Applications of Portfolio Variety

Dan diBartolomeo

June 2006

Abstract
Investment managers spend large amounts of time and money to assess the potential volatility of their portfolios, as measured by the time series variation in the returns. However, the asset management mandates of most institutional investors focus on returns relative to some benchmark index. In this context, the cross-sectional dispersion, or variety, of returns within the set of permissible securities is the predominant influence on the range of potential outcomes for active management, and hence the risk of underperforming the benchmark. We will review a number of investment applications of variety including estimating the correlations of assets, risk management and performance analysis. In addition, we will also show that common equity management strategies can be characterized as active bets on the variety of returns.
**Introduction**

Investment managers spend large amounts of resources to assess the potential volatility of their portfolios, as measured by the time series variation in the returns. However, the asset management mandates of most institutional investors focus on returns relative to some benchmark index. In this context, the cross-sectional dispersion, or *variety*, of returns within the set of permissible securities is the predominant influence on the range of potential outcomes for active management, and hence the risk of underperforming the benchmark. Obviously, if all securities within an investor’s universe produce the same return then the benchmark-relative return for all possible portfolios is zero. Active management is meaningless in such a situation, as no active return is possible, nor is there any active risk to be endured.

The concept of variety has many important uses in investment management practice. Among these is to provide a means of estimating with the correlations among a set of assets with much more limited data than is possible by the usual time series analysis. Another important use of variety is to improve the statistical robustness of the analysis of a manager’s past performance by adjusting for the changing levels of volatility in the market.

The concept of variety also plays other important roles with respect to active management of equity portfolios. Many strategies such as “value” and “momentum” strategies can be viewed as active bets on the magnitude of variety within a given universe of securities. Of potential interest is that by using the variety concept, it is possible to reconcile the “value premium” that has been the subject of voluminous research in equity markets, with the broadly accepted notion of market efficiency.

**Some Applications of Variety**

Solnik and Roulet (2000) consider the use of cross-section dispersion as a means to estimate the correlations of equity market returns across countries. Obviously, the level of correlation is an important component in determining an appropriate level of portfolio diversification and measuring the economic benefits of such diversification. They find the traditional approach of measuring the time series correlation of different markets problematic. Using monthly return data, sample periods must be so long (say five years), that the estimates of correlation do not adapt quickly enough to changes in actual market conditions. However, using more frequent data (e.g. daily) is polluted by the non-synchronous nature of the returns due to the time differences around the world.

To avoid the problems they propose to use the cross-sectional dispersion of the returns across markets as a measure of the lack of correlation of markets. If markets are all moving together, their returns will be similar and the spread of returns achieved across markets in any one period will be small. If the market returns are less correlated we would expect that the variety of returns would be greater in each period observed.

In order to make the concept mathematically tractable, they make simplifying assumption that market return volatilities are constant through time, and hence changes in variety across time periods arises solely from changes in the correlation structure. They first
derive an expression that assumes that every country return has the same correlation to a world index, and an equal level of residual risk, leading to equal total volatility for each market. They then extend this expression to allow different countries to have different correlations to the world index, while retaining the assumption of equal levels of residual risk. Under these assumptions, the correlation between any two assets (markets) is given by Equation 1.

\[ P_{ij} = \frac{1}{\left(1 + \frac{\sigma_e^2}{B_i^2 \sigma_w^2}\right) \cdot \left(1 + \frac{\sigma_e^2}{B_j^2 \sigma_w^2}\right)^5} \]  \hspace{1cm} (1)

Where

- \( P_{ij} \) = the correlation of asset i with asset j
- \( \sigma_e \) = the residual standard deviation of the return for each asset
- \( \sigma_w \) = the standard deviation of the return for the world index
- \( B_i \) = the beta of asset i with respect to the world index

Solnik and Roulet present an empirical analysis of fifteen country returns during the period of 1971 through 1998. They show that the average conditional correlation among countries derived from the cross-sectional dispersion and the world index return, and the average correlation computed via the traditional times series approach agree to the third decimal place, given credence the realistic nature of their underlying assumptions.

