the output information in a form consistent with analytical norms in public security markets.
As the entire process associated with our technique involves the interaction of several analytical models, each of which is individually complex, we present the method only in descriptive form, and provide some key details in the appendix.

**Literature Review**

Real estate is a particularly difficult challenge because of its lack of liquidity. Typically, even published indices in real estate are based on annual appraisals of large properties, not actual transactions. The lack of transaction based pricing, and the long periods between transactions have the statistical effect of smoothing the ups and downs of the real estate market, as compared to the actual but unobservable real market conditions. Traditional real estate indices such as NCREIF typically underestimate the true volatility of real estate, and have a high degree of serial correlation that arises from the smoothing. The problems with real estate measurement are well documented in Graff and Young (1996) and Geltner (1998). Partial solutions to these problems are proposed in Fisher (2000) and Fisher and Geltner (2000), wherein available transaction prices on repeated sales of the same property, and concurrent appraisal data are used to adjust the published time series of index returns.

In a similar vein, time series of returns can be modeled as a first-order autoregressive process in order to neutralize the serial correlation. Sample statistics regarding real estate index returns are then based on the adjusted time series. Unfortunately, even if we were able to fully resolve all the statistical problems of broad real estate indices, this is no help with respect to measuring the marginal impact of individual properties on portfolio returns.

Another approach to measuring the unobservable true returns in the real estate market has been to create a “synthetic” real estate return series, by thinking of unobservable real estate returns as being the returns of Real Estate Investment Trusts, with the influence of the general stock market hedged away. The idea of measuring real estate returns through a hedged REIT index began with Giliberto (1993), and was put into the context of asset allocation decisions by Liang and Webb (1996). Chatrath (1999) and Clayton and MacKinnon (2001) extended this line of research to related REIT returns to those of other financial assets, providing at least a partial foundation for our approach. Once again, the “hedged REIT” approach may be helpful when considering the inclusion of real estate in an asset allocation exercise, but is of limited value when considering decisions at the individual property level.
**Risk Model Basics**

Modern methods of portfolio risk analysis typically rely on a linear factor model of assets returns.

\[ R_{it} = \sum_{m=1}^{n} [B_{im} \times F_{mt}] + E_{it} \]  

Where

- \( R_{it} \) = the return on asset \( i \) during period \( t \)
- \( n \) = the number of factors in the model
- \( B_{im} \) = the exposure of asset \( i \) to factor \( m \)
- \( F_{mt} \) = the return to unit exposure of factor \( m \) during period \( t \)
- \( E_{it} \) = the asset specific return of asset \( i \) during period \( t \)

By the usual algebra we can extend the linear model for the return of a single asset to describe the variance of return to an entire portfolio of assets.

\[ V_p = \sum_{j=1}^{n} \sum_{k=1}^{n} [B_{pj} \times B_{pk} \times S_{j} \times S_{k} \times P_{jk}] + \sum_{\iota=1}^{z} (W_{\iota}^2 \times S_{\iota}^2) \]  

Where

- \( V_p \) = the variance of portfolio returns
- \( B_{pj} \) = the exposure of the portfolio to factor \( j \)
- \( S_{j} \) = the standard deviation of returns to factor \( j \)
- \( P_{jk} \) = the correlation of returns to factor \( j \) and factor \( k \)
- \( W_{\iota} \) = the weight of asset \( \iota \) in the portfolio
- \( S_{\iota} \) = the standard deviation of asset specific returns to asset \( \iota \)

Finally, we should note that the factor exposures of a portfolio are simply the weighted average of the factor exposures of the individual assets.

\[ B_{pj} = \sum_{\iota=1}^{z} (W_{\iota} \times B_{\iota j}) \]  

In applying this sort of linear model to the financial assets, two types of specifications are popular. In *economic* models, the factors are defined to be exogenous variables such as interest rates or oil prices, such that the factor returns (the \( F \) values) can be observed in the real world. A separate time series regression is generally then used to estimate the factor exposures (the \( B \) values) of each asset. In such regressions, the independent variable is the periodic returns to the particular asset, and the independent variables are the observable returns to the factors.

Alternatively, we could use a *fundamental* model, where endogenous characteristics of the assets (e.g. market cap of a stock) are used to specify
observable values of the factor exposures (the B values) and the factor returns
(the F values) are estimated for each period in a separate cross-sectional
regression for each time period. In these cross-sectional regressions, the
independent variable is the vector of asset returns for the period, and the
independent variables are the factor exposures for all the assets at the beginning
of the period. Normally, we distinguish between the two types of models via
notation. In fundamental models, the factor exposures can be time varying, so
the factor exposures (the B values) would also carry a time subscript.

One particular model of the economic type that is widely used by institutional
investors to evaluate the risk of their marketable securities portfolios is the
“Everything, Everywhere” (EE) model. This model links global public security
performance to over 50 factors including: stock and bond market performance
across five global geographic regions and six broad economic sectors. There are
also factors meant to measure investor confidence (e.g. the spreads in yields for
different qualities of bonds), and macroeconomic conditions (interest rates, energy
costs, exchange rates). It breaks discount rate risk into two components; the risk
of treasury curve movements and the risk of changes in credit related yield-
spreads. Bond risk is estimated by measuring a bond’s price sensitivity to both
the credit factors and the treasury factors using a binomial model that
incorporates prepayment options. As of this writing, the model covers
approximately 35,000 global equities and 270,000 fixed income securities. An
extensive discussion of the specification of the EE model is provided in Appendix A.

