



## **Half-Day Seminar on Asset Allocation, Style Analysis and Manager Search**

Friday, September 24, 2004, 8:30am – 12:00 Noon  
Hotel Monaco Chicago 225 North Wabash Chicago, IL 60601

### **AGENDA**

**Using the Analytical Hierarchy Process for Asset Class and Managed Fund Selection**

**Style Analysis With and Without Confidence Intervals**

**The CUSUM Technique Applied for Manager Search**

**Returns: Historic, Adjusted, Expected, and Implied**

# Using the Analytic Hierarchy Process to Assist Asset Allocation and Manager Search

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Northfield Asset Allocation Seminar  
September 2004

# How to Quantify a Qualitative Process

- ◆ Institutional asset managers and consultants face the task of selecting and assigning assets to money managers to satisfy the needs of the beneficiaries.
- ◆ Academic theory says that this is accomplished by using mean-variance analysis to maximize utility, usually a quadratic function of active return and risk.
- ◆ Investment practice is very different from theory in this case, and the process is much more qualitative than theory assumes.

# The Analytic Hierarchy Process: Background

- ◆ Thomas Saaty, a professor at the University of Pittsburgh, developed the AHP as a way to improve complex decision making and to identify and weight selection criteria.
- ◆ AHP is a methodology that arises from operations research literature. AHP is used as a non-parametric method for making complex, often qualitative decisions in a robust, consistent fashion.
- ◆ AHP provides a proven, effective means to deal with analyzing the data collected for the decision criteria and expediting the decision-making process.
- ◆ A wide body of literature indicates the AHP is useful when making complex decisions involving multiple criteria.

# Analytic Hierarchy Process: Mechanics

- ◆ For *each* evaluation criterion, usually expressed as a multiple choice question, the AHP creates a comparison matrix.
- ◆ The upper triangle holds the relative ratings (1-9, with 1 being best) of the alternatives: asset classes or fund managers.
- ◆ The diagonal of the matrix is ones – every fund compared with itself is a 1!
- ◆ The lower triangle is the reciprocal of the upper triangle:  $x(i, j) = 1 / x(j, i)$ 
  - If A is 9 times as good as B, then B is 1/9 as good as A

# Analytic Hierarchy Process: Mechanics

- ◆ When the comparison matrix has been filled, the matrix's first eigenvector will contain the weights to assign to each choice.
- ◆ For this application we use these weights as the asset class or manager allocation for that criterion.
- ◆ The portfolio weights for each criterion are then averaged using the weight for each criterion.
- ◆ It's a form of "importance weighted" average score.

# Literature: Using the AHP in Investment Management

- ◆ Bolster, Janjigian, and Trahan, "Determining Investor Suitability Using the Analytic Hierarchy Process," *Financial Analyst's Journal*, July/August 1995
- ◆ Saraoglu and Miranda Lam Detzler, "A Sensible Mutual Fund Selection Model," *Financial Analysts Journal*, May/June 2002
- ◆ Khaksari, Shahriar, Ravindra Kamath and Robin Grieves. "A New Approach To Determining Optimum Portfolio Mix," *Journal of Portfolio Management*, 1989, v15(3), 43-49.

# Institutional Asset Allocation

- ◆ If we used mean-variance optimization, we would:
  - Choose the appropriate liability (benchmark):
    - ❖ The CPI (to preserve spending power)
    - ❖ A bond of known duration
    - ❖ A model portfolio that represents typical peer group policy
  - Develop return expectations for each asset class relative to liabilities.
  - Estimate the co-variance between each asset class
  - Use optimization to determine the efficient frontier
  - Pick the position on the efficient frontier that fits the beneficiaries' risk tolerance relative to liabilities.

# Example from HBS Case Study of Harvard Management Company

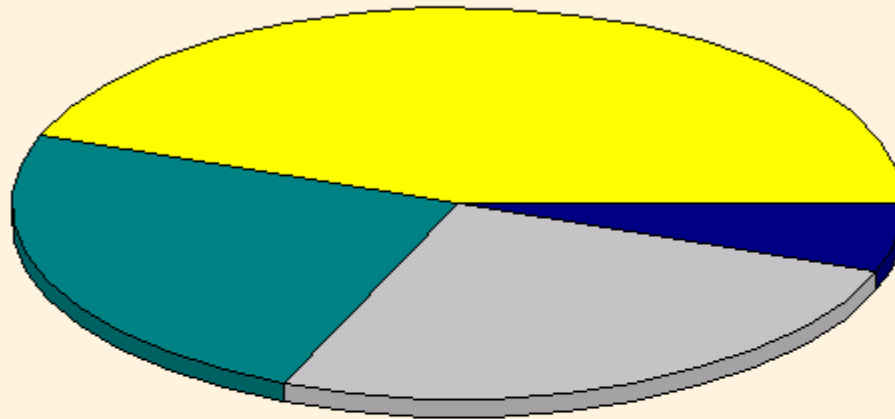
Asset Class	Policy Wt.	Real Return	Risk	r, US Stock	r, US Bond
Cash	-3%	3.0	1.0	0.10	0.10
Inflation Indexed Bonds	7%	3.6	3.0	0.10	0.40
Non US Bonds	4%	4.0	8.0	0.10	0.30
US Bonds	10%	4.0	7.0	0.35	1.00
Real Estate	7%	5.0	12.0	0.20	0.20
Commodities	6%	5.3	10.0	-0.15	-0.10
High Yield Bonds	3%	5.2	12.0	0.55	-0.40
Absolute Return Strategies	5%	5.3	12.0	0.70	0.25
Private Equity	15%	9.5	20.0	0.50	0.15
Emerging Market Bond & Stock	9%	8.5	20.0	0.50	0.05
Non US Stock	15%	6.5	17.0	0.60	0.15
US Stock	22%	6.5	16.0	1.00	0.35

# An Asset Allocation Example

- ◆ We have returns data on twelve reasonable asset class proxies that can model Harvard's asset allocation.
- ◆ We estimate returns:
  - Using historic returns and a Bayesian adjustment.
  - Using the returns from the case study in the previous slide
- ◆ We estimate the co-variance matrix using historic returns.
- ◆ We estimate the risk and return of the policy portfolio and compare it to the efficient frontier.