Use of the term variety for the cross-sectional dispersion of returns begins with Lilo, Mantegna, Bouchard and Potters (2001). In addition to this syntactic contribution this paper presents the concept of idiosyncratic variety, which they define as the degree of cross-sectional dispersion which cannot be explained by the dispersion of beta values across assets and the contemporaneous realization of return on the market portfolio. They reason that much of the dispersion of returns can be readily explained by the differing levels of systematic risk (beta) across assets. The unexplained portion or idiosyncratic variety is can be used (see Equation 2) to provide an instantaneous estimate of the average correlation across a set of assets, based on a single observation period.

\[ C_t = \frac{1}{\left[1 + \left(\frac{v_t^2}{r_m t^2}\right)\right]} \]  \hspace{1cm} (2)

Where

- \( C_t \) = the average pair-wise correlation across the set of assets during period t
- \( v_t \) = the idiosyncratic variety during period t
- \( r_m t \) = the return on the market portfolio during period t

Implicit here is the assumptions that beta coefficients for assets have already been determined and are stable over time.
deSilva, Sapra and Thorley (2001) provides an extensive survey of the implications of cross-sectional dispersion for active management of equity portfolios. Rather than use variety as a way of inferring the correlation between assets, they derive an expression for the cross-sectional variation of returns across a set of securities, which is used as a metric for the opportunity set available for active managers. In the spirit of Lilo, et al. they derive the following expression for the cross-section variance conditional on a one period market return:

\[
E [D^2] = \sigma^2_B (r_m - r_f) + \sigma^2_e
\]  

(3)

Where

\- D = the cross-sectional standard deviation of asset returns during the single period
\- \sigma_B = the cross-sectional standard deviation of beta values
\- r_m = the return on the market portfolio during the single observation period
\- r_f = the risk-free return during the single observation period
\- \sigma_e = the residual standard deviation of the return for each asset

Equation 3 may be extended to provide an expectation of the return dispersion across the differing portfolios held by a set of active managers. However, the extension requires use of the very common assumption that residual risks are uncorrelated across securities. To the extent we now have a measure for the probable dispersion across manager returns this study proposes that for the performance evaluation, the benchmark excess returns achieved by managers be rescaled to account for heteroscedasticity over a multiple period sample of observations. As the scaling measure for a given period, they propose the ratio of the sample time series average of variety to the one period value of variety for that period. The intuition is that it is easier for a manager to randomly achieve a particular magnitude of benchmark excess return when conditions are noisy (high variety). The proposed adjustment for heteroscedasticity give more weight to excess return observations during low variety periods and place less weight on high variety periods when attempting to evaluate a manager’s true value-added.

**Empirical Research on Variety**

In recent years there has been considerable empirical research that focused on variety. As previously noted, Solnik and Roulet found that the average correlation of equity market returns across countries estimated from a variety measure agreed very closely with the estimated correlation using the tradition calculation over their sample period.

deSilva, Sapra and Thorley provided two important empirical findings in their study. The first is that the degree of cross-sectional dispersion observed during the late 1990s was a historically unprecedented event for equity markets around the world. They speculate that the dramatic increase in variety they observe may be related to either an increase in the general level of firm specific risk or the emergence of a new common factor in security returns (e.g. new economy / old economy divergence) that was not properly accounted for in their specification of common factor risk.
They also studied a large number of US mutual funds from 1981 to 2000, and found that the dispersion across fund returns was highly linearly related over time to the observed variety of security returns. This suggests that fund return dispersion arises from changes in the opportunity set available to active managers, rather than temporal changes in manager aggressiveness or dispersion of skill levels. Interestingly, they find that among US equity managers, the dispersion of fund returns is typically around one fourth as great as for the security universe. This suggests that the diversification level in a typical mutual fund is equivalent to each fund holding an average of sixteen completely uncorrelated securities. Since almost all equity securities in a given market will display some positive correlation to one another, the number of securities needed to achieve this degree of diversity is much greater than sixteen, as is evident from their examination of the actual portfolios.