In that the EE model is of the economic type, the estimation of factor exposures is
normally carried out by time series regressions. However, since real estate
investments do not typically have observable periodic returns, we have no
information to use as the independent variable in our regressions. Instead we
take advantage of various techniques available to estimate the exposure of a
financial asset’s returns to the factors in closed form. For example, one might
calculate the sensitivity of a bond’s return to changes in the level of interests rates
by the sort of time series regression discussed here. However, there are well
known closed form methods for calculating the duration of a bond, the sensitivity
measure that would have arisen as the result of our regression. We will therefore
endeavor to include real estate within the EE framework by use of such closed
form methods.

**Application to Real Estate**

Traditional real estate appraisals utilize one of three basic methods to value a
property: (a) replacement cost, (b) comparable sales and (c) capitalizing the
expected income. One way that real estate investors have tried to evaluate risk of

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1 The Everything Everywhere Model is made commercially available through Northfield Information Services, Inc. and is widely used by large institutional investors to assess risk across multinational, multi-asset class portfolios.
individual properties in the past is to do Monte Carlo simulations of their valuation models. By varying the valuation inputs across their expected range, we can obtain an expectation of the range of property values at any future moment in time. This allows us to estimate the uncertainty of return on that property over a known time horizon. However, such procedures are not tractable over large portfolios, nor do the "bottom up" estimates of real estate specific variables such as rents or operating expense allow for any insight into the interrelationships between real estate properties and other asset classes.

Our methods for estimating factor exposures for real estate are closely related to the third method. From the perspective of typical real estate analysis, we are using financial market data external to real estate to forecast the possible range of inputs to such a valuation process across time, and thereby derive a direct assessment of risk. For example, we can use observed volatility of bond interest rates to frame the range of potential capitalization rates for a property. Further, we can do things like assess potential demand for office space in lower Manhattan based on the recent strength of the stock market performance of the financial services sector of the economy. The more volatile the expected stock market performance of the financial services sector, the greater the uncertainty of demand for such office space. By direct use of information from the stock and bond markets, the model automatically provides the institutional investor with consistency of assumptions across all asset classes.

Our model first takes a complicated problem and breaks it down into its parts. We do this by disaggregating a portfolio into buildings, and buildings into their constituent sources of risk including the cash flow from tenants, tenant credit risk, rent volatility and the property's financing structure. Each of these sources of risk is represented by a hypothetical proxy portfolio of marketable securities that we believe will have the same economic payoff properties as the concerned aspect of the real estate property. Once we have done this, we can apply our existing model used for traded securities to value and estimate the risk of each piece. Having done that, we can reassemble the components and examine risk at any level we choose.

Given the model’s framework, we can examine the sensitivity of each contributor to changes in property value to the common set of underlying factors. Armed with estimates of the potential range for factors (e.g. how volatile do we expect oil prices to be?) the mathematics of a risk assessment for a single property or entire portfolio is simple algebra. The risk estimate contains the future range of interest rates, existing cash flows streams, rent volatility, and financing structure risk.
**Required Property Level Input Data**

In order to make the model useful, we must define a parsimonious set of input data that can be practically collected for actual real estate portfolios in order to compute the various aspects of the model. Below is a listing of the inputs the model uses for each property. For even large property portfolios, this information can be maintained in a single spreadsheet.

1. Dominant Property Use (Apartment, Hotel, Industrial, Office, or Retail)
2. Current Occupancy
3. Anchor Tenants
   a. Square Footage
   b. Lease Renewal Date
   c. Credit Rating
4. Debt
   a. Property Only or Cross Collateralized
   b. Duration
   c. Fixed or Variable
   d. Coupon Rate
   e. Prepayment Options/Penalties
5. Expected Capital Expenditures
6. Current Effective Rent
7. Current Estimated Property Value

It should be noted that the estimated property values are not used as part of the individual property analyses, but rather are used to compute portfolio weights for portfolio level calculations.

In addition to the information on individual properties, we also collect data on local real estate market conditions for each area in which a property is located. Our real estate data set is completed from commercially available databases of real estate statistical data, and regional economic data. The EE model encompasses a wide range of information on both individual securities and financial market conditions.

**Estimating Factor Exposures for Real Estate**

Our first step is to estimate the Interest Rate Risk of Incoming Cash Flows. This is done by forecasting the time series of a property’s cash flow in a deterministic fashion, without considering rent volatility. There are two components to the cash flow, the inward cash flows provided by net operating income, and the outgoing cash flows required by the mortgage financing (if any). Using the framework of fixed income securities markets, we can consider tenant leases like long positions in bonds. These pseudo-bonds are subject to credit risk, and have other bond like
characteristics such as fixed expiration dates, and embedded options (e.g. tenant renewal options).

Incoming cash flows are based on projected net operating income (NOI) which changes with projected income taking into consideration expected vacancy levels. Our approach to forecasting cash flows is similar to that found in real estate software packages such as Argus and Circle, but with less detail and precision. In addition to rental growth, operating margins also depend on occupancy levels since vacant space is generally costly to the landlord. A building’s long-term vacancy is assumed to move from its current level to a long-term equilibrium structural vacancy over time unless there are convincing idiosyncratic factors to do otherwise. Lease renewal rates for existing tenants are also modeled in a manner that makes them inversely related to vacancy. Therefore, in order to calculate a property’s cash flow the following inputs are needed:

- Current rent and expenses (NOI)
- Current occupancy/vacancy
- Market-level structural vacancy and reversion
- Down-time between leases
- Rent and expense growth over time
- Useful life of the building (assumed to be 50 years in examples)

With that data, we use current NOI and projected NOI forward based solely on assumed rental inflation and expenses, lease renewal schedules, probability of renewal, credit risk, downtime, as well as changes due to whether the building’s occupancy level is above or below the market average. For buildings whose current vacancy is below the market’s long-term average there will be a downward trend in NOI as current vacancy reverts to the structural vacancy estimates. Given the projected pattern, a building’s cash flow can be valued and its exposure to the treasury curve risk factors can be measured in a fashion that is consistent with common bond market practice.