# Asset Allocation: Optimal Portfolio

Optimal (Portfolio 006)



Fund	Init	Opt
Citi Treasury-InflationLkd	13.0	44.3 %
Citi ESBI Index USD!	3.0	24.3 %
NAREIT Equity!	7.0	25.7 %
Goldman Sachs Commodity!	6.0	5.7 %

**This does not seem to be a well diversified portfolio.**

**Returns were estimated using Bayesian adjustment over a ten-year sample period.**

## Portfolio # 6

Investment Horizon	5.0
5 Years Return	3.9
5 Years Std Dev	3.3
Single Year Return	3.9
Single Year Std Dev	7.4
Annual Yeild	-0.1

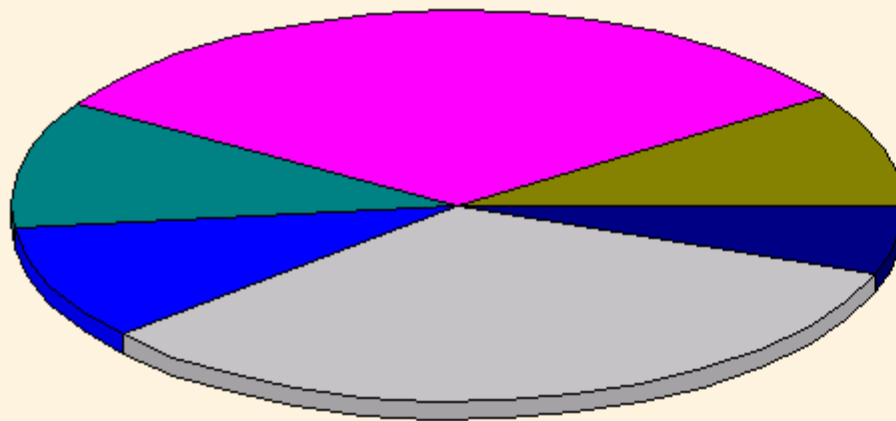
## Return

1 Year(+)	11.3
5 Years(+)	7.2
Expected	3.9
5 Years(-)	0.6
1 Year(-)	-3.5



# Asset Allocation: Optimal Portfolio

Optimal (Portfolio 006)



Fund	Init	Opt
Lehman High Yield(US Corp)	6.0	9.5 %
Citi CurHgd Non-USD WGBI	12.0	31.7 %
Citi ESBI Index USD!	3.0	10.6 %
FTSE All-Wld Adv Emerg USD!	3.0	9.7 %
NAREIT Equity I	7.0	32.9 %
Goldman Sachs Commodity I	6.0	5.6 %

**This is still not a well diversified portfolio.**

**Returns based on HBS case study**

## Portfolio # 6

Portfolio # 6		Return	
Investment Horizon	5.0	1 Year(+)	14.4
5 Years Return	6.0	5 Years(+)	9.8
5 Years Std Dev	3.7	Expected	6.0
Single Year Return	6.0	5 Years(-)	2.3
Single Year Std Dev	8.4	1 Year(-)	-2.3
Annual Yeild	-0.1		



# Mean Variance Results

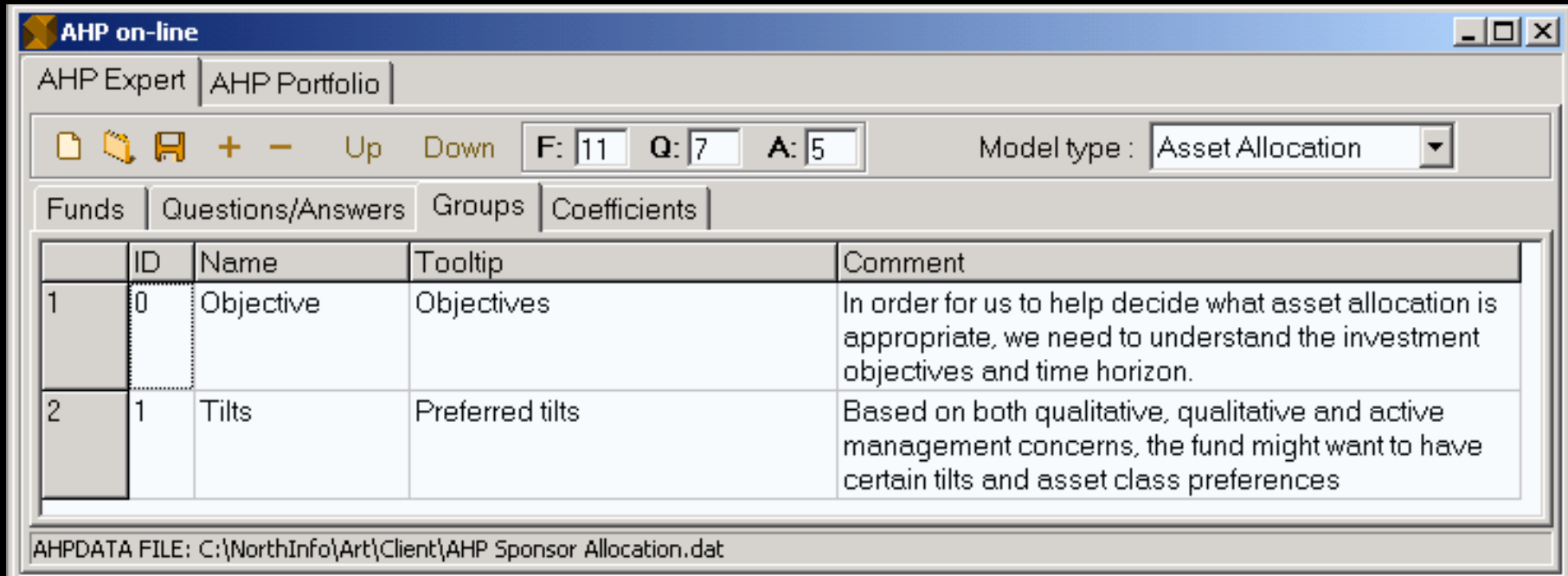
- ◆ Using Bayesian adjustment, the optimal portfolio gets only four of twelve asset classes.
- ◆ Two chosen asset classes, emerging market and TIPS bonds, are not typically given much weight in policy portfolios.
- ◆ The return estimates from the HBS case study are an improvement, but the portfolio is still “unusual” and poorly diversified.
- ◆ Clearly, there must be a better way to develop a reasonable strategic allocation.

