

# **Comprehensive Risk and Performance Attribution**

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# Highlights


- A theory of attribution
- Returns-based risk decomposition
- Portfolio-based performance attribution



# 1. Theory

# A new approach to attribution (Feldman, 2007)

- Start with a model of utility maximizing behavior
- Consider a random order model of the relative importance of *the factors* contributing to utility
- Look for a distribution over orders to describe the probability of being ordered by relative importance
- Assume
  - Random order consistency
  - Functional separability
  - Exclusion
- Proportional marginal attribution (PMA) is
  - The unique resulting expectation of factor contributions
  - A powerful and theoretically consistent attribution method

A blurred background image featuring a vibrant rainbow arching over a city skyline. The colors of the rainbow are soft and diffused, blending into the greyish tones of the city buildings. The overall effect is a soft, atmospheric scene.

## **2. Returns-based risk attribution**

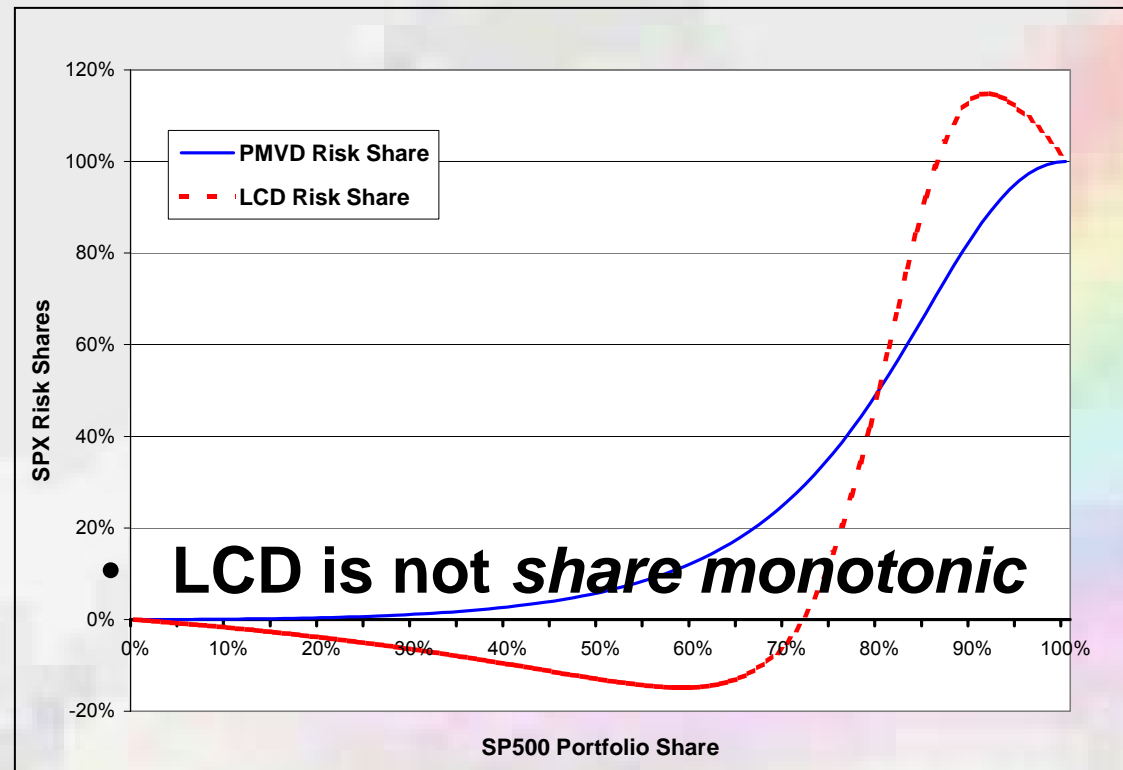
# Returns-based risk attribution / decomposition

- Let the utility function of an analyst be the  $R^2$  of a factor model and use PMA
  - This is PMVD
- Let the utility function of an analyst be based on the  $|R|$  of a quantile regression and use PMA
  - This is PMQD
- *Linear covariance decomposition* or *LCD* is the standard method of variance decomposition
- The key advantage of PMA over LCD and statistical significance measures is that PMA is *designed* to take factor correlation into account

The background of the slide is a blurred image. On the right side, there is a vibrant rainbow with all the colors of the spectrum. On the left side, there is a dark silhouette of a city skyline with several tall buildings. The overall image is out of focus, creating a soft, artistic effect.

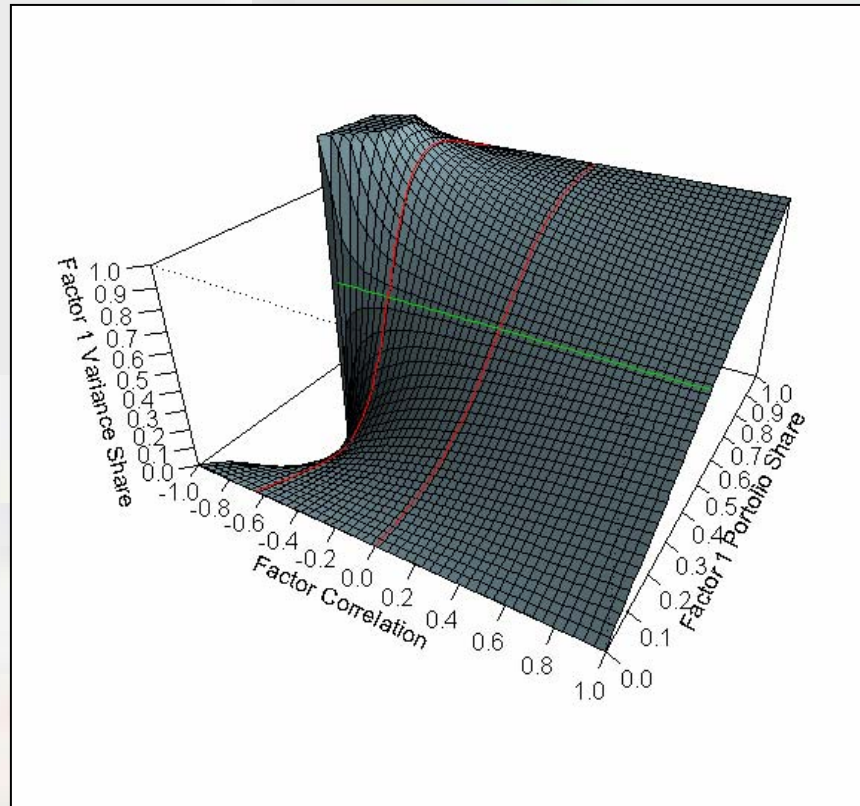
### **3. Variance decomposition: The two-factor case**