The exhibit below shows a building whose current vacancy rate is well below the market vacancy. As the building trends towards the market equilibrium, cash flows decline to reflect the loss of tenants to other buildings. Once it approaches market occupancy, it is assumed to trend with the market thereafter.
Lease renewal rates for existing tenants are also inversely related to market vacancy levels. Tenants have more options in a market where the rate of vacancy is high, so the probability of renewal is presumed lower. Downtime between leases is also incorporated and like lease renewals are a function of the vacancy at the time of renewal. Finally, rents are assumed to move with inflation in the long run and we also assume the useful life of a building is 50 years from the start of the analysis.

The credit risk of tenants is incorporated into the model in two ways. First, expected levels of lease defaults are incorporated into projections of incoming cash flow in terms of vacancy level, and down time between leases. We will incorporate the impact of credit risk on discount rates in a later step. For anchor tenants with published credit ratings (e.g. Moody’s or S&P), the default rates associated with these ratings are taken into account in forecasting cash flows.

For each property we also estimate the creditworthiness of a non-credit rated, generic tenant. Noncredit tenants are thought to be “typical” inhabitants of their local economy and their credit risk is determined by weighting the credit risk parameters of a low rated, high yield bond by the employment-based sector share of their metropolitan area. This allows generic in Houston (i.e. local economy concentrated in the energy sector) to be differentiated from tenants in San Jose (i.e. concentrated in the high tech sector). It should be noted that corporate credit ratings provide a conservative measure of tenant risk since “salvage values” in bond defaults are lower than can be recovered from situations of tenants defaulting on leases. For properties with a large number of small tenants, some of this risk will diversify away (the tenant specific portion), while the credit risk arising from the potential for a general economic downturn will not.
It should be noted that the migration of credit risk across time is also taken into account. It is assumed that when a credit rated tenant’s lease expires, that tenant may renew their lease (with some assumed probability) or be replaced by a generic tenant. As such, cash flow expectations of a property (frictional vacancy rates, downtime, NOI) slowly migrate toward greater influence of generic tenants, as leases turn over. The credit migration process is modeled as a binomial tree with expected renewal rates of leases used to define the probabilities at each decision node.

For most properties, the projected NOI stream associated with each individual lease are then analyzed within the EE model’s term structure process, in the same fashion as we would consider each separate bond in a fixed income securities portfolio. Factor exposures to the three factors that define potential variation in the term structure (shift, twist, butterfly) are then calculated using EE’s binomial type OAS model. The factor exposure to the “shift” potential of the term structure is comparable to duration.

Our next step is to consider the impact of credit risk on discount rates used to compute the present value of future cash incoming cash flows. The rate of discount applied to incoming cash flows can be thought of as having two components, the term structure portion having purely to do with the time value of money, and a credit spread. Time series changes in the credit spread are like a parallel shift in the yield curve, so the sensitivity of a cash flow stream to a change in the spread is given by its duration. The expected volatility and correlations of the credit spreads are modeled separately as a function of the EE model factors, for each credit rating level and economic sector (see Appendix A for details).

So far, our detailed projections of future operating cash flows have assumed future rents will be the current rent level adjusted for inflation. We will now explicitly incorporate the future uncertainty of levels of rents and occupancy. Over time, changes in the level of rents and occupancy are driven by both demand and supply. Each property “package” includes a set of risk exposures to represent rent and occupancy volatility. The supply of commercial space changes very slowly having little correlation with the EE model’s broad economic factors and is largely a function of local market conditions. In contrast, demand for commercial space is elastic and can be effectively captured in our framework by relating percentage changes in rents to the broad economic factors of the EE model.

For each property type and each metropolitan area we run a time series regression which models the percentage change in rents. The independent variable for this equation is a demand variable specific to each metropolitan area. It is a weighted average of the stock market returns for each of the six EE industrial sectors, weighted by the employment shares in each individual market. In that retail and personal services employment are a significant percentage of total employment,
the consumer sector typically receives the biggest weight. Variation does occur across metro areas since each market has its own employment profile given the makeup of the local economy.

In addition to demand, we also incorporate the change in metro-level building stock for each property type as well as an initial conditions variable in the form of vacancy rates. Changes in demand and supply have less of an impact when the market is 25% vacant than when it is 5% vacant. This is important because most markets were not in equilibrium at the beginning of the observation period. Initial market conditions affect the impact of the weighted EE factors on rent volatility since it is the cumulative effect of supply and demand factors that impact rent growth, and hence the expected variation of rents through time.

The impact of rent volatility is incorporated into risk assessment in a novel fashion. Our cash flow analysis assumed future rents are known with certainty. We incorporate the uncertainty in rents by assuming property owners have entered into a forward contract with tenants to keep future rents constant in real terms. The coefficients arising from the regression equation above represent the EE factor exposures of this contract. The portion of a building’s rents subject to the forward contract at any one moment in time is presumed to be combination of the expected percentage of vacant space plus the expected percentage of leases turning over in the current year. As this expected value varies from year to year, this value is projected out over the expected life of the building and a present-value weighted average is taken as the final coefficient. The residuals from the rent volatility estimation equation are also scaled by this rent volatility exposure coefficient and used as an estimate of the idiosyncratic risk of a particular building.