# The Analytical Hierarchy Process

## First Steps

1. Develop question categories to help focus the client on the purpose of this group of questions.
2. Develop a number of questions for each category.
3. Split the responses for each question into levels, five being typical.
4. Assign weights to each question.
5. Select the asset classes that will be appropriate for the investor, in this case we use the ones in the HBS case study.

# Step 1: Develop Question Categories



The screenshot shows the 'AHP on-line' application window. The title bar reads 'AHP on-line'. Below the title bar, there are two tabs: 'AHP Expert' and 'AHP Portfolio'. The main interface includes a toolbar with icons for file operations and navigation, and a set of controls for 'F: 11', 'Q: 7', and 'A: 5'. A dropdown menu for 'Model type' is set to 'Asset Allocation'. Below these controls are four tabs: 'Funds', 'Questions/Answers', 'Groups', and 'Coefficients'. The 'Questions/Answers' tab is active, displaying a table with the following data:

	ID	Name	Tooltip	Comment
1	0	Objective	Objectives	In order for us to help decide what asset allocation is appropriate, we need to understand the investment objectives and time horizon.
2	1	Tilts	Preferred tilts	Based on both qualitative, quantitative and active management concerns, the fund might want to have certain tilts and asset class preferences

At the bottom of the window, the status bar displays the file path: 'AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat'.

## Step 2: Develop the Questions for Each Category

**AHP on-line** [AHP Expert | AHP Portfolio]

Model type: Asset Allocation

F: 11 | Q: 7 | A: 5

Group: All

Question N6:							Commodities Exposure			
	Question	Group	H/V	Rap	Weight	Comment		Answer	Hidden	DefA
1	Duration of Liabilities	0	0	0.0100	0.2000	What is the duration of your liabilities?	1	No Exposure	<input type="checkbox"/>	<input type="checkbox"/>
2	Spending Requirements	0	0	0.0010	0.2000	What are your current spending requirements?	2	Low Exposure	<input type="checkbox"/>	<input type="checkbox"/>
3	Inflation Protection	0	0	0.0100	0.2000	How important is inflation protection for your spending requirements?	3	Moderate Exposure	<input type="checkbox"/>	<input checked="" type="checkbox"/>
4	Domestic tilt	1	0	0.0100	0.1000	What level of domestic/foreign tilt is appropriate?	4	Overweight	<input type="checkbox"/>	<input type="checkbox"/>
5	Real Estate Exposure	1	0	0.0100	0.1000	What level of real estate exposure do you want?	5	Maximum	<input type="checkbox"/>	<input type="checkbox"/>
6	Commodities Exposure	1	0	0.0100	0.1000	What level of commodities exposure do you want?				
7	Emerging Markets	1	0	0.0100	0.1000	What level of emerging market exposure is appropriate?				

AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat

# Step 3: Selection of Asset Class Proxies

AHP on-line

AHP Expert | AHP Portfolio

ReCalc StartDate: 1985/01 EndDate: 2004/07

	Code	Name	Proxy Fund	Fund Name
1		Cash	115291	Citi 3 Month CD !
2		Investment Grade Bonds	114401	Lehman Aggregate !
3		High Yield Bonds	150339	Lehman High Yield(US Corp)
4	IDB	TIPS	115770	Citi Treasury-InflationLkd
5		Non US Bonds: Developed	115321	Citi CurHgd Non-USD WGBI
6	IIB	Non US Bonds: Emerging	106823	Citi ESBI Index USD!
7	IDE	US Equities	112302	Russell 3000 TR !
8	IFT	Non US Equity: Developed	122738	FTSE World Ex USA USD!
9	IFT	Non US Equity: Emerging	102012	FTSE All-Wld Adv Emerg USD!
10	IDE	Real Estate	117203	NAREIT Equity !
11	IHF	Commodities	117280	Goldman Sachs Commodity !

AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat

# Now for the Hard Part

- ◆ For each combination of asset class and question response level, we assign a suitability ranking.
- ◆ The suitability ranking is an integer ranging from 1 (most suitable) to some chosen upper limit. Normally the upper limit is 9, but sometimes we use 99 to ensure minimal exposure.
- ◆ For twelve asset classes, five response levels and seven questions, we have:
  - Ratings =  $12 \cdot 5 \cdot 7 = 420$  suitability judgments

# Suitability Judgments

AHP on-line

AHP Expert | AHP Portfolio

F: 11 Q: 7 A: 5 Model type: Asset Allocation

Funds | Questions/Answers | Groups | Coefficients

Sectors		Fund	US Equities	Question:	Emerging Markets	A1				
	Name		Questions			A1	A2	A3	A4	A5
1	Cash	1	Duration of Liabilities			7	3	1.5	1	1
2	Investment Grade Bonds	2	Spending Requirements			7	3	1.5	1	1
3	High Yield Bonds	3	Inflation Protection			5	5	5	5	5
4	TIPS	4	Domestic tilt			1	3	5	7	9
5	Non US Bonds: Developed	5	Real Estate Exposure			9	9	9	9	9
6	Non US Bonds: Emerging	6	Commodities Exposure			9	9	9	9	9
7	US Equities	7	Emerging Markets			9	9	9	9	9
8	Non US Equity: Developed									
9	Non US Equity: Emerging									
10	Real Estate									
11	Commodities									

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# Questions to Determine Objectives

**AHP on-line**

AHP Expert | AHP Portfolio

Questionnaire | Portfolio

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Objectives | Preferred tilts

**In order for us to help decide what asset allocation is appropriate, we need to understand the investment objectives and time horizon.**

Duration of Liabilities  
What is the duration of your liabilities?

Less than 2 Years     2 to to 5 Years     5 to 10 Years     10 to 15 Years     More than 15 Years

Spending Requirements  
What are your current spending requirements?

More than 5%     Less than 5%     Less than 4% of assets     Less than 3%     Less than 1%

Inflation Protection  
How important is inflation protection for your spending requirements?

