Optimal Trading Strategy
With Optimal Horizon

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Trading – An Integral Part of Investment Process

- Alpha forecasting
- Portfolio construction
- Trading – portfolio implementation
- Performance attribution
Conflicting Objectives in Trading

Immediacy and costs
- Alpha capture
- Risk reduction
- Labor costs
- Opportunity costs
- Market impact
Optimal Trading Strategies

- Optimal trading path (sequence) with minimum costs for a given level of risk
  \[ h^*(t), t \in [0, T], \quad T \text{ is the trading horizon}. \]

- Previous researches (Grinold & Kahn 1999, Almgren & Chriss 2000) used a fixed horizon \( T \)

- Extension to optimal trading strategy with optimal horizon (Qian 2008 JOIM, Qian, Hua, Sorensen 2007)
  \[ h^*(t), t \in [0, T^*]. \]
**Optimal Horizon**

- Horizon is not known in advance
  - Single stocks versus baskets
- It is optimal along two dimensions
- Flip-floping in optimal trading with fixed horizon
Mathematical Model - Inputs

➤ **Trade weight** \( \Delta w \) and **trade path** \( h(t) \Delta w, \ h(0) = 0 \) and \( h(T) = 1 \)

➤ **Trade shortfall** \( h(t) \Delta w - \Delta w = \Delta w [ h(t) - 1 ] \)

➤ **Return shortfall** \( f \Delta w [ h(t) - 1 ] \, dt \)

➤ **Shortfall variance** \( \sigma^2 (\Delta w)^2 [ h(t) - 1 ]^2 \, dt \)

➤ **Fixed cost** \( c |\Delta w| T, \ c > 0 \)

➤ **Market impact** \( \psi (\Delta w)^2 [ \dot{h}(t) ]^2 \, dt, \psi > 0 \)
Mathematical Model – Objective Function

- Find path and horizon $h^*(t), t \in [0, T^*]$, that maximize

$$J = \int_0^T f \Delta w \left[ h(t) - 1 \right] dt - \frac{1}{2} \sigma^2 \int_0^T \left( \Delta w \right)^2 \left[ h(t) - 1 \right]^2 dt - c |\Delta w| \int_0^T dt - \psi \int_0^T \left( \Delta w \right)^2 \left[ h(t) \right]^2 dt$$

- Similar to MV optimization that maximizes expected return for a given level of risk
Mathematical Model – Solutions

- **Method of calculus of variation**
  - Find optimal function instead of optimal parameter

- **Ordinary differential equation for** $h(t)$

- **Boundary condition for** $h(t)$
Solution – No Risk Aversion

Three different expected returns (s)
Solution – No Risk Aversion

Optimal horizon

\[ T^* = \frac{2\sqrt{\psi}}{\sqrt{c + \sqrt{c + f}}} \]

Horizon should be longer if

- Market impact is high
- Fixed cost is low
- Return is low (if it agrees with the trade)
**Numerical Examples**

*Base parameter assumption. Optimal horizon = 0.52 day*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1% / day</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4% / day</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2 day / %</td>
</tr>
<tr>
<td>$c$</td>
<td>0.1% / day</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5 % day</td>
</tr>
<tr>
<td>$s = f / 2\psi$</td>
<td>1 / day$^2$</td>
</tr>
<tr>
<td>$g = \sqrt{\lambda \sigma^2 / 2\psi}$</td>
<td>5.7 / day</td>
</tr>
<tr>
<td>$p = \sqrt{c / \psi}$</td>
<td>0.45 / day</td>
</tr>
</tbody>
</table>
Numerical Examples

Changing parameters – case I
Numerical Examples

Changing parameters – case II

Optimal Trading Horizon

- $0.75-1$
- $1-1.25$
- $1.25-1.5$
- $1.5-1.75$
- $1.75-2$

- $f$
- $c$
Summary

➢ There is often an optimal trading horizon with optimal trading strategy

➢ Our analytic solution shows the optimal horizon depends on
  • Expected return
  • Stock volatility
  • Fixed cost
  • Market impact

➢ The solution can be extended to portfolios of stocks

➢ Practical applications hinge on modeling of multiple processes
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