# Example: Portfolios of SPDRs and VIX futures



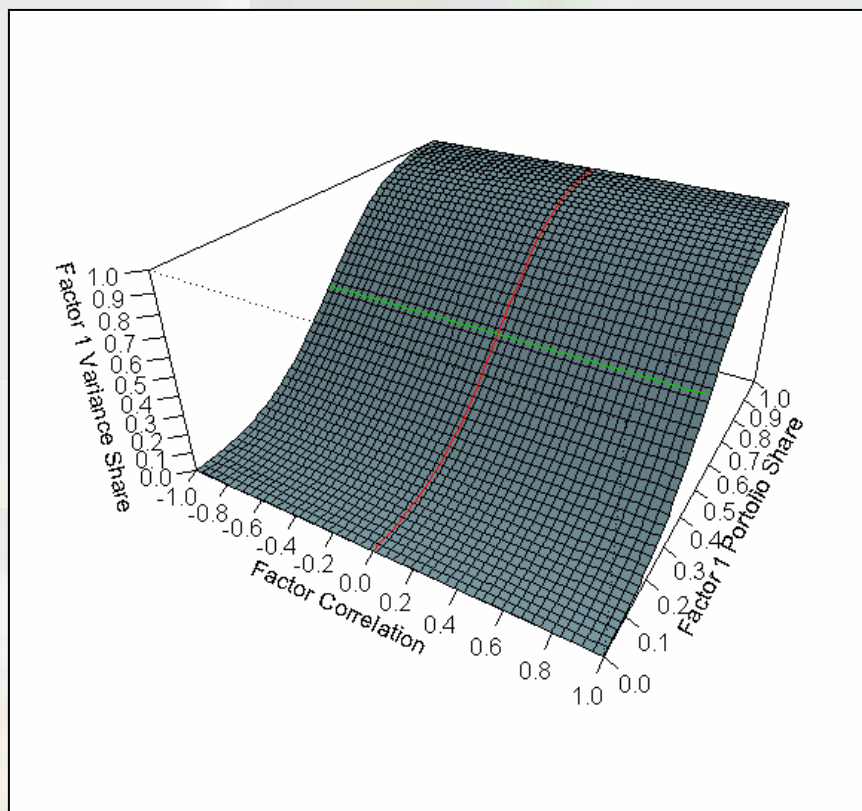
- PMVD SPDR attribution in blue, LCD SPDR attribution in red
- Horizontal axis shows SPDR portfolio percentage
- LCD gives negative attributions and those over 100%
- Correlation between SPDRs and VIX: -0.64

# Standard variance decomposition (LCD) in the general two-factor case

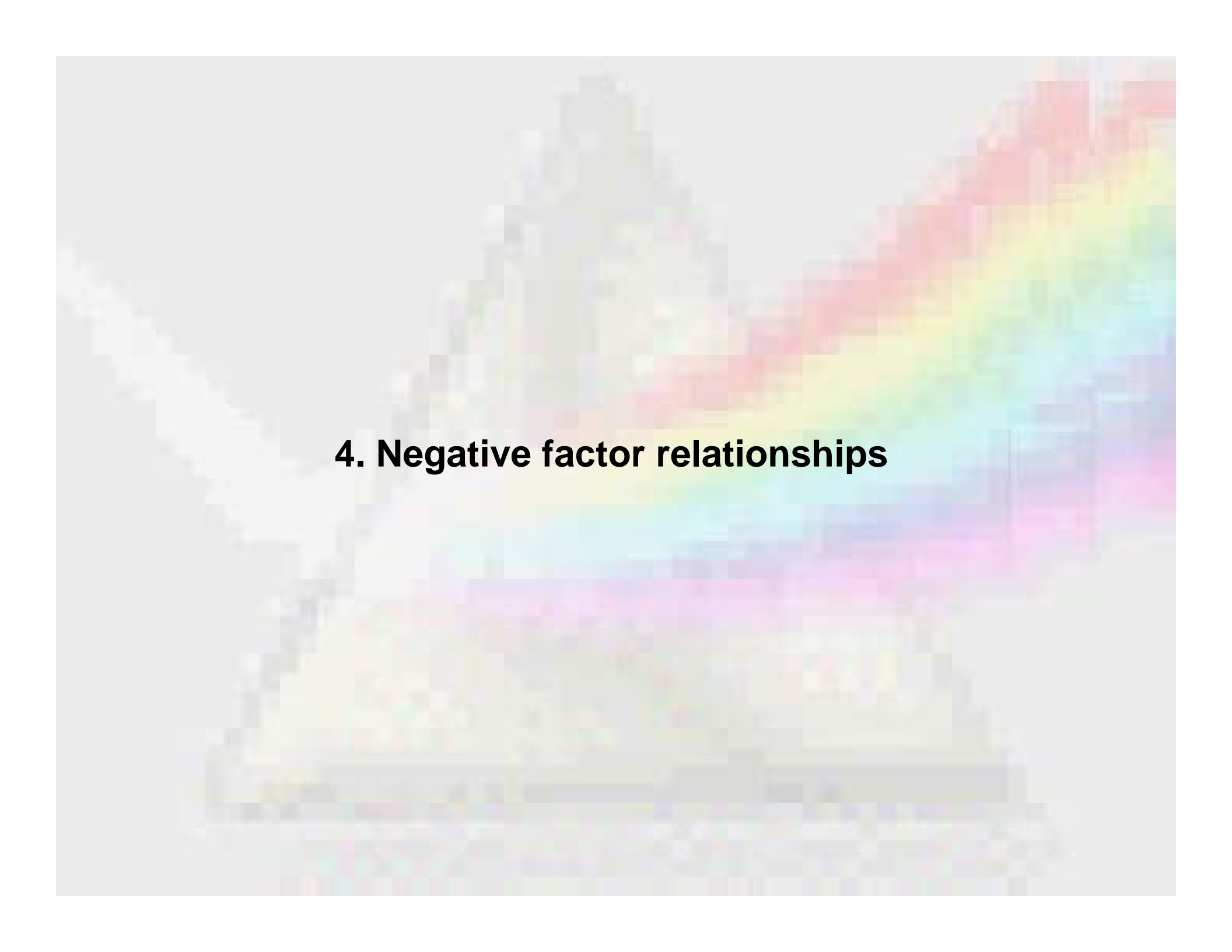


- Attributions diverge to plus and minus infinity as factor correlation approach -1.0

# PMVD in the general two-factor case



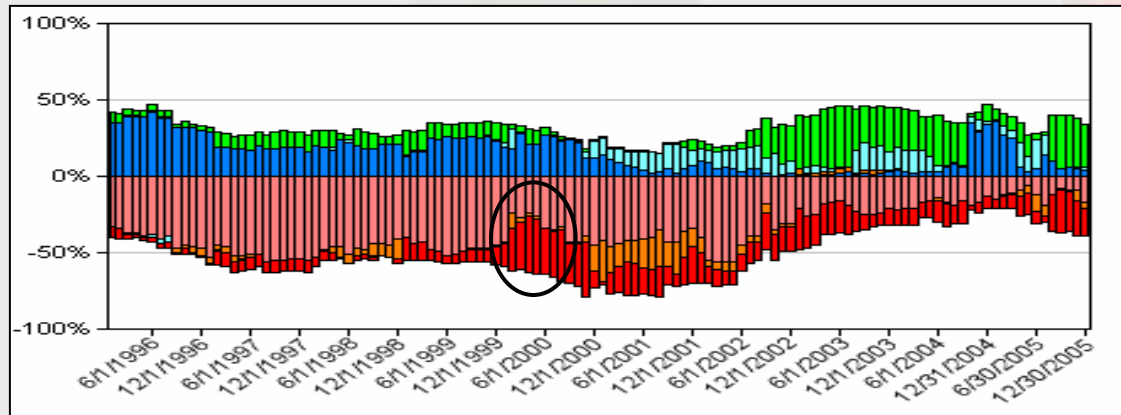
- PMVD attributions are independent of correlation in the two-factor case