No Inflation Protection     Minimal Protection     Moderate Protection     Significant Inflation     Maximal Protection

## Questions to Assess Desired Tilts

**AHP on-line** [AHP Expert | AHP Portfolio]

Questionnaire | Portfolio

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Objectives | Preferred tilts

**Based on both qualitative, quantitative and active management concerns, the fund might want to have certain tilts and asset class preferences**

**Domestic tilt**  
What level of domestic/foreign tilt is appropriate?

Maximum US exposure    Strong US tilt    Moderate US tilt    Moderate Non US tilt    Strong Non US tilt

**Real Estate Exposure**  
What level of real estate exposure do you want?

Low exposure    Moderate Exposure    Typical Exposure    High Exposure    Very high exposure

**Commodities Exposure**  
What level of commodities exposure do you want?

No Exposure    Low Exposure    Moderate Exposure    Overweight    Maximum

**Emerging Markets**  
What level of emerging market exposure is appropriate?

No Exposure    Low Exposure    Typical    Above Average    High Exposure

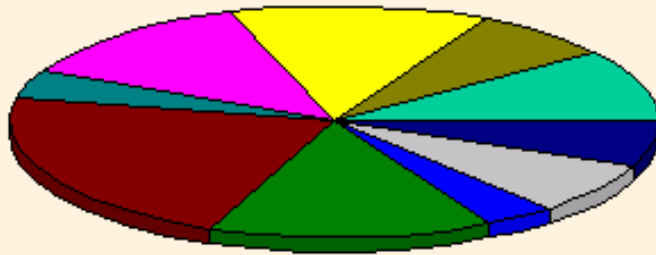
# What Does the AHP Do?

- ◆ Let's assume that we have a plan sponsor that has average liability duration and spending requirements and no desired tilts away from a reasonable policy portfolio.
- ◆ What is the asset allocation? Our sample approximates the HBS case study.
- ◆ What is the portfolio's expected return and risk?
- ◆ How does the AHP portfolio compare to the efficient frontier?

# Policy Portfolio

## Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)



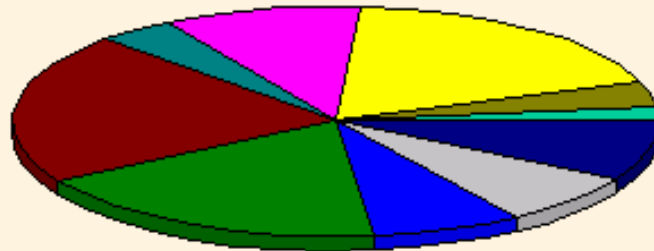
Fund	Init	Opt
Citi 3 Month CD !	0.2	0.0 %
Lehman Aggregate !	10.2	1.8 %
Lehman High Yield(US Corp)	6.9	0.0 %
Citi Treasury-InflationLkd	13.0	0.0 %
Citi CurHgd Non-USD WGBI	12.4	42.3 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR!	22.1	12.5 %
FTSE World Ex USA USD!	14.2	23.3 %
FTSE All-Wld Adv Emerg USD!	3.6	6.4 %
NAREIT Equity!	7.3	13.7 %
Goldman Sachs Commodity!	6.2	0.0 %

**Returns estimated by averaging ICAPM and Black Litterman estimations**

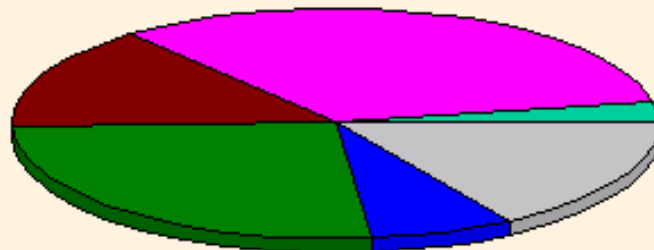
# Low Spending, High Duration and Maximum Inflation Protection

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)



Fund	Init	Opt
Citi 3 Month CD !	0.1	0.0 %
Lehman Aggregate !	1.9	3.2 %
Lehman High Yield(US Corp)	3.5	0.0 %
Citi Treasury-InflationLkd	18.2	0.0 %
Citi CurHgd Non-USD WGBI	9.8	32.8 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR !	20.9	14.5 %
FTSE World Ex USA USD!	18.4	26.4 %
FTSE All-Wld Adv Emerg USD!	7.5	7.3 %
NAREIT Equity !	7.3	15.9 %
Goldman Sachs Commodity !	8.2	0.0 %

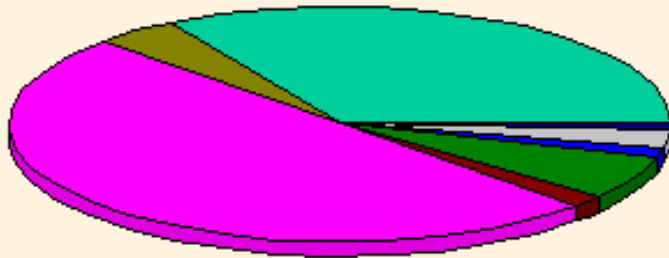
# High Spending, Low Duration and Minimum Inflation Protection

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)

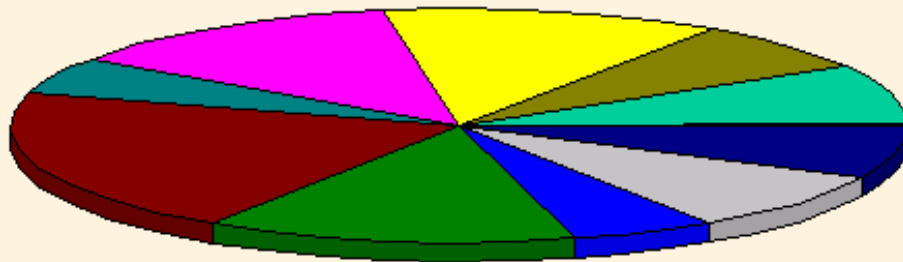


Fund	Init	Opt
Citi 3 Month CD !	5.1	0.0 %
Lehman Aggregate !	26.3	33.5 %
Lehman High Yield(US Corp)	2.7	4.2 %
Citi Treasury-InflationLkd	9.1	0.0 %
Citi CurHgd Non-USD WGBI	10.0	50.0 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR !	15.6	1.6 %
FTSE World Ex USA USD!	11.5	6.0 %
FTSE All-Wld Adv Emerg USD!	3.6	1.3 %
NAREIT Equity !	8.1	2.6 %
Goldman Sachs Commodity !	4.0	0.8 %