Having measured a property’s underlying cash flow structure, the model still needs to analyze and quantify the risk associated with its financing structure. Mortgages are modeled using the EE term structure model. This process is comparable to analyzing a short position in mortgaged backed securities. Provisions of a particular financing vehicle may include fixed or variable rate mortgages. Prepayment options are taken into account and are similar to having embedded calls, where the exercise price is the prepayment penalty plus the outstanding loan amount at the particular point in time. If a property is part of a cross-collateralized pool of assets we build a synthetic security which is the sum of all the properties under the same financing umbrella. We also take a conservative approach with respect to the borrower by assuming that intentional default is not an option.
An Empirical Example
Much of the motivation to create the model came from Investcorp, a prominent institutional investor based in Bahrain. Investcorp sought a methodology to measure real estate risk from various perspectives including the individual property and portfolio levels, as well as the amount of risk arising on its own balance sheet as a co-investor. With a statistically more accurate way to aggregate real estate risk relative to its other investments (private corporate equity and hedge fund of funds) it would be able to evaluate capital risk allocation for deals from acquisition and placement through to disposition. Additionally, the Basle II capital accord permits a closer alignment between economic and regulatory risk capital for institutions able to use a model approach for its risk assessment, something which Investcorp will now be able to extend to its real estate portfolio.

To illustrate the use of the model, we now report on how the model was applied to their large and diverse portfolio of US commercial real estate. Two analyses were conducted, one in early 2004 and another in early 2005. At both moments in time, the portfolio contained more than forty different properties, including more than twenty-five separate combinations of property type (office, retail, hotel and residential) and geographic location. The aggregate purchase cost of the properties was in excess of $1.5 billion, with considerably greater market value. Almost all the properties had significant financing in place, and in the case of several sub-groups, the financing was cross-collateralized.

As shown in the chart, for the overall portfolio the expected return volatility was approximately 10% per annum on an un-levered basis. Given the current financing structures the expected volatility of return on ownership equity was 14% per annum. These values compare to approximately 13% for the NAREIT index, 5% for corporate bonds, and 1.3% for the NCREIF (quarterly returns annualized). These volatility values were measured over the precedent five years to the analysis date, which is comparable to the EE model estimation sample period.

At the individual property level, the forecast return volatilities were between 8% and 12% annually on an un-levered basis, with forecasts of between 9% and 30% annual volatility for the returns on equity. As an important check of the reasonableness of the results, we formed two ranked lists of the properties from least risky to most risky. One list was done based on the un-levered risk values and the other list inclusive of financing. When the lists were presented to the investment professionals who had acquired and managed the properties, the relative risk rankings were found to be highly intuitive and consistent with management beliefs.
The last aspect of our exercise will be to review the risk decomposition of a single property. In this case, the property is a medium sized shopping center. Each row of the table below describes each contributing influence on the property return of a particular factor in the EE model. The exposure column represents the sensitivity of the property’s return to the factor, while the factor variance column describes the volatility (in variance units) of the returns associated with this factor. For example, the annual volatility of oil prices is listed at a variance of about 1800 %². The product of the exposure squared and the factor volatility would sum to the value of the variance contribution column if the returns to the factors over time were uncorrelated. To the extent that correlations exist amongst the factors, the variance contribution column also includes the respective covariance terms. The factor contributions then sum to a total of 89.96%. A small amount of specific tracking variance is then added that accounts for risk aspects of the property that could not be included in the model. In variance units, we now have a total of 90.38%² or 9.51% per annum in standard deviation terms.
Risk Decomposition of a Shopping Mall

Risk Decomposition

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Factor Tracking Variance 89.96
Specific Tracking Variance 0.42
Total Tracking Variance 90.38
Return Standard Deviation 9.51

Conclusions and Contributions

In this paper, we have put forward a new approach to estimating the risk of real estate that does not rely on typical appraisal based benchmark indices. The proposed model decomposes real estate risk into four components: operating cash flow valuation risk, financing structure, credit, and rent/occupancy volatility. Each of these risks is then expressed as functions of factors that are observable in financial markets or in the general economy. Our approach is congruent with methodologies used for risk management of securities market portfolios allowing for seamless integration of risk assessment in multi-asset class portfolios. Most importantly, the model provides a framework for determining how much of the risk of investing in a property arises from characteristics of the specific property, and how much of the risk arises from common influences across all properties such as interest rates and levels of economic activity. We believe this new level of transparency with respect to real estate risk will encourage investors to be confident in their understanding of real estate, and consequently be more willing to allocate more of their resources to property investments.
Our work with specific real estate portfolios has led to a number of useful empirical conclusions. First, even our limited results suggest that widely published real estate indices do understate the true (but unobservable) volatility in real estate returns. The volatility forecasts from our model seem consistent with both comparative statistics for other asset types such as REITs, and are also consistent in property by property comparisons with the beliefs of the investment professionals concerned.

One more interesting use of the model would be the potential creation of synthetic "real estate" by creating securities portfolios that are constructed to have comparable exposure to the economic forces that drive returns. Such synthetic products may be an important step forward in overcoming the inherently illiquid nature of real estate. This new approach to estimating real estate risk also provides an avenue for many innovations in real estate practice, including better benchmarks, the active hedging of interest rate risk to property values, and the more appropriate inclusion of real estate in optimal asset allocation.
References


Appendix A: Review of the EE Model

The EE model was designed to provide risk estimates for a wide variety of equity and fixed income securities across more than seventy countries. The model is estimated as a three part linear factor model. The first part of the model includes exogenously specified factors believed appropriate to the risk analysis of global equities. The second part of the model uses factor analysis to estimate statistical (blind factors) arising from variables that may have been omitted from the set specified variables, whether arising from oversight, impact on only a subset of the securities universe (e.g. something specific to stocks in Norway), or changes in market conditions over time such as the Internet bubble period of the late 1990s. The factors in the third section of the model represent the evolution of the terms structure of interest rates. The covariance of factor returns to all three sets of factors comprises the factor covariance matrix.