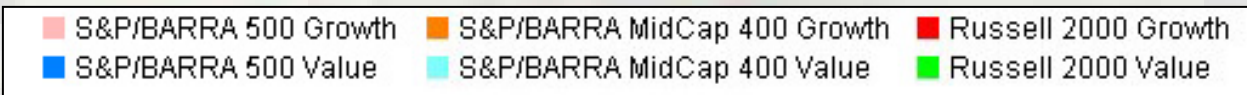
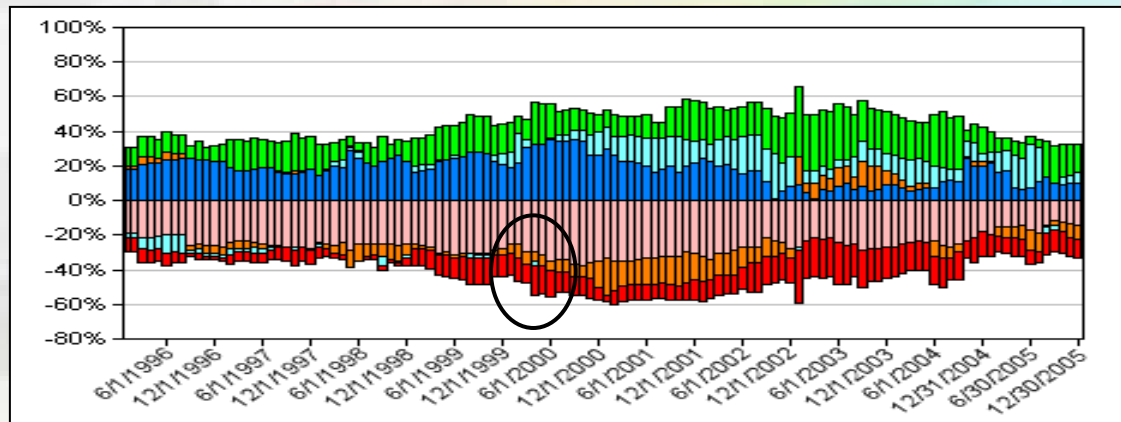
## **4. Negative factor relationships**

# Portable alpha example: A portfolio long the RAFI 1000 and short the Russell 1000

PMVD



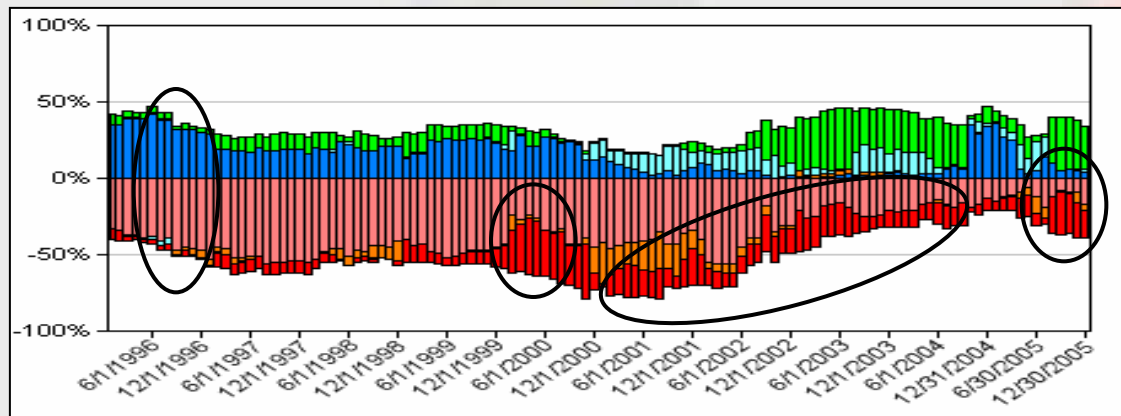
Factor Model



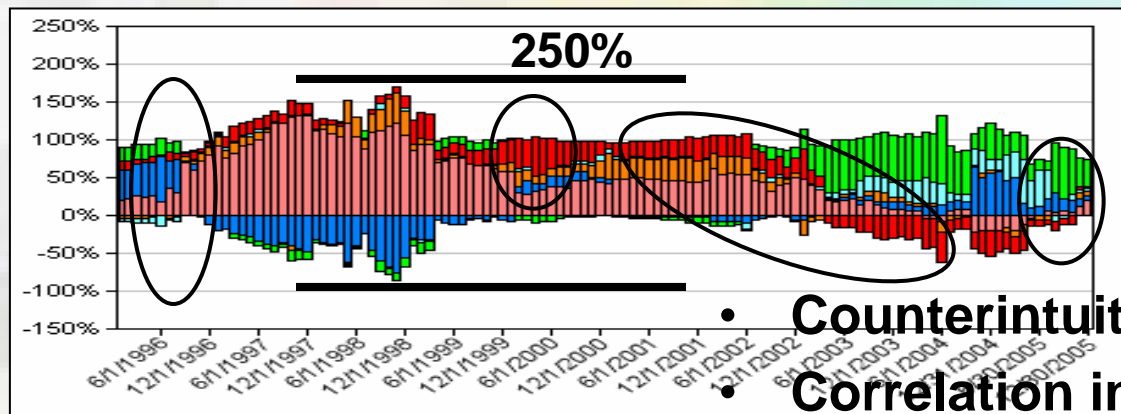
- PMVD exposures consistent with the factor model