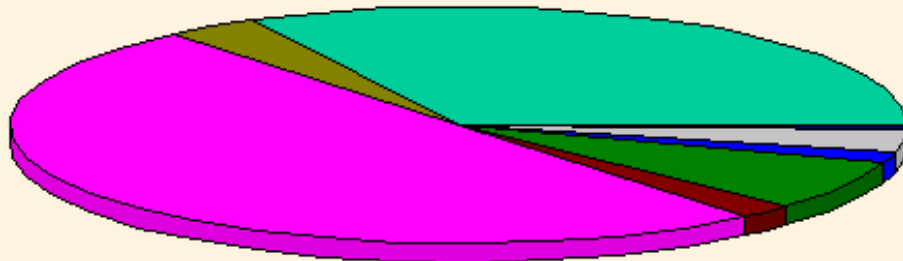
# Non US, Real Estate, Commodity and Emerging Markets Tilt

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)



Fund	Init	Opt
Citi 3 Month CD !	0.1	0.0 %
Lehman Aggregate !	8.4	32.6 %
Lehman High Yield(US Corp)	6.9	3.4 %
Citi Treasury-InflationLkd	12.3	0.0 %
Citi CurHgd Non-USD WGBI	12.4	50.0 %
Citi ESBI Index USD!	5.3	0.0 %
Russell 3000 TR !	20.3	2.0 %
FTSE World Ex USA USD!	13.4	6.7 %
FTSE All-Wld Adv Emerg USD!	5.3	1.6 %
NAREIT Equity !	8.3	3.0 %
Goldman Sachs Commodity !	7.2	0.6 %

# Portfolio Construction

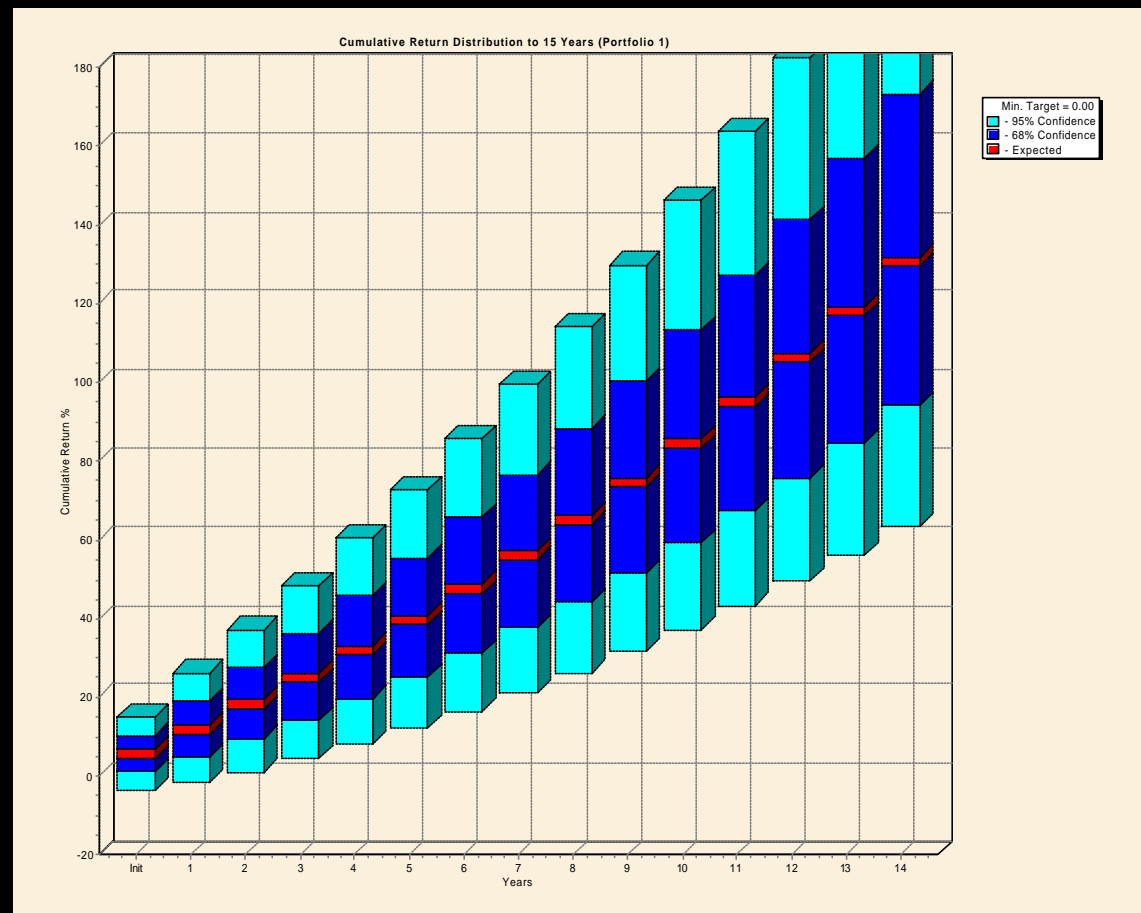
- ◆ Using “best guess” return assumptions, the suitable portfolios are within 15 to 25 b.p. of the efficient frontier.
- ◆ Next step: Choose asset managers accounts to implement the portfolio.
- ◆ Use optimization to minimize the funds’ tracking error vs. the asset allocation
  - This does not require developing expected returns for the “implementation portfolio”

# Portfolio Construction, Continued

- ◆ During the optimization process, sensible constraints (such as minimum and maximum holdings) can be used.
- ◆ After portfolio construction, return assumptions can be developed using historic averages, Bayesian adjustment, CAPM estimation or implied returns (Black-Litterman)

# Portfolio Construction, Completed

- ◆ After creating return expectations for the portfolio, we can create portfolio cumulative return expectations and confidence intervals.



# Traditional Manager Selection

- ◆ Let's assume that we have ten managers.
- ◆ How would we assign them weights in the portfolio?
- ◆ If we only used mean-variance optimization, we would:
  - Determine an appropriate benchmark, which could be either actuarial liabilities or a model portfolio.
  - Develop benchmark relative expected returns for each manager
  - Estimate the co-variance between each manager pair.
  - Use optimization to determine the efficient frontier
  - Pick the position on the efficient frontier that fits the beneficiary's risk tolerance.

