

# Estimating Returns for Asset Allocation

Northfield Asset Allocation Seminar  
September 2004

# Expected Returns

- ◆ Optimum portfolio weights are very sensitive to return expectations.
  - Small changes in expected returns → large weight changes.
- ◆ Historic returns are often used, but they are not usually good predictors of future returns.
  - Domestic Equity returns are upwardly biased by an expansion of P/E ratios.
  - Bond returns are upwardly biased by decreases in interest rates.
  - Active returns are upwardly biased because of survivorship bias, particularly with hedge funds.

## Literature: Historic and Expected Returns

- ◆ Bradford Cornell, *The Equity Risk Premium*, 1999
- ◆ William Bernstein, *The Birth of Plenty*, 2004
- ◆ Jeremy Siegel, *Stocks for the Long Run*
- ◆ Dimson, Marsh, & Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns*
- ◆ Arnott and Bernstein, "What Risk Premium is Normal," *Financial Analyst Journal*, Mar/Apr 1992
- ◆ Dempsey, "The Nature of Market Growth, Risk and Return (DOGMA)," *Financial Analyst Journal*, May/June 2002
- ◆ Ibbotson & Chen, "The Supply of Stock Market Returns," [http://icf.som.yale.edu/pdf/Supply\(v5\).pdf](http://icf.som.yale.edu/pdf/Supply(v5).pdf)
- ◆ Harvard Business School Case Study: "The Harvard Management Company (2001)"

# Inflation Adjusted U.S. Stock Market Returns

## Jeremy Siegel: Stocks for the Long Haul

Time Period	Dividend Yield	Compound Total	Standard Deviation
		Yearly Return	Annual Returns
1802-1870	6.4%	7.0%	16.9%
1871-1925	5.2%	6.6%	16.8%
1926-1997	4.6%	7.2%	20.4%

# Historic Returns

- ◆ When calculating historic returns, even long histories show returns that are much higher than what probably can be expected in the future.
- ◆ The US is the “surviving economy” and was an emerging market in 1802, the point at which real per capita GDP growth began to grow at 2% for what is now the “developed world.”

# Capital Asset Pricing Model

- ◆ Expected Return is:

Expected Return = Risk Free Rate +  $\beta$  \* Market Return

- ◆ What is the Risk Free Return for a reasonable investment horizon:

- Risk Free Return = Inflation + Embedded Productivity
- Market Return: The “equity risk premium”
- Beta = Systematic risk, proportional to covariance between asset and market portfolio.

# What is the Market Portfolio

- ◆ It should include the entire US stock market. The Russell 3000 or Wilshire 5000 are reasonable proxies. The MSCI World index does not include small cap stocks.
- ◆ It should include corporate bonds, since the market should be (mostly) independent of the market's capital structure. Analogous to Modigliani Miller.
- ◆ It should include foreign stocks since they are part of a well diversified portfolio.
- ◆ It should *not* include government or agency bonds, since they are related to the supply and demand for current vs. future consumption. See Robert Fuhrman, Northfield Newport Conference 2004.

# Expected Returns Using CAPM Model

$$R_f = 4\%, R_m = 3.5\%$$

Asset Class	Beta	Expected Return
US Bonds	0.12	4.42
Non US Bonds	0.19	4.67
US High Yield Bonds	0.34	5.19
US Value Stocks	0.99	7.47
Non US Stocks, Developed	1.23	8.31
Emerging Market Stocks	1.26	8.41
US Growth Stocks	1.39	8.87

# Bayes-Stein Adjustment

- ◆ The reduction in errors arising from Bayesian shrinkage estimators is a mathematically provable result.
- ◆ Bayesian shrinkage jointly minimizes the errors in the return expectations over the portfolio, rather than trying to minimize the errors in each asset class return expectation separately.
- ◆ The “inadmissibility” of sample mean as an estimator for multivariate (portfolio) problems was proven by Stein in 1955.

# Bayes-Stein Methods

- ◆ We need to distinguish between using these techniques:
  - To explicitly compensate for “error maximization” in mean-variance optimization.
  - For improving the quality of return forecasts that we make. This technique provides better numbers for what we actually believe, but requires providing more information than just historical data.

# Literature: Bayesian Adjustment

## ◆ Compensating for Error Maximization

- <http://www-gsb.uchicago.edu/fac/arnold.zellner/more/CURRENT-PAPERS/bayshrin.pdf>
- Jorion, “International Portfolio Diversification with Estimation Risk,” *Journal of Business*, July 1985
- Jorion, P., “Bayes-Stein Estimation for Portfolio Analysis,” *Journal of Financial and Quantitative Analysis*, September 1986
- Jorion, P., “Bayesian and CAPM Estimators of the Means: Implications for Portfolio Selection,” *Journal of Banking and Finance*, June 1991

## ◆ Requires Additional Data:

- Black and Litterman, “Global Portfolio Optimization,” *Financial Analyst Journal*, 1992
- He and Litterman, “The Intuition Behind Black-Litterman Model Portfolios,” Goldman Sachs

# Bayesian Priors

- ◆ There are really two dimensions of priors.
  1. Whether the prior is diffuse or centralized.
  2. Whether the prior is “uninformative” or “informative” or somewhere in between: “semi-informative.”
- ◆ Imagine we have some historical data for 3 asset classes: stocks, bonds, cash.
- ◆ The sample period return for stocks is -3%, bonds 4%, cash 9%.

# Diffuse Priors

- ◆ A diffuse prior would be one where the prior could take on a variety of values.
- ◆ Imagine we just picked some random number between -3 and 9. This would be a diffuse and uninformative prior.
- ◆ This is the technique Markowitz and Ussman used in their recent paper that compared Bayesian methods and re-sampling.