# Portable alpha example: PMVD and LCD compared

PMVD

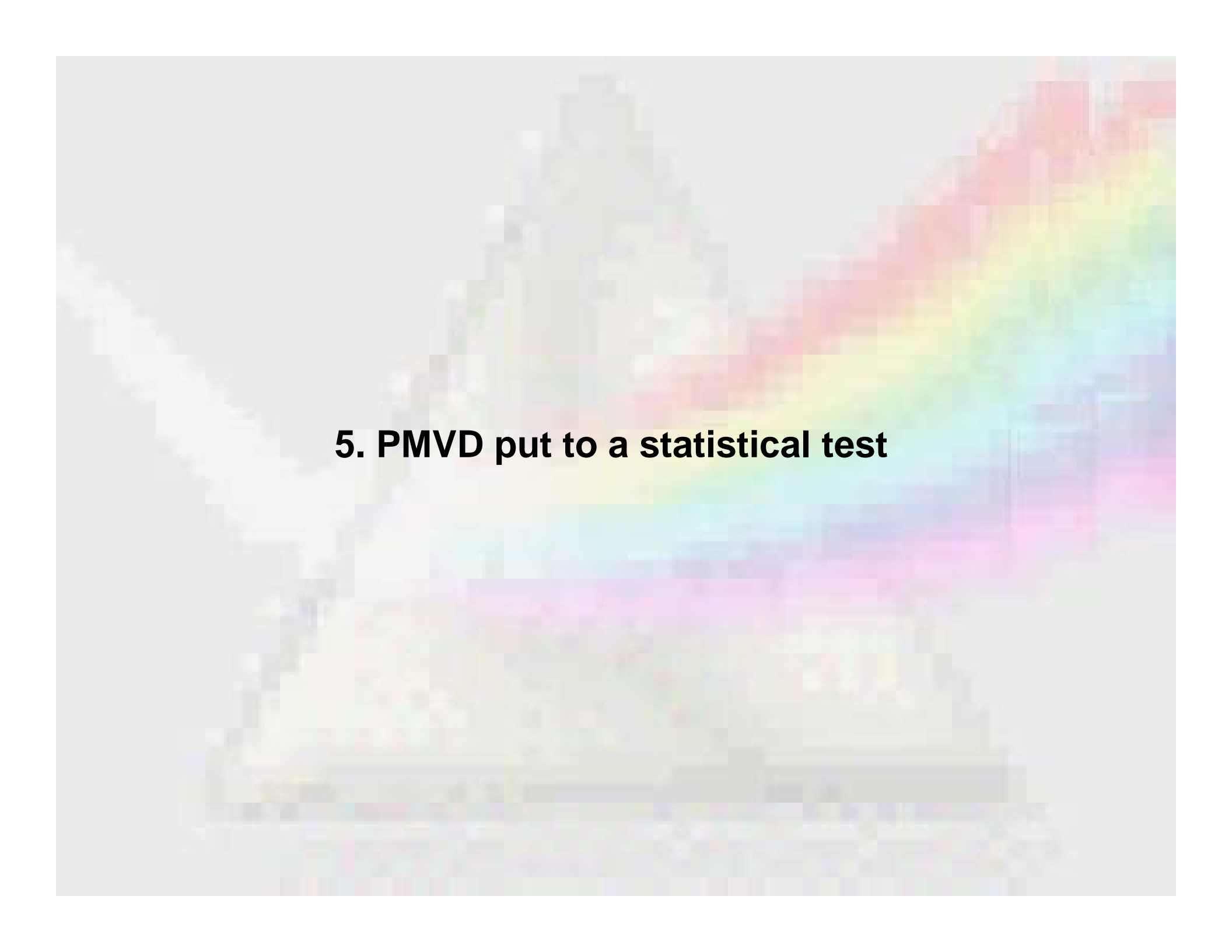


LCD



- Counterintuitive exposures
- Correlation induced bias
- Colinearity inflation

■ S&P/BARRA 500 Growth	■ S&P/BARRA MidCap 400 Growth	■ Russell 2000 Growth
■ S&P/BARRA 500 Value	■ S&P/BARRA MidCap 400 Value	■ Russell 2000 Value



## **5. PMVD put to a statistical test**

# Laudus Rosenberg Long/Short Fund

## Is the LCG PMVD attribution reasonable?

Basic factor model

Factor	Beta	T-Stat	p-value	PMVD
LCG	-18%	-1.23	0.221	-9.2%
LCV	-20%	-0.94	0.351	-6.4%
MCG	36%	2.50	0.015	5.8%
MCV	-20%	-0.88	0.382	-0.9%
SCG	-52%	-3.76	0.000	-26.7%
SCV	36%	1.78	0.079	3.6%
INTL	9%	0.67	0.503	0.7%
EMERG	2%	0.20	0.839	0.1%
HighYld	-8%	-0.47	0.643	-0.1%
LT	16%	1.15	0.255	1.1%
TBILL	-252%	-1.18	0.242	-1.0%

$R^2$ : 55.52%

Combined LC factor

Factor	Beta	T-Stat	p-value	PMVD
LC	-35%	-2.11	0.038	-16.6%
MCG	36%	2.59	0.011	6.2%
MCV	-22%	-1.12	0.265	-1.2%
SCG	-52%	-3.83	0.000	-25.1%
SCV	37%	1.87	0.066	2.8%
INTL	9%	0.64	0.522	0.8%
EMERG	2%	0.22	0.829	0.1%
HighYld	-9%	-0.52	0.608	-0.2%
LT	16%	1.24	0.219	1.2%
TBILL	-254%	-1.21	0.232	-1.0%

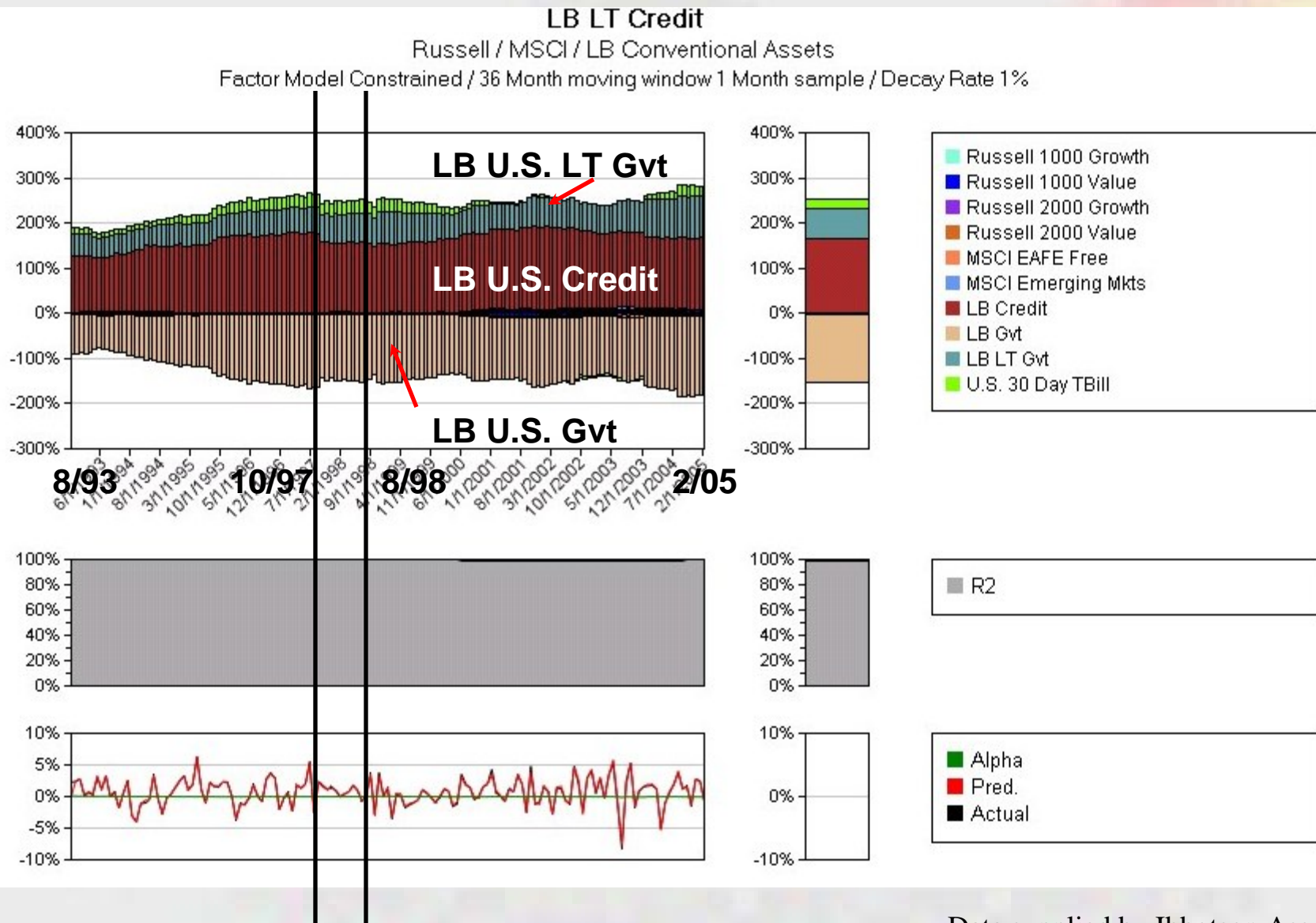
$R^2$ : 55.19%

- $F$ -test constraint  $p$ -value = .47
- Correlations degrade LCG stat. sig. levels