Campbell, Lettau, Malkiel and Xu (2001) provide an analysis of the changes in the level of security volatility over time. They conclude increased variety levels evident in the late 1990s arose from a combination of both asset specific and industry sector effects. Their analysis suggests that correlations among securities declined as variety increased, as would be consistent with Solnik and Roulet. A final result was that the asset specific returns to securities appeared to increase.

Akrim and Ding (2002) studied the variety among active manager returns across several equity markets. They confirm the result of daSilva, et. al. that the dispersion across manager returns is very closely related to the dispersion of security returns. It was noted that for some markets such as Japan, the high variety of the late 1990s did not represent all time high values, but rather one of a number of high variety periods that have been historically observed. They conclude that the dramatic increase in the magnitudes of variety for equity markets in the late 1990s was driven largely by increased variety across business sectors, with a very large portion of the increase attributable to the behavior of the technology sector.

In contrast to the high observed values of variety in the late 1990s, Senechal (2004) documents a persistent downward trend in variety for the period of 2000 through 2004. His analysis using the Barra E3 risk model of US equities suggests that the explanatory power of the model declined significantly during the late 1990s but subsequently recovered. He shows that during the late 1990s, both increased volatility of factor returns and increased asset specific risk contributed to the upward spike in variety. However, the increase in the asset specific portion was proportionately greater, leading to a large but temporary decline in the in-sample explanatory power of the risk model.

diBartolomeo (2000) presents a formulation of idiosyncratic variety similar to Equation 3, but asserts a different empirical view from Senechal. This paper argues that increases in variety do not arise from an actual increase in the level of asset specific risk, but rather from the emergence of a new factor of common behavior that is not accounted for in the specification of a given risk model. To the extent that risks of the new factor are not accounted for within the risk model being used, the returns attributable to the new factor
are incorrectly presumed to be asset specific (and therefore uncorrelated), when they in fact arise from an unrecognized common element. If any positive serial correlation is present in the returns to the common factors, particularly the unrecognized factor, high magnitude variety will also result. Put differently, there is an important difference between calculating returns that are unexplained by a given model (i.e. residual) and demonstrating that those returns are actually asset specific.

**Variety and Risk Estimation**

The level of idiosyncratic variety (net of market effects) may change over time from three sources. First, the correlations among securities can increase or decrease as described in Solnik and Roulet. Secondly, the degree of security volatility can change as discussed covered in Campbell, et. al. Finally, serial correlation in security returns can be involved, as positive serial correlation will tend to increase dispersion (what goes up keeps going up and what goes down keeps going down), while mean-reversion (negative serial correlation) would drive variety down as things that go up come back down and vice versa.

Even if our empirical research cannot disentangle these effects, the variety measure may still be useful in improving the estimation of future portfolio risk levels. A mathematical treatment of this issue is presented in Hwang and Satchell (2004). For example, let us assume that we are using a common factor risk model that is estimated over some sample period (e.g. the past 60 months). If we have information about the variety of returns during the sample period, a simple scaling similar to the heteroscedasticity adjustment proposed in deSilva, et. al. might allow us to capture more of the short-term dynamics of the risk level and therefore improve ex-ante forecasts over some near horizon. diBartolomeo and Baig (2006) explore one such formulation:

$$E \left[ \sigma_{pc} \right] = \sigma_{pu} \times \left( \sum_{t = n-x}^{n} \left[ \frac{v_t}{x} \right] \right) / \left( \sum_{t = 1}^{n} \left[ \frac{v_t}{n} \right] \right)$$  \hspace{1cm} (4)

Where

- $\sigma_{pc}$ = the risk of portfolio $p$ conditioned on recent variety
- $\sigma_{pu}$ = the unconditional risk of portfolio $p$ from the model
- $v_t$ = idiosyncratic variety for period $t$
- $x$ = the number of periods in the “recent sample”
- $n$ = the number of periods in the full sample

For example, if we set $X = 12$ and $N = 60$, then we are taking the original risk estimate for a given portfolio and scaling it by the ratio of the twelve month average of variety to the sixty month average of variety. If recent variety has been higher than the full sample period, our risk estimate is adjusted upward. If recent variety has been lower than the full sample our risk estimate is lower. A preliminary investigation of this simple adjustment has been promising, with a twenty percent decline in the time series volatility of the extent to which the unconditional risk models overestimated or underestimated the
median level of risk across a broad range of portfolios and time periods. The improvement in risk forecasts for the US market is presented in Figure 1.