For equity securities, factor exposures are estimated for both the first and second set of EE model factors. This combination of exogenously specified and statistical factors is referred to as a hybrid model. For fixed income and derivative securities deemed to have no credit risk, only factor exposures to the third set (term structure factors) are estimated.

Bonds with credit risk are treated as a long position in a zero credit risk bond and a short position in a credit swap. The time series of returns to the credit swap is treated as an equity security. As such, bonds with credit risk have exposures to all three sets of factors. Securities such as convertible bonds that have an explicit combination of fixed income and equity characteristics have exposure to all three sets of factors.

Estimation of Equity Risk Factors
The EE model measures every security's exposure to its regional market return and global sector market return, global interest rates, oil prices, and other pervasive factors. etc. It also takes into account the portfolio's exposure to currency bets relative to its home currency. There are 90 factors in total, including currency dummy variables. Importantly, however, each security is exposed to only 13 factors which are:
1. Region
2. Sector
3. Interest rates
4. Oil prices
5. Company Size
6. Value / growth
7. Market development
8. Currency
9. “Blind factor” 1
10. "Blind factor” 2
11. “Blind factor” 3
12. “Blind factor” 4
13. “Blind factor” 5

Mathematically, the model's form is:

\[ R_{it} = \text{Int}_i + \beta_{1i} \text{SectRet}_{st} + \beta_{2i} \text{RegRet}_{ct} + \beta_{3i} \text{BR}_t + \beta_{4i} \text{Oil}_t + \beta_{8i} \text{Size}_t + \]
\[ \beta_{6i} \text{ValGrow}_t + \beta_{7i} \text{MktDev}_t + \beta_{9i} \text{FX}_c + \beta_{10i} \text{BF1}_t + \beta_{11i} \text{BF2}_t + \beta_{12i} \text{BF3}_t + \beta_{13i} \text{BF4}_t + \epsilon_{ti} \]

where:

- \( R_{it} \) = the return for company i in period t, calculated in the base currency
- \( \text{Int}_i \) = the intercept
- \( \text{SectRet}_{st} \) = the return for the sector s index (s = the sector for company i) in period t
- \( \text{RegRet}_{ct} \) = the return for the region c index that the company i’s country belongs to in period t
- \( \text{BR}_t \) = the return on the Salomon Brothers World Government Bond Index (a proxy for global interest rates) in period t
- \( \text{Oil}_t \) = the % change in oil prices in USD terms in period t
- \( \text{Size}_t \) = company size in period t (The difference between a return index of the 10% of the companies with the largest capitalization and a return index of the 10% of the companies with the smallest capitalization)
- \( \text{ValGrow}_t \) = A two part factor capturing equity market value/growth returns spreads (see discussion below for details)
MktDev\textsubscript{t} = market development return spread in period t (The difference between an index of returns of companies in developed countries and an index of returns of companies in emerging countries)

FX\textsubscript{c} = return on currency c; \beta\textsubscript{Si} is a dummy variable with value 1 for the currency in which the security is denominated, and zero for all others

BF1\textsubscript{t} – BF5\textsubscript{t} = A series of principal components of “blind factors” estimated on the residuals of the rest of the model using principal components analysis.

\varepsilon\textsubscript{ti} = the error term for company i in period t

The model is estimated on a universe of approximately 30,000 securities across seventy countries. It should be noted that the composition of regions from countries and the composition of sectors from industry groups is determined by complete linkage cluster analysis of the returns over the sample period.

**Value / Growth Factor Specification:**

The value/growth factor has two parts: the dividend yield (spread) factor and the correlation of stocks to “idiosyncratic variety”. The latter part of the factor is an innovative way of applying academic research to determine value and momentum effects in securities markets.

The dividend yield (spread) factor time series is defined as:

\[ D\textsubscript{t} = D\textsubscript{hi} - D\textsubscript{0t} \]

Where

\[ D\textsubscript{t} = \text{the return for the Dividend Yield factor for month T} \]

\[ D\textsubscript{hi} = \text{the average return of stocks in the highest 10\% of dividend yield of the estimation universe during month T} \]

\[ D\textsubscript{0t} = \text{the average return of stocks with zero dividend yield in the estimation universe during month T} \]

The logic here is that stocks that tend to do well when high yielding stocks (value) are doing better than low yielding stocks (growth) produce positive coefficients.

The correlation to the cross-sectional standard deviation of stock alphas during each period is introduced as a further measure of a stock’s value / growth qualities. The standard deviation of stock alpha is called “idiosyncratic variety”.

\[ J\textsubscript{t} = \text{Standard Deviation (A\textsubscript{kt})} \]

\[ A\textsubscript{kt} = R\textsubscript{k} - (R\textsubscript{ft} + B\textsubscript{k} \times R\textsubscript{mt}) \]

Where

\[ J\textsubscript{t} = \text{idiosyncratic variety during month T} \]
\( A_k = \text{alpha on stock K during period T (under CAPM)} \)
\( R_k = \text{return on stock K during period T} \)
\( R_f = \text{risk free rate of return during period T} \)
\( B_k = \text{beta of stock K} \)
\( R_m = \text{return on the market portfolio during period T} \)

Stocks with “value” characteristics tend to do well when idiosyncratic variety is low, and stocks with “growth” or momentum characteristics tend to do well when idiosyncratic variety is high.