## 6. Extreme positive colinearity

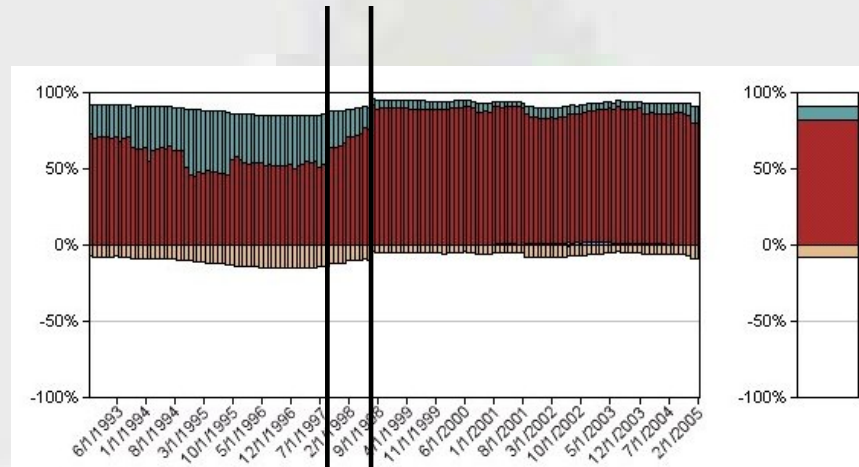
# Example from Feldman (2006): LB US Long Term Credit



Data supplied by Ibbotson Associates.

# PMVD shows evidence of a 1997-1998 structural break

PMVD



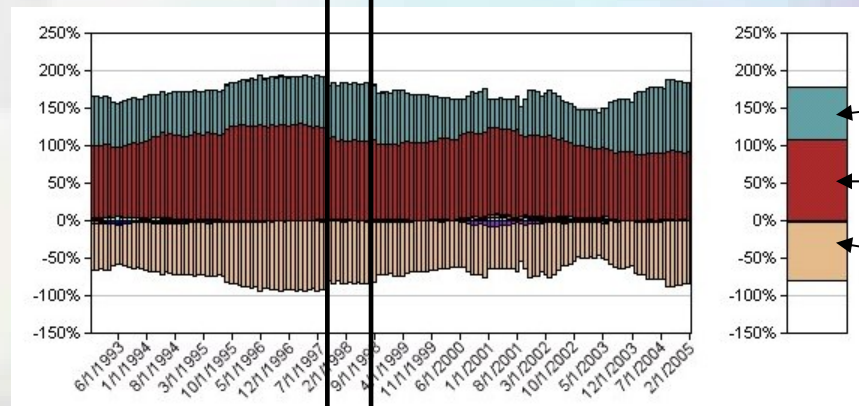
8/93

10/97

8/98

2/05

LCD

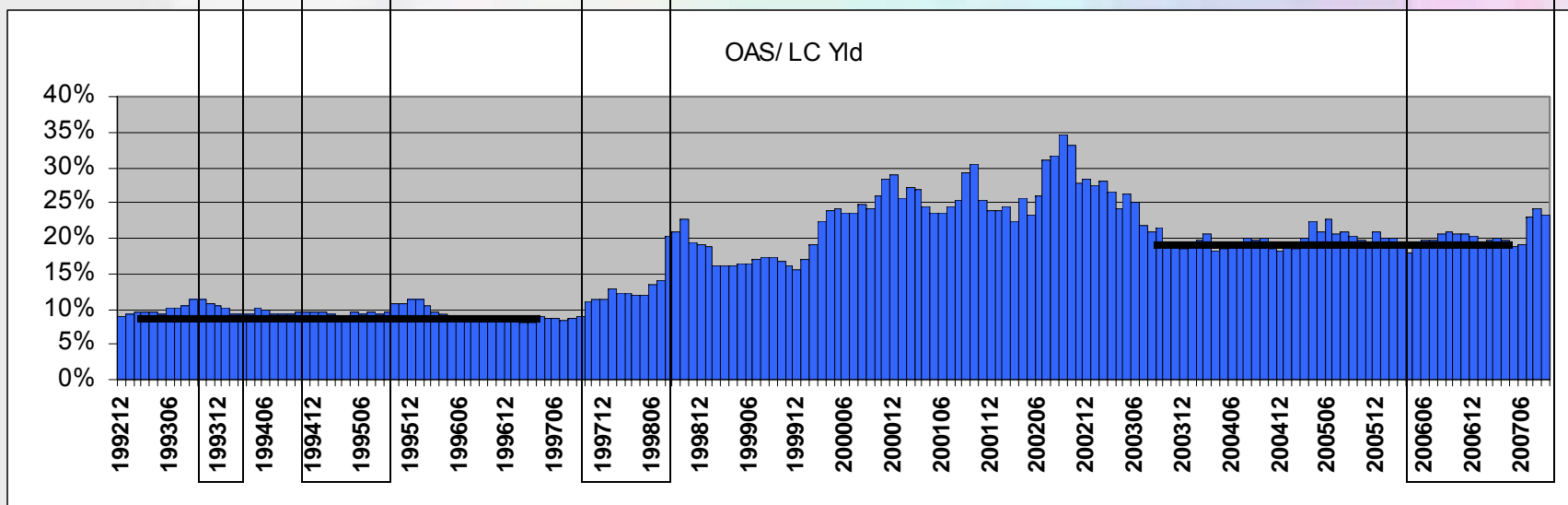
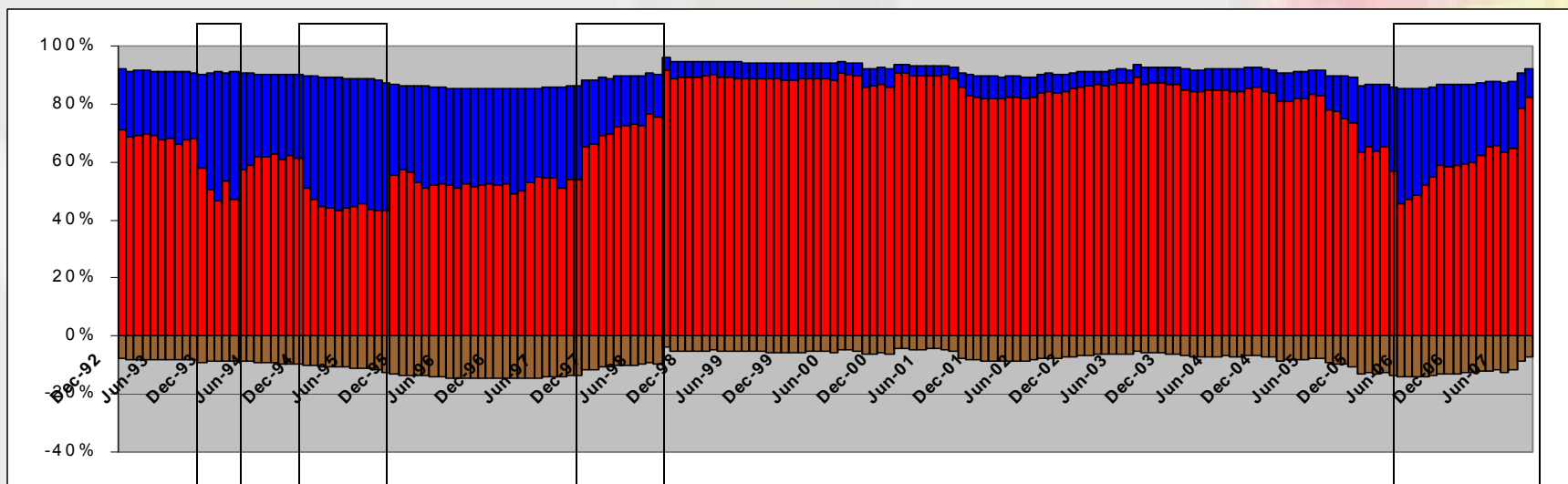