Variety as an Explanation of Active Management Styles

One intriguing possible usage for variety is in characterization of active equity management styles. Such approaches to active equity management are described as being oriented toward “value”, “growth” or “momentum”.

Momentum investing is obviously related to the past price movements of a stock in which the investor favors stocks that have had recent price increases. To the extent that security prices exhibit greater volatility than the underlying fundamentals of firms as argued by Shiller (1989), then “value” investing can be summarized as having a preference for stocks that have gone down in price recently. Conversely, a “growth” strategy can be likened to an investor’s preference for stocks that have gone up in price recently, an implicit momentum strategy.

Wilcox (2000) notes that “Price-sensitive active management strategies can be replicated by option payoffs”. Momentum strategies buy stocks on price strength and sell on price weakness. This is similar to a Constant Proportion Portfolio Insurance (Black and Perold, 1992) applied to the cross-section of stock returns. CPPI mimics being long a put option on the underlying asset (plus a long position in the underlying). Option buyers are advantaged when realized volatility (variety in this case) is greater than the volatility expected when the option was established and priced. If momentum strategies are comparable to being long an option, then anti-momentum strategies (i.e. value) must be comparable to being short an option. This suggests that value investors would be advantaged when the realization of variety is lower than expectations when the securities were purchased.

There is precedent for this option-like characterization of active equity management styles among practitioners. Value approaches are often referred to among hedge funds...
and trading desks as “convergence strategies” as they depend on the convergence between the market price and a manager’s definition of the fair price of some security. The process of convergence will be impeded by high levels of noise in the market environment, as evidenced by high levels of variety. Strongin, Petsch, Segal and Sharenow (2002) find value strategies work best when confined within sector (small cross-sectional dispersion), while growth strategies work best with no sector constraints (high dispersion).

To test the assertion that value, growth, and momentum strategies could be thought of as “bets” on the future value of variety with a universe of securities, we conducted some simple experiments also described in diBartolomeo and Baig. Over a sample period of monthly observations from January, 1998 through September of 2002, the values of idiosyncratic variety for the US and UK equity markets were regressed against the spread in returns between the Citigroup Primary Growth and Value indices for the two markets. In both cases, the coefficient was of the expected sign, with a greater than 99% confidence level. Subsequent tests on a larger time sample from January 1990 through December 2005 provided similar results for the US and Global equity markets with confidence levels beyond 95%, while the European coefficient was of the correct sign, but only at the 90% confidence interval. A chart of idiosyncratic variety for the global equity market is presented in Figure 2.

Figure 2

This variety-based characterization of equity styles may also render an important clue in asset pricing theory. Many studies such as Fama and French (1992) have argued that value investors receive higher than average long term returns. If value and momentum investing styles can be said to replicate option, then the distribution of the returns to these styles should also share the skewed nature of option payoffs. If investors are averse to negative skew as suggested in Harvey and Siddique (2000), then markets may be wholly efficient while still offering value investors higher long term returns as compensation for being the risk of a negative skew in the return distribution as is intrinsic in short option positions.
Summary
As long as the practice of measuring investment returns on a benchmark index relative basis is prevalent, the cross-sectional dispersion or *variety* of returns is of paramount importance to investors. The most basic concepts of portfolio diversification revolve around the effective estimation of asset correlations, and the variety measure offers us an important tool in adapting such estimates to current conditions in a timely fashion. Other important uses of the variety measure include improving the statistical robustness of ex post performance analysis and in the forecasting of portfolio risk.

In addition, there is at least a theoretical framework for characterizing popular active equity management styles as involving bets on future level of variety within a universe of securities. Under this framework, a return premium for value investors is economic compensation for enduring negatively skewed returns, and is compatible with a highly efficient market.
References