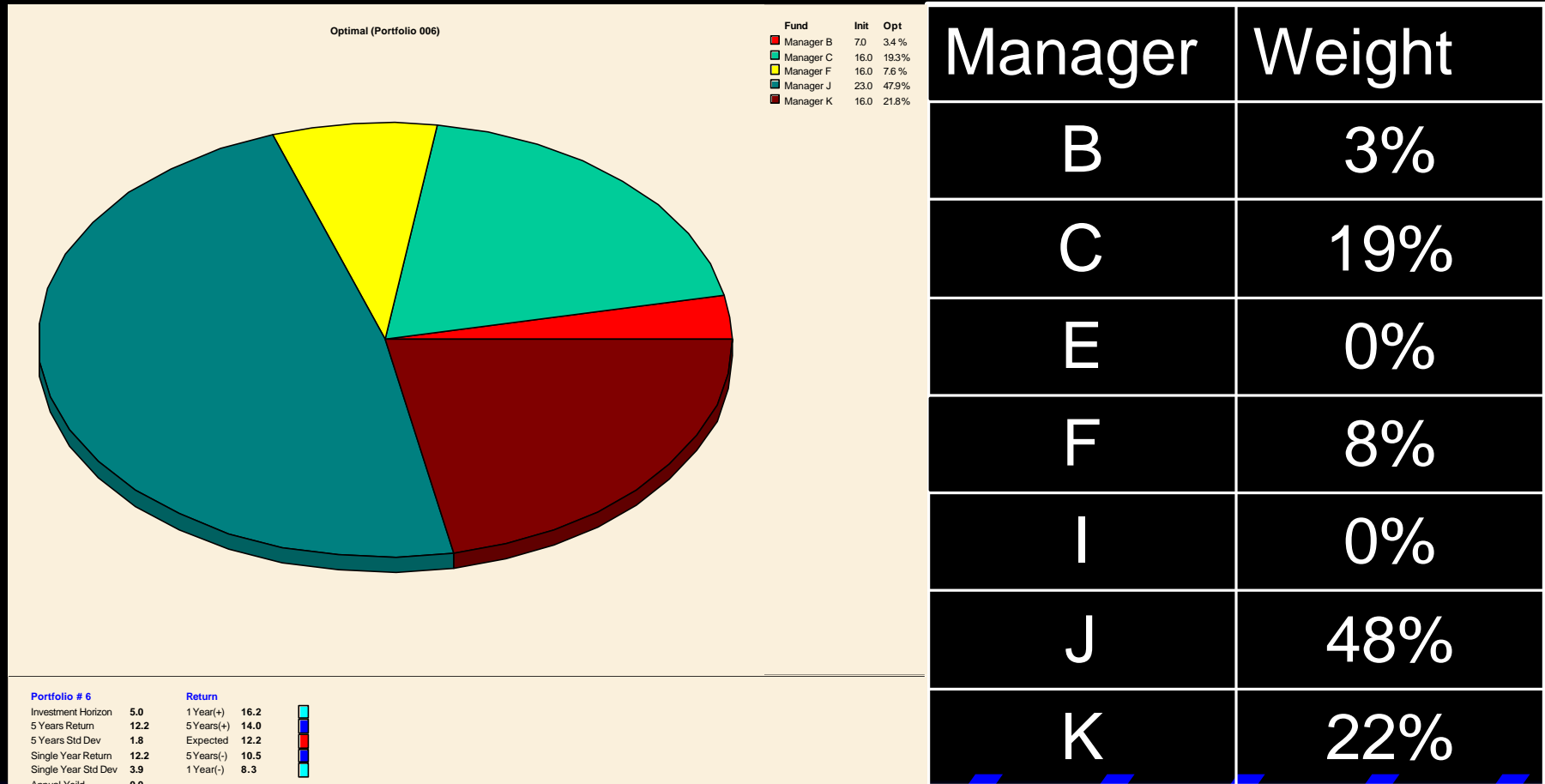
# A Manager Allocation Example

- ◆ We have returns data on seven managers that a consultant wants to evaluate and assign assets to manage.
- ◆ All managers are using an “absolute return strategy” and have been identified as “good to excellent.”
- ◆ We estimate returns using the Bayesian adjustment.
- ◆ We estimate the co-variance matrix using historic returns.
- ◆ We optimize and pick a point on the efficient frontier whose risk is similar to an equally weighted portfolio.

# Mean Variance Results

- ◆ Two “good to excellent managers” get no allocation.
- ◆ Two other managers get to share 10% of the allocation.
- ◆ Two managers share 40% of the allocation.
- ◆ One manager gets almost half of the allocation
- ◆ If all of these managers are “good to excellent,” this allocation is not reasonable.

# An Example: Manager Selection



# Let's Try the Analytic Hierarchy Process

- ◆ First we need a set of criteria on which to judge managers.
- ◆ Saraoglu and Detzler propose a set of criteria for selecting mutual funds, but we want something more applicable to institutional manager selection.
- ◆ At [www.ennisknupp.com](http://www.ennisknupp.com) (EK) we find a set of criteria for choosing asset managers.

# Manager Selection Ratings

AHP on-line

AHP Expert | AHP Portfolio

Managers | Ranking

+ - [Icons] C:\NorthInfo\Art\Client\Projects\Kate\AHP\ahpmanqa.qa View: Q/A Panel

	Code	Name	Comment
1	99001		
2	99002		
3	99003		
4	99004		
5	99005		

Selected: **Manager N1**

Fund Selection

**This sample questionnaire is based on a paper by Saraoglu and Detzler "A Sensible Mutual Fund Selection Model," for the May/June 1992 Financial Analysts Journal**

**Long Term Performance**  
The long term (10 to 15 year) total return of the fund measures how well the fund does over a few market cycles.

Poor     Below Average     Average     Above Average     Excellent

**Fees**  
The lower the fee, the higher the future returns are likely to be.

High Fee     Above Average Fee     Average     Below Average Fee     Low Fee

**Manager Tenure**  
A longer manager tenure is likely to be related to higher skill and better performance in different market conditions.

Short Tenure     Below Average     Average     Above Average     Long Tenure

**Risk Adjusted Performance**  
Treyrnor ratio or Sharpe ratio measure the fund performance relative to its risk.

Poor     Below Average     Average     Above Average     Excellent

# Let's Re-do the Manager Selection using the AHP Weights

- ◆ We estimate the Sharpe ratio for each of the 7 managers and assign performance of each of the managers for the last 3 years (the time history for this database).
- ◆ We do a long-short style analysis and observe the alpha, tracking error, style drifts and CUSUM statistics.

# Manager Allocation using AHP

- ◆ Based on the Sharpe ratio and other statistics, we rate the managers fair to excellent on the Performance and perceived skill.
- ◆ We leave the other rankings at “average” since we don’t have the information to make these judgments.