LB U.S. LT Gvt

LB U.S. Credit

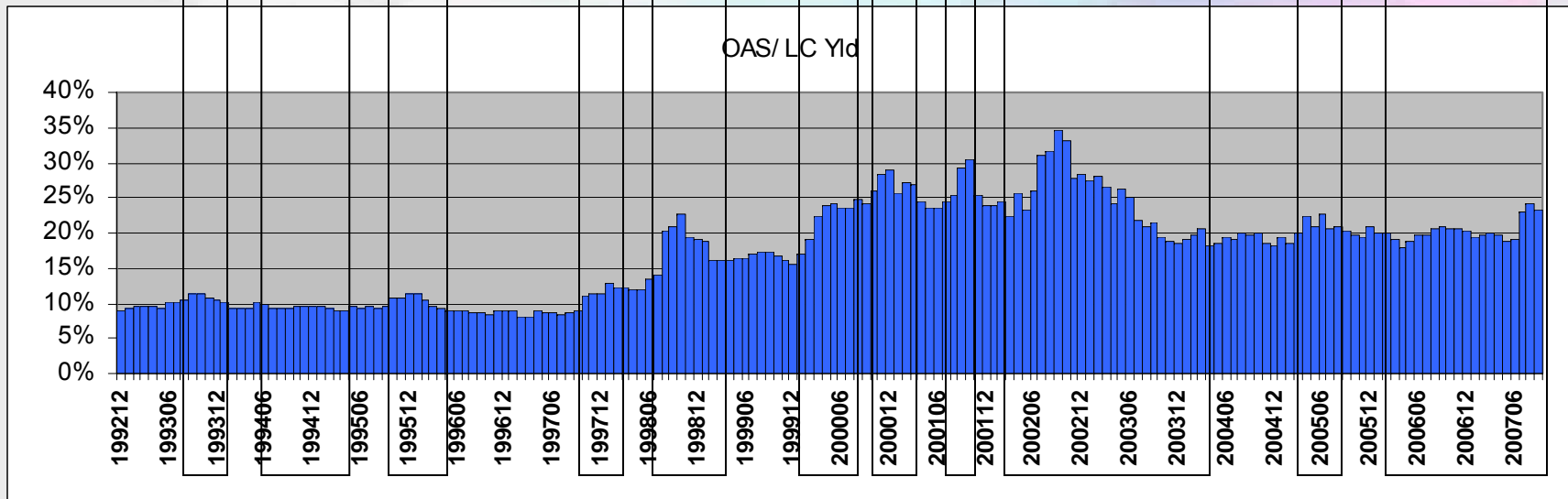
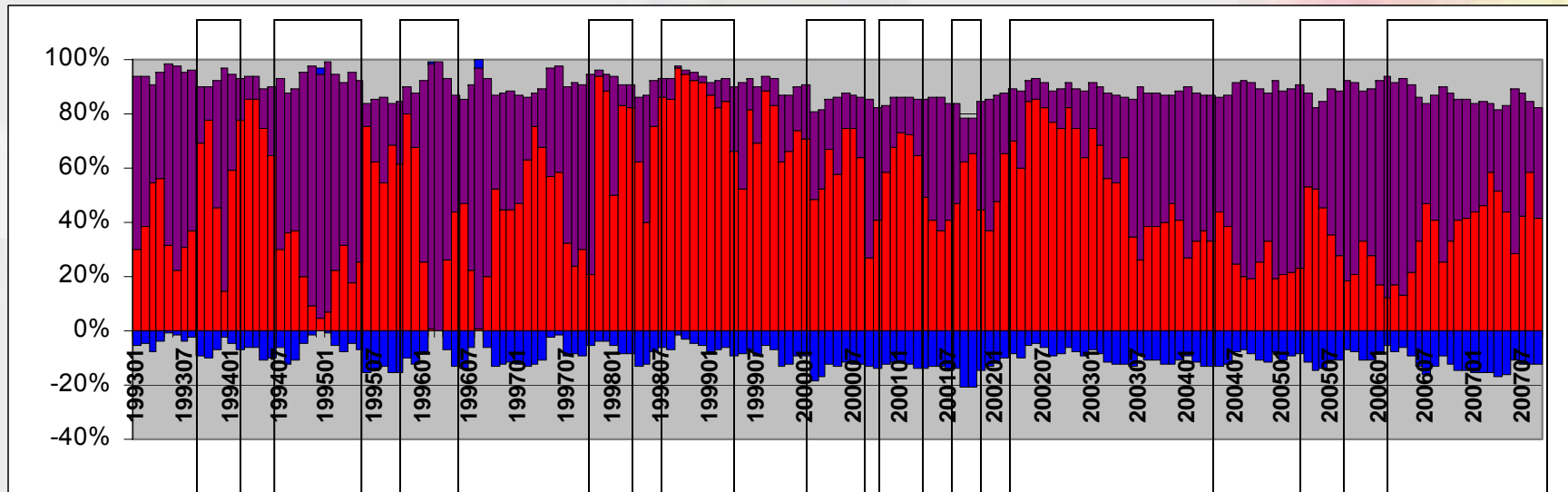
LB U.S. Gvt