# Semi Informative Prior

- ◆ A “slightly informative” prior would be to assume that the prior was random, but normally distributed between 0 and 9 (implying mean 4.5)
- ◆ It is economically irrational for any investor to invest in a risky asset with negative expected (excess) returns.

# Centralized Prior

- ◆ Let's say the government regulated interest rates on bonds to always be 6%.
- ◆ In such a case, it might be sensible to always use 6% as a prior. The prior would have a single value rather than a distribution.
- ◆ Grinold's alpha scaling rule of thumb is very similar to a Bayesian shrinkage where  $W = IC$  and  $U_p = 0$  (a single fixed value).

# Time Horizon

- ◆ While we normally talk about “long term” returns in asset allocation, real world investors usually have intermediate horizons over which they view things.
- ◆ For example, a 30 year zero coupon Treasury bond has wildly volatile annual returns but the return over 30 years is known with a high degree of certainty if we hold one bond all the way.

# Jorion's Bayes-Stein Adjustment

All Bayesian estimates have the form:

$$U_{bs} = W (U) + (1-W) U_p$$

where:

$U_{bs}$  = the Bayes adjusted expectation of return

$W$  = is the weight given to the return based on the data we observe

$U$  = return based on the data we observe (history)

$U_p$  = the return "prior"

# Stein Shrinkage Estimator

Jorion's Technique estimates W:

$$W = 1 - [(n-2) / (T-n-2)] - 1 / [(U-U_p)^T Q^{-1}(U-U_p)]$$

where:

T = number of time periods of data

n = number of asset return time series

Q = covariance matrix of return time series

# Single Value Prior

- ◆ Jorion's method that has a single value prior for all asset classes addresses estimation risk in optimization.
- ◆ Our actual expectations have not changed based on the Bayesian adjustments
- ◆ This is the way our beliefs ought be presented to an optimizer. Rationale:
  - When our beliefs about returns are uncertain, the uncertainty makes the asset classes less distinct from one another.
  - Efficient return estimators shrink the return spreads amongst the asset classes.

# Jorion's Bayes-Stein Adjustment

- ◆ The prior is the historical return on minimum variance portfolio formed of all the assets, excluding a risk free asset.
- ◆ If you have risky assets, the minimum return you would expect from any one of them individually would be the return that could be achieved by investing in the portfolio that yields the lowest risk.

# Bayesian Adjustment: Risks

- ◆ Once we've adjusted the means away from the sample means, the standard deviations will increase slightly
- ◆ The adjusted correlations *may* also differ *slightly* from the sample values. In practice, correlations do not change when using two significant digits.
- ◆ From an optimization standpoint, we are biasing the covariance matrix to a slightly more conservative posture.

# Jorion's Adjustment 1979-2004

Asset Class	Historic Return	Bayesian Return	Risk
Cash	4.94	4.93	0.58
US Bonds	8.62	7.60	4.54
Non US Bonds	10.60	9.04	10.46
US Equity	13.18	10.91	15.83
Non US Equity	10.08	10.08	17.76

# Currency Hedge Non US Bonds

Asset Class	Historic Return	Bayesian Return	Risk
Cash	4.93	4.93	0.58
US Bonds	8.62	7.61	4.54
Non US Bonds	<i>8.10</i>	<i>7.23</i>	<i>3.66</i>
US Equity	13.18	10.92	15.83
Non US Equity	10.08	10.09	17.76

# Bayesian Adjustment

- ◆ Bayesian Adjustment reduces expected return for bonds by about 1%.
- ◆ Bayesian Adjustment reduces expected return for stocks by about 3%.
- ◆ US \$ depreciation caused Non US Bonds.
  - 2% extra return
  - Increase in volatility 3.6 to 10.5

# Informative Priors

- ◆ Black-Litterman uses an informative prior, where we are bringing a lot of new information into the problem: in this case the CAPM.
- ◆ We estimate what we think the values of the returns should be, given each asset's weight in the global market portfolio

# Black-Litterman Model

- ◆ This model is used for active asset allocation at Goldman-Sachs: as described in He and Litterman, “The Intuition Behind Black-Litterman Model Portfolios,”
- ◆ The model’s key action is the estimation of the vector of implied returns based on estimations of the:
  - Market’s risk tolerance
  - The weights of each asset in the market portfolio
  - The covariance between the assets in the market

# Black-Litterman Model

- ◆ An asset's expected returns should vary from the implied returns depending on the relative outlook for that asset and the confidence in the active prediction.
- ◆ The implied returns are calculated:
  - $\Pi = S w / \lambda$
  - $\Pi$  is the vector of implied returns
  - $w$  is the portfolio weight vector
  - $S$  is the covariance matrix
  - $\lambda$  is the market's risk tolerance parameter (RAP)

# Black-Litterman Return Estimation

Asset Class	Weight in Market Portfolio	Implied Return
Cash	0	4.56
US Bond	20	5.00
Non US Bond	30	5.11
US Stock	50	8.02
Non US Stock	20	7.89

# What About Actively Managed Funds?

- ◆ Use style analysis vs. benchmarks for which you have “good” expected return estimates.
- ◆ Use the style factor weighted average of the asset class returns to establish a baseline expected return.
- ◆ Adjust the alpha for selection (you are not likely to pick something that hasn’t been good) and survivorship bias.
- ◆ Add the alpha to the expected returns due to beta.

# Conclusions

- ◆ Historic returns, even over long time histories, are poor predictors of future (and hence expected) returns.
- ◆ Bayesian adjustment accounts for the uncertainty of return forecasts. Jorion's Bayes Stein adjustment and Black-Litterman are both examples of Bayesian adjustment.
- ◆ CAPM and Black-Litterman are equilibrium models to estimate expected returns from historic covariance data.