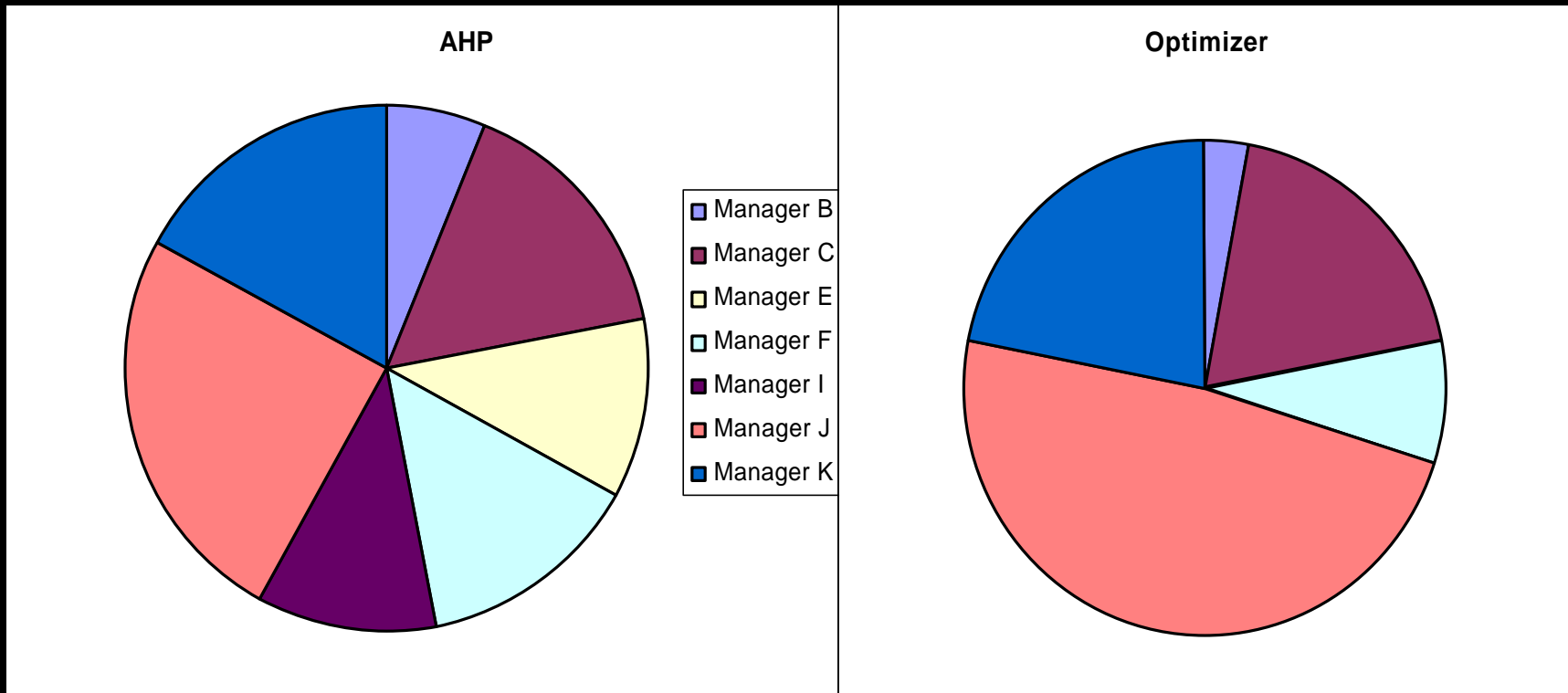
# Manager Allocation using AHP

- ◆ We then work out a new allocation, and then estimate the expected return and risk of the new allocation.
- ◆ We also estimate the implied returns for each manager using the AHP allocation and an estimation of the appropriate risk tolerance for the AHP portfolio.

# Comparison of the AHP and MV Optimal Portfolios

Fund Name	AHP Weight	Optimal Weight	Expected Return	Historic Return	Implied Return	Risk
Manager B	6	3	6.0	4.5	13.3	16.2
Manager C	16	19	9.3	9.7	9.8	5.3
Manager E	11	0	6.6	5.5	9.8	3.6
Manager F	14	8	16.3	20.8	23.5	16.2
Manager I	11	0	7.7	7.2	10.6	4.0
Manager J	25	48	14.5	18.0	11.5	6.7
Manager K	17	22	9.5	10.0	9.2	3.9

# Comparison of AHP and Optimizer Manager Allocations



# Conclusions

- ◆ AHP is a methodology that arises from operations research literature that is used as a non-parametric method for making complex, often qualitative decisions in a robust, consistent fashion.
- ◆ AHP has now been adapted as a tool in the selection of, and the allocation of capital to, investment managers.
- ◆ We think AHP is the way to go for many problems in investment decision making where quantitative and qualitative criteria must both play a role.

# Robust Returns-Based Style Analysis for Manager Selection

Asset Allocation, Style Analysis and Manager  
Search Seminar  
September 2004

# Quantifying a Manager's Performance

- ◆ Institutional asset managers and consultants face the task of selecting and assigning assets to money managers to satisfy the needs of the beneficiaries.
- ◆ The questions are:
  - What is the manager doing?
  - How well is the manager doing it?
  - What is an appropriate benchmark for the manager?

# Issues in Manager Selection

- ◆ Is the manager's style as advertised?
- ◆ Is one manager's style distinct from another manager?
- ◆ Is the manager's style consistent over time or has it varied?
- ◆ If we don't have a record of the manager's actual holdings, we can only make inferences about these questions
- ◆ Like any statistical inference, style analysis results should be presented with confidence intervals or we're going to get lots of false "positive" conclusions

# Concept of Returns Based Analysis

- ◆ The basic idea was based in Sharpe's original name "Effective Asset Mix Analysis"
- ◆ The idea is to form a portfolio of indices such that this portfolio has a return behavior most closely related to the returns on the fund being analyzed
- ◆ Was originally conceived for "long only" portfolios

# Mathematics of Return Based Analysis

- ◆ Mathematically the process is equivalent to a “least squares” regression analysis, subject to constraints:
  - All weights must be between zero and one
  - All weights must sum to one
- ◆ Basically, we’ve got a “pie chart” of indices that mimics a particular fund over time