Two time series are now available to represent the potential return differences relating to the “value/growth” property. Unfortunately, they are, at this point, in different units and structured in opposite directions! In the “Dividend Yield” factor time series, high numbers are associated with “value” doing better than “growth”. In the “Idiosyncratic Variety” factor time series, high numbers are associated with “growth” doing better than “value”.

In order to combine these two series in a single factor, they must be transformed into similar units. This is accomplished by standardizing each time series. This gives each time series a mean of zero and a standard deviation of one.

\[ E_t = D_t - \frac{\text{Average}_{[t= 1 \text{ to } n]} D_t}{\text{Standard Deviation}_{[t= 1 \text{ to } n]} D_t} \]

\[ F_t = J_t - \frac{\text{Average}_{[t= 1 \text{ to } n]} J_t}{\text{Standard Deviation}_{[t= 1 \text{ to } n]} J_t} \]

Where

\( E_t = \text{standardized Dividend Yield factor return during month T} \)
\( F_t = \text{standardized Idiosyncratic Variety factor return during month T} \)
\( n = \text{number of months in the time series} \)

Clearly, it is important that the direction of the effect be the same in both series, or they would tend to counter-act each other. Simply reversing the sign on the Idiosyncratic Variety time series sets its values in the same direction as the dividend yield spread part of the factor.

\[ G_t = -F_t \]

Where
\( G_t \) = the reversed value for the Idiosyncratic Variety during month \( T \)  

The two factor time series are combined by simply taking the average  

\[ V_t = \frac{(E_t + G_t)}{2} \]

Where \( V_t \) = return to the “Value/Growth” factor during month \( T \)  

The \( V_t \) factor return time series can then be included in the normal exposure estimation process. High exposures imply a “value” orientation and low exposures imply a “growth” orientation. 

**Estimating Factor Exposures Via Regression**  
The above model is estimated separately for the time series of returns of each security exposed these factors. The estimation time series-regression is done as a four stage, stepwise process (sector, region, other specified factors and blind factors) in a weighted least squares fashion. 

The nature of financial data and securities returns in particular gives us cause to be concerned about the potential for undue influence from outliers. Weighted least squares (WLS), a variant of generalized least squares, is used to estimate securities’ factor exposures in the Global model. WLS is appropriate “whenever the error terms are heteroskedastic with variances known to a multiplicative constant and not correlated with one another” (MacKinnon and Davidson, "Estimation and Inference in Econometrics", Chapter 9).  

Estimating the coefficients using WLS is essentially a two-stage process. For example the intention in the following is to estimate coefficients \( a, b, c... \)  

\[ R_{it} = aF_{1t} + bF_{2t} + cF_{3t} + \ldots \ldots + E_{it} \]

The above equation generates the time series of residual (\( E_{it} \)) values. The standard error of regression \( S_i \) is the standard deviation of the \( E_{it} \) series.  

The regression is then re-run giving each observation \( (t) \) a different weight according to the scheme below:  

\[ (W_{it})R_{it} = a'(W_{it}F_{1t}) + b'(W_{it}F_{2t}) + c'(W_{it}F_{3t}) + \ldots \ldots + E'_{it} \]

where \( a', b', c' \), etc. are the robust coefficients and \( E'_{it} \) are the residuals of the GLS-like estimation  

\( W_{it} \) is the weight given to each observation and \( E'_{it} \) is the residual for the weighted observation.
\[ W_{it} = \min \left( 1, \frac{1}{\text{abs}(E_{it}/S_i)} \right) \]

This formulation of \( W_{it} \) provides that any observation with residual less than one standard error (i.e. a less than one standard deviation event) is given weight one, while outlying events are given a proportionately lower weight (i.e. a two standard deviation event gets a weight of 1/2, a three standard deviation event gets a weight of 1/3, etc.)

The estimated coefficients \( a', b', c' \), etc. are now the factor loadings. The residual risk for the stock is derived using the residuals in the original, not weighted space so:

\[ S''_i = \text{StdDev} \left( R_{it} - \text{Sum} \left[ a'F1_t + b'F2_t + c'F3_t + \ldots \right] \right) \]

where \( S''_i \) is the standard deviation of the residual for stock

**Estimating the Factor Return Covariance Matrix**

The covariance matrix of the factor returns is computed over the trailing sixty months. The observations are exponentially weighted with a decay of .02 per period.

The factor return time series variances are computed as the squared value of the factor returns rather than using the squared differences from the mean. This is equivalent to assuming that the mean is zero in the usual formula.

Efficient market theory suggests that mean alphas (returns net of market risk) to a particular factor should be close to zero over time. However, in a bubble or trending market, a particular factor may exhibit a high mean return, with low variance around the mean for a substantial period of time. The failure of normal theoretical assumptions to hold is an additional source of uncertainty that the traditional factor return variance calculation does not capture.

Empirically, most factor returns do have a mean close to zero, so the change will not be noticeable. However, when a factor return is consistently large and of one sign (i.e. positive returns to the internet factor during tech bubble), this procedure will inherently bias the factor variance values upwards to provide a warning of the unusual factor behavior. The two contrasting examples below show the difference the change can make when a trend is apparent. More of this issue can be found in.