# PMVD: December 1992 to August 2007



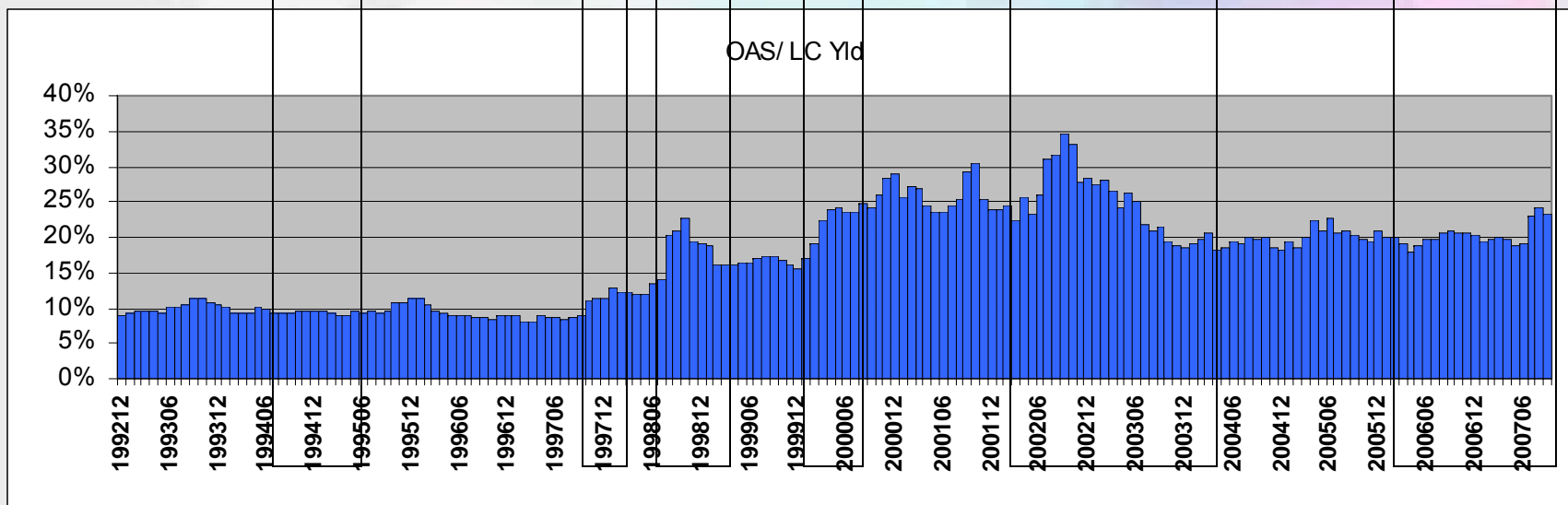
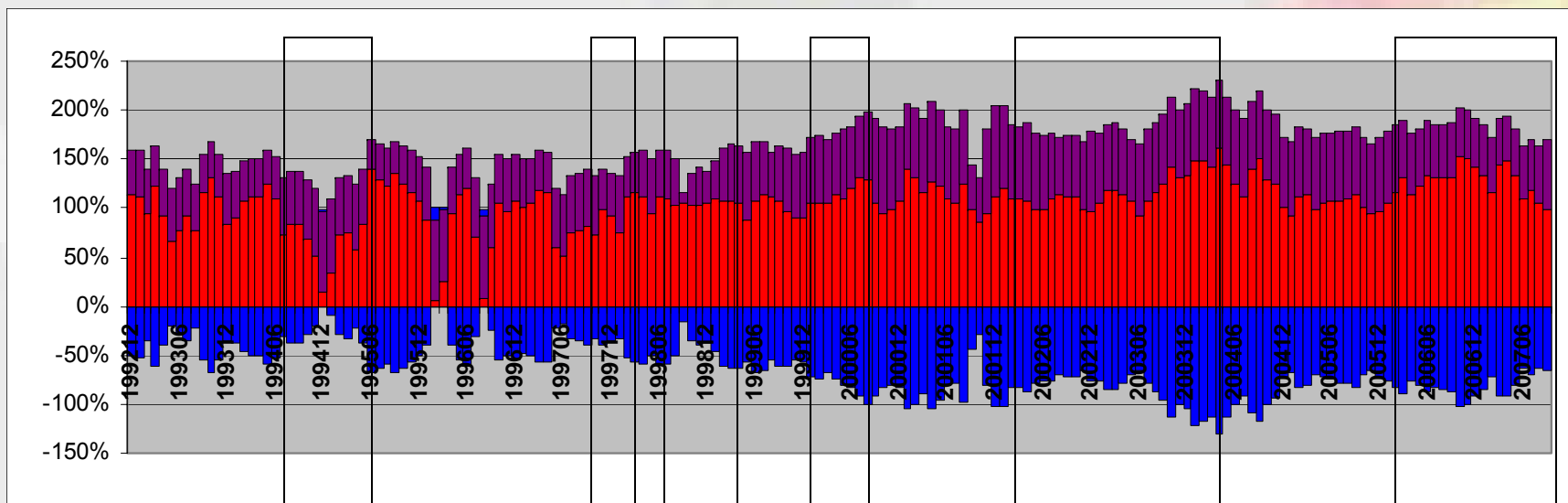
Data courtesy of Morningstar.

# PMVD analysis based on daily data



Data courtesy of Morningstar.