Rationale for the Added Use of Blind Factors
In the hybrid model approach, the usual observable-factor model is built with whatever known factors are most appropriate to the observed market data. By re-examining the residuals to estimate the temporary “blind factors”, the hybrid model addresses one of the key limitations of traditional risk models - the fact that they cannot “learn” or adapt as markets change. The application of a principal components analysis on the residuals from the main observable factor model allows automatic adaptation to changes in the set of factors that is influencing market behavior at a particular moment in time.

An implicit assumption of any factor model is that the factors specified account for all the sources of correlation between securities. Unfortunately, factor models can only be built from past data. As such, the model will provide an imperfect representation of future correlations as conditions change, potentially, or often, even, resulting in an omitted variable bias. This can be addressed by performing a principal components analysis on the residuals from the observable factor model. To be clear, the principle components are only run on that part of the securities’ returns not explained by the model's factors. Using the output from this analysis, temporary factors can be added to the model. These new factors can quantify the existence of whatever new or transient forces are influencing the market currently.

Term Structure Factors
The EE model evaluated fixed income securities based on the term structures of five currencies (US, UK, Euro, Japan, Swiss).

A global yield curve is formed as a weighted average of the five major yield curves. The weights are proportional to the market value of traded fixed income securities denominated in that currency.

At each month end, each term structure is fitted to an equation of the form:

\[ Y_m = a + bm + cm^2 \]

Where

\( Y_m \) = the zero coupon yield at maturity \( M \)

Month to month changes in the \( a \) coefficient represent changes in the level of interest rates (“level”, TF1), changes in the \( b \) coefficient represent changes in the slope of the term structure (“twist”, TF2), and changes in the \( c \) coefficient represent changes in the curvature (“butterfly”, TF3) of the term structure.
Within the model, the volatility and correlation of the time series of coefficient changes for the global yield curve are used in the factor covariance matrix.

TF1 is comparable to duration in traditional fixed income analysis. However, the values are adjusted for the fact that the volatility of the level of interest rates for a particular currency and maturity, maybe greater or lesser than the volatility of global interest rates. The scaling factor is equal to the ratio of two volatility levels involved.

Yield curves for securities issued in currencies other than the five major or “core” currencies are inferred from the five core currency curves. The non-core curves are inferred by forming a new curve constructed from the yield curves of the five major currency curves weighted by the inverse of their exchange rate volatility with the currency under consideration.

**Calculating Term Structure Factor Exposures**

The arbitrage free term structure model allows for the three factor exposures to be calculated inclusive of the effect of embedded options such as call, put and sinking fund options.

Floating rate fixed income instruments and the potential prepayments of mortgage related securities is handled by conducting Monte Carlo simulations down the interest rate paths established by the term structure model. Expectations for mortgage prepayments are derived from a simple exogenous model.

The term structure factor exposures for each fixed income security are evaluated under the term structure for the currency in which that security is denominated. These factor exposures are then scaled to reflect the differences in factor volatility between the yield curve of the currency of denomination and the global yield curve that is part of the factor covariance matrix. In addition to the factor exposures, an important output of the term structure model is the “option adjusted spread”. The OAS represents the portion of a fixed income instrument’s yield that is being paid to the investor as compensation for non-interest rate risks such as credit risk and limited liquidity.

**Incorporating Credit Risk**
Once OAS values have been computed for all securities, these values are averaged for a series of sub-sets of the fixed income securities universe (currently 270,000
instruments). The “buckets” are based on a three-dimensional sorting by geographic region, sector and published credit rating.

Over time, the average OAS value for each bucket will increase or decrease, corresponding to a widening or narrowing of credit spreads in the fixed income markets. This time series behavior is modeled as if it were an equity security using the same regression procedure. These coefficients can be interpreted as the equity factor exposures of a unit of credit spread volatility.

Changes in credit spreads are comparable to a parallel shift in the yield of a fixed income instrument. To obtain the equity factor exposures related to the credit risk of a stream of cash flows, we multiply the exposures of the appropriate credit bucket times the duration of the cash flow stream.

Appendix B: Real Estate Factor Exposures

Projecting NOI
As previously described the interest rate risk of incoming cash flows is accomplished by forecasting the net operating income of associated with revenue stream of a property over the expected life. This cash flow stream is then analyzed like a fixed income security in the OAS portion of EE model to estimate factor exposures to the terms structure factors.

The basic estimation of net operating income operates as follows (minor details omitted):

Let

\[ R = \text{the rental revenue of the lease} \]
\[ M = \text{the expected term of future leases} \]
\[ E = \text{the fraction of rent spent on operating expenses} \]
\[ D = \text{the expectation of rent lost to defaults} \]
\[ G = \text{the remaining term on the current lease} \]
\[ Y = \text{the expected vacancy rate} \]
\[ I = \text{the expected long term inflation rate} \]
\[ P = \text{the expected rate of renewal for existing tenants} \]
\[ U = \text{the useful life of the building} \]
\[ T = \text{the time increment (years)} \]

For periods up to time \( G \):

\[ C_T = R_T \times (1 - D - E) \]
At the end of G periods, two things can happen. There will be probability P of the least being renewed by the existing tenant. If the lease is renewed by the existing tenant, the new rent from period G+1 to G+M is

\[ R_{G+1} = R_1 \times (1 + I)^G. \]

Cash flows from period G+1 to G+M are still equal to:

\[ C_T = R_T \times (1 - D - E) \]

If the lease is not renewed by the existing tenant, we assume there is \( (1-P) \) probability that the space will set empty for a while waiting for a new tenant. The length of this fallow period would be the expected length of the lease times the assumed vacancy rate. This zero rent period would be create negative cash flow from operations. So cash flow from period G to period G + (M*V) would be

\[ C_T = 0 - (R_T \times E) \]

Once the lease has been renewed, the cash flows for the period from G + (M*V+1) to G+M would be proceed as before to assume new rents and new net cash flows.