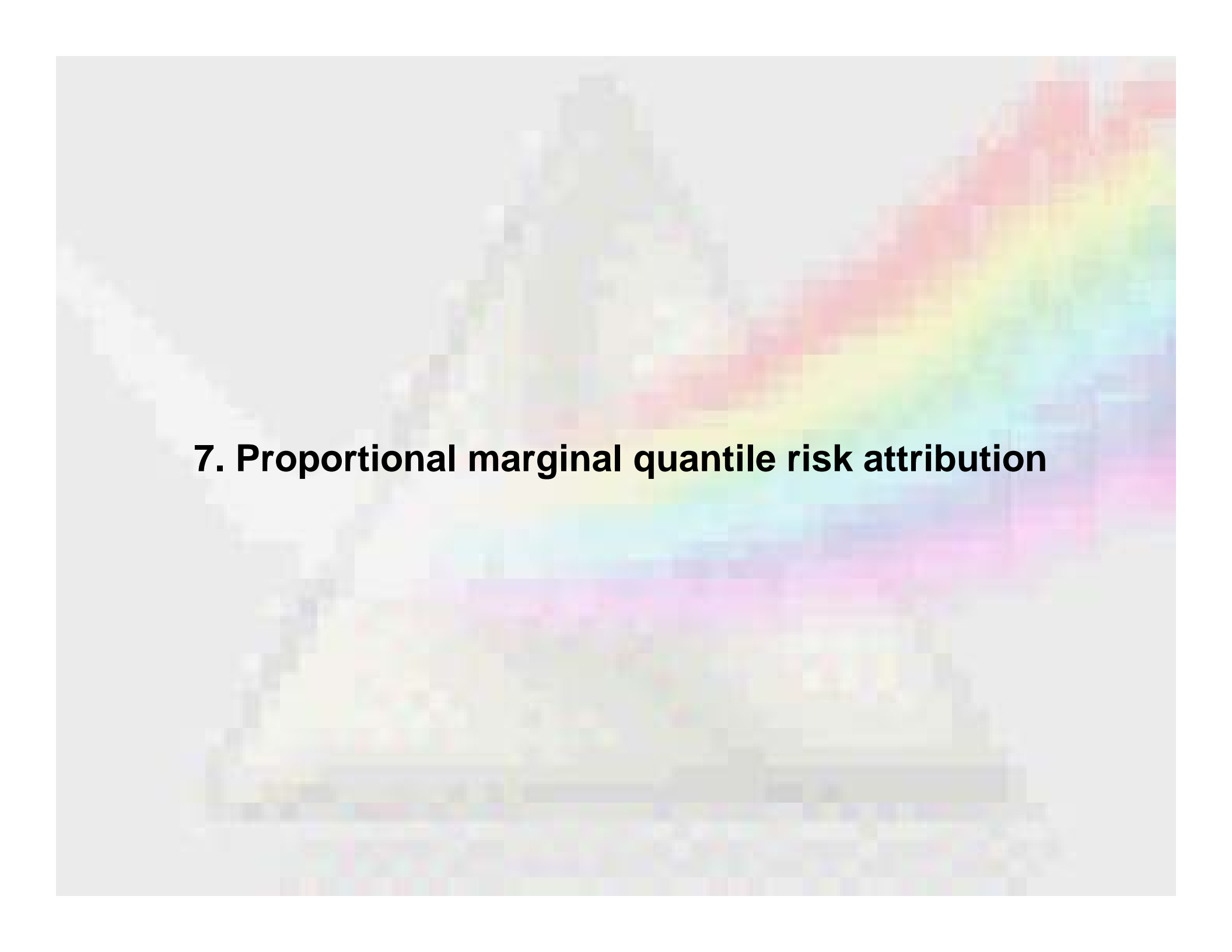
# LCD with daily data



Data courtesy of Morningstar.

## Long-term credit example conclusions

- PMVD appears to capture information in credit markets, sometimes before it is reflected in spreads
- Many historical events clearly identified
- Other factors also reflected in PMVD results
- Noise likely also present (Gromping, 2006)
- LCD appears to capture little information

The background of the slide is a blurred image. On the right side, there is a vertical rainbow with colors transitioning from red at the top to purple at the bottom. On the left side, there is a dark silhouette of a city skyline with several tall buildings. The overall background is a light, hazy grey.

## **7. Proportional marginal quantile risk attribution**

# Quantile regression

- Koenker and Bassett (1978) and Bassett and Chen (2001)
- Exact analogy to least squares
- Estimate conditional quantile (e.g. median) instead of conditional mean
- Minimize the sum of absolute quantile deviations
- Attribution approach is exactly analogous to OLS
- Operational difference with OLS: choice of quantile

## PMQD example: Analysis of CTA performance

- Based on Premia Capital Management daily P/L for 2005

Sector	PMVD	Quantile		
		1%	50%	99%
Agriculture	20%	66%	16%	10%
Energy	13%	31%	10%	58%
Metals	19%	1%	26%	7%
Livestock	37%	2%	34%	5%
Precious	1%	0%	8%	19%
UST5TR	10%	1%	7%	0%
R1 or R2	1.9%	24.5%	1.5%	3.2%

- PMVD and median quantile PMQD components similar
- Agriculture and energy dominate 1% quantile downside risk
- Energy dominates 99% quantile upside potential
- Metals and livestock show median quantile risk but little tail risk

P/L data courtesy of Premia Capital Management.

The background of the slide is a blurred image of a city skyline with a prominent rainbow arching across the sky. The colors of the rainbow are visible but soft and out of focus. The city buildings are also blurred, creating a bokeh effect. The overall tone is bright and colorful.

## **8. Portfolio performance attribution**

# Attribution of portfolio performance

- Use a utility function to quantify the benefit of historical or forecast investment performance of an optimized portfolio of assets
  - Determine the utility share of each asset
  - Divide the utility share of an asset by its portfolio weight
- This is proportional marginal performance attribution (PMPA)

# Standard portfolio optimization problem

MANAGER	Historical Return	Return Adjustment	Adjusted Return	Std. Dev.	Annualized Sharpe Ratio
DFA U.S. Large Company Institutional	0.90%	0.40%	1.30%	3.34%	1.19
Robeco WPG 130/30 Large Cap Core Intl	0.75%	0.60%	1.35%	3.93%	1.06
Bridgeway Ultra-Small Company	1.96%	-0.20%	1.76%	5.44%	1.02
ProFunds Real Estate UltraSector Svc	1.67%	-0.30%	1.37%	6.78%	0.63
AllianceBernstein Intl Growth C	1.70%	-0.10%	1.60%	3.93%	1.28
Oppenheimer Emerging Growth A	1.28%	0.30%	1.58%	6.23%	0.80
Matthews China	2.15%	-0.50%	1.65%	5.02%	1.04
Putnam High Yield M	0.87%	-0.10%	0.77%	1.43%	1.51
Vanguard Long-Term Bond Index	0.55%	0.25%	0.80%	2.57%	0.88
Mellon Short Term U.S. Govt Secs M	0.20%	0.20%	0.40%	0.39%	2.24

- 10 Managers
- Historical returns, standard deviations and correlations based on 60 months of date to June 2007
- Return adjustments constitute one relatively optimistic assessment of the coming year
- MVO: Quadratic utility with  $\lambda=10$

# Performance attribution example: Assets and performance characteristics

MANAGER	MVO Portfolio Allocation	PMA	PMPA	Annualized Sharpe Ratio
DFA U.S. Large Company Institutional	5.2%	6.3%	1.20	1.19
Robeco WPG 130/30 Large Cap Core Intl	11.8%	13.1%	1.11	1.06
Bridgeway Ultra-Small Company	8.5%	7.2%	0.85	1.02
ProFunds Real Estate UltraSector Svc	1.9%	0.3%	0.17	0.63
AllianceBernstein Intl Growth C	15.7%	16.3%	1.03	1.28
Oppenheimer Emerging Growth A	0.0%	0.0%	--	0.80
Matthews China	21.7%	25.0%	1.15	1.04
Putnam High Yield M	0.0%	0.0%	--	1.51
Vanguard Long-Term Bond Index	32.6%	31.3%	0.96	0.88
Mellon Short Term U.S. Govt Secs M	2.7%	0.6%	0.21	2.24

- PMPA: Proportional marginal portfolio attribution
- PMPA takes manager correlations to the portfolio into account
- PMPA results surprisingly similar to Sharpe ratio for some managers



## 9. Limitations

# PMA Limitations

- Computing time
  - Increases sharply with number of factors
  - Solution: approximation methods
- Attribution precision
  - Examined in Feldman (2005) and Gromping (2007)
  - PMA is estimated consistently
  - Bootstrapped confidence intervals

A large, colorful, pixelated rainbow graphic that serves as a background for the text. The rainbow is composed of many small, square pixels in various colors, creating a vibrant, multi-colored effect. The colors transition from red at the top to purple at the bottom, following the traditional rainbow spectrum. The background is a light gray color.

**10. Your turn**



## 11. Wrap up

# Get more information from your data

- PMA methods have a strong theoretical foundation
- PMA is share monotonic
- PMA methods work
- LCD and statistical significance measures have serious limitations as risk attribution methods
  - LCD and statistical significance measures are vulnerable to correlation-driven risk attribution distortions
  - LCD risk exposures can be completely suppressed

# References

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