Given probability P, we can combine these two possible scenarios to produce an expected cash flow stream over the period of the second lease. After the second lease is up, we can repeat the whole procedure again and again until we have estimate a forward stream of cash flows out to year U. This stream of cash flows is presumed certain, but has positive and negative periods and an upward drift (inflation).

**Interest Rate Risks of Financing**

The expected outward cash flows from mortgage debt service are modeled directly within the OAS term structure model. Prepayments are modeled as embedded call options with a strike price equal to the then current principal balance plus a prepayment penalty. The variation in expected cash outflows on a adjustable rate mortgage are handled directly within the term structure model.

**Incorporation of Tenant Credit Risk**

Now let us proceed to consider the volatility associated with the widening or narrowing of credit spreads. Out to period G, this is pretty easy. We know who the tenant is, what their creditworthiness is, what sector they are in, etc. Essentially we handle this just like a junk bond. Within the existing EE framework we provide loadings to the EE factors that correspond to the various combinations of credit rating and sector. Let us call this vector of factor loadings \( \Sigma B_{\text{initial}} \)
However, there is $1-P$ probability that when the lease is renewed the tenant will be different and have unknown properties. To account for this we introduce the idea of a "generic" tenant for each property type in each location. For example, we might know that the average store in a shopping center is leased to a tenant of "B" credit and is the consumer sector. For office buildings, we might have partial exposures to multiple sectors (e.g. an office building in Detroit might be heavily influenced by the auto industry, while an office building in New York might have a high exposure to finance, while in San Jose the predominant exposure would be from high tech.

$\Sigma B(generic)$

So in the second leasing period, we have $p$ probability of having factor exposures $\Sigma B(initial)$ and $(1-p)$ probability of having factor exposures $\Sigma B(generic)$. For the third leasing period, we would have $P^2$ probability of having exposures $\Sigma B(initial)$ and $(1-p^2)$ of having exposures $\Sigma B(generic)$. We can repeat this process for each likely lease rollover out to year $U$. Once we have established the vector of credit related factor exposures for each period of time, we can combine them into a single set of factor exposures by taking the present-value (of the impacted cash flows) weighted average of the various combinations of $\Sigma B(initial)$ and $\Sigma B(generic)$ applicable to different time periods.

Now we should have a both a time series of cash flows and a set of credit related factor exposures. We should now be able to compute the exposure of the risky cash flow stream to our three term structure factors, completing the factor exposures for each lease (cash flow stream). It should be noted that this approach is applicable to most commercial property situations. For multi-family housing (apartment blocks), the individual tenant risk is so diversified that the default and vacancy rates may be highly certain with any remaining uncertainty being driven by local employment conditions, which again can be represented by the "share of employment" scheme already used to represent our demand function.

**Exposure to Rent Volatility**

The remaining risk exposure that we must estimate for a property is the volatility of rents. We do this by introducing a synthetic security, conceptually similar to a forward contract that represents one unit of exposure to the volatility of rents for that particular property type in a particular geographic locale.

Essentially we will be looking to estimate the volatility per unit time of the percentage changes in rents as a function of the EE factors. Different properties will have a lesser or greater to the "rent volatility" security depending on the frequency of lease rollovers and vacancy rates. For example, if a New York building is rented to the US government at a fixed rent for 100 years, it would
have a very low exposure to the rent volatility for New York office buildings, whereas an apartment complex in Chicago might have a very full exposure to the rent volatility of Chicago apartment buildings.

We estimate the relationship between percentage changes in rents as a function of our EE factors via a time series regression of the type described for equities. In this case, the independent variable is a weighted combination of the return series for the EE global industrial sectors. For retail tenant in a shopping mall, the independent variable would be the return on the global consumer products sector. For an office tenant, the independent variable would be a weighted average of the global sector return series, with the weights arising from the shares of local employment designated within each EE sector. So we would calculate

\[ Z_{ijt} = \frac{(R_{ijt} - R_{ijt-1})}{R_{ijt-1}} \]

and

\[ Z_{ijt} = \sum B(Rent_{ij}) + \text{(other)} + \epsilon_{ijt} \]

Where

\[ R_{ijt} = \text{is the average rent for property type } i, \text{ in area } j, \text{ during period } t \]

\[ \epsilon_{ijt} = \text{residual rent for property } i, \text{ in area } j, \text{ during period } t \]

(\text{other}) = location specific independent variables such as the existing vacancy rates, estimated share of new buildings under construction and a measure of the restrictiveness of zoning

So for a each property type in a give locale, we would have a vector of exposures to the EE factor combination.

If we think of \( Z_{ijt} \) as having unit scale (exposure for $1 of rent) we can express the factor exposures of the rent risk for building \( N \) as

\[ X_n = \text{Rents}_n \times A_n \times (\sum B(Rent_{ij}) + \epsilon) \]

Where

\[ A_n = V + 1/M \]

The standard error of the regression times \( A_n \) is carried through to the model as a proxy for the idiosyncratic risk of property \( n \).
Appendix C: Data Sources
Data on equity securities characteristics and returns, and currency returns are obtained from FactSet.

Yield curve information is collected directly from various central banks. Pricing and security characteristics of fixed income securities are obtained from Reuters.

Mortgage backed securities data is obtained from INTEX.

Real estate data on rents and general vacancy levels is obtained from the Torto-Wheaton database.

Economic statistics are obtained from Economy.com.