# The Problem of Linear Combinations

- ◆ If we use three indices to explain the returns of a manager, we can't attribute manager returns to the indices effectively, if the indices can also be explained by each other
  - Style analysis can only reliably attribute portfolio returns to the portion of market index returns that are themselves not attributable to the returns of other indices”
- ◆ In traditional regression this problem is called multicollinearity
- ◆ It was a not big deal for asset classes because bond returns don't explain stock returns and so forth, so Sharpe never worried about it

# The Problem with “Style” Analysis

- ◆ To get at “styles” within an asset class, the indices will be highly correlated with one another, so the results will be imprecise
- ◆ The question of exactly how imprecise the results are was solved in:
  - Lobosco, Angelo and DiBartolomeo, Dan, “Approximating the Confidence Intervals for Sharpe Style Weights,” *Financial Analyst’s Journal*, July/August 1997, Volume 53 Number
- ◆ Interestingly, no commercial style analysis package (until now) has included the confidence interval calculation

# Potential Errors using Returns Based Style Analysis

- ◆ We can be wrong about whether a manager is, or is not, following stated style preferences. The fund acts like 25% small cap value
  - The fund acts like 25% small cap value, plus or minus 50%
- ◆ We can be wrong about whether styles have changed over time
  - The fund used to act like it was 25% small cap value, now it acts like its 35% small cap value. That's a big difference
  - The fund used to act like it was 25% small cap value, now it acts like its 25% small cap value, but both numbers are estimated to plus or minus 50%. We can't tell if anything has really changed
- ◆ The long only requirement distorts portfolios that really do take short positions

# 7 Hedge Fund Managers using Diverse Styles

- ◆ We were supplied seven manager's monthly return series and told the following things:
  - All of the managers are “good to excellent”
  - All of the managers are running a market neutral strategy, although some have more risk (volatility) than others.
- ◆ We're going to relax the constraint that all index weights must be positive, but they still must add to one.

# The Managers' Basic Statistics: 2001-2003

<b>Fund Name</b>	<b>Return vs.</b>		<b>Sharpe Ratio</b>
	<b>Cash</b>	<b>Risk</b>	
Manager B	2.20	16.21	0.06
Manager C	7.58	5.30	1.17
Manager E	3.26	3.56	0.55
Manager F	18.59	16.20	1.07
Manager I	5.04	3.96	0.94
Manager J	15.80	6.74	2.15
Manager K	7.84	3.87	1.69

# Style Indices: Popular Choices for Domestic Equity Managers

- ◆ Domestic Equity Size and Style
  - S&P Large Cap Growth and Value
  - S&P Mid Cap Growth and Value
  - S&P Small Cap Growth and Value
  - Cash – 3 Month t-bill

# Manager B

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-52.0</b>	<b>41.5</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>35.6</b>	<b>45.2</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>-1.1</b>	<b>53.3</b>
<b>S&amp;P/BARRA Midcap 400 Growth</b>	<b>53.6</b>	<b>49.2</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>-47.0</b>	<b>53.7</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>0.0</b>	<b>62.6</b>
<b>Citi 3 Month Treasury Bill</b>	<b>110.8</b>	<b>13.1</b>
<b>T.E.</b>	<b>10.2</b>	
<b>Alpha</b>	<b>9.9</b>	

# Manager B: Observations

- ◆ None of the style weights are statistically different from zero, and are also not statistically different from 100%
- ◆ We don't know anything about this manager on average, except they are roughly market neutral.
- ◆ Maybe they are taking big tactical asset allocation shifts, so we may be able to get a better read by sub-samples of time, but then our confidence intervals will get even bigger

# Manager C

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-6.6</b>	<b>16.4</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>3.9</b>	<b>17.8</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>16.6</b>	<b>21.0</b>
<b>S&amp;P/BARRA Midcap 400 Growth</b>	<b>-49.7</b>	<b>19.4</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>-4.5</b>	<b>21.2</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>40.1</b>	<b>24.7</b>
<b>Citi 3 Month Treasury Bill</b>	<b>100.1</b>	<b>5.2</b>
<b>T.E.</b>	<b>4.0</b>	
<b>Alpha</b>	<b>6.3</b>	

# Manager C: Observations

- ◆ The alpha (about 6.3%) is lower than the manager's excess returns (about 7.5%). Therefore, security selection added to performance.
- ◆ The style indices only reduce the tracking error to about 4% compared to the fund's absolute volatility of about 4.6%.
- ◆ Even though some of the weights are significantly different from zero or one (or minus one), the portion of fund volatility that is explained is quite low. This manager is taking big individual security bets, or doing "something else"

# Manager C Continued

- ◆ Maybe this fund isn't doing long/short equity at all
  - Add in in a Corporate Bond Index (-30%  $\pm$ 15% weight)
  - Add in a Convertible Bond Index (32%  $\pm$ 17%)
- ◆ Indicates that the manager may have been using a convertible arbitrage strategy – long the convert, short the stock and the bond.

# Manager E

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-8.1</b>	<b>8.9</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>9.2</b>	<b>9.7</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>5.4</b>	<b>11.4</b>
<b>S&amp;P/BARRA Midcap 400 Growth</b>	<b>-4.5</b>	<b>10.6</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>-12.2</b>	<b>11.5</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>21.6</b>	<b>13.4</b>
<b>Citi 3 Month Treasury Bill</b>	<b>88.5</b>	<b>2.8</b>
<b>T.E.</b>	<b>2.1</b>	
<b>Alpha</b>	<b>2.7</b>	

# Manager E: Observations

- ◆ The style indices reduce the tracking error to about 2.1% compared to the fund's absolute volatility of about 3.1%.
- ◆ Other than cash, none of the style weights are statistically significant.

# Manager F

Fund Name	Style Wt. %	Error
S&P/BARRA 500 Growth !	-53.3	41.9
S&P/BARRA 500 Value !	46.4	45.6
S&P/BARRA Midcap 400 Value	-44.1	53.8
S&P/BARRA Midcap 400 Growth	29.2	49.6
S&P/BARRA 600 SmCap Value	30.6	54.1
S&P/BARRA 600 SmCap Growth	-2.6	63.2
Citi 3 Month Treasury Bill	93.8	13.2
T.E.	10.1	
Alpha	18.1	

# Manager F: Observations

- ◆ The alpha (about 18%) is about the same as the manager's excess returns.
- ◆ The style indices reduce the tracking error to about 10% compared to the fund's absolute volatility of about 16%.

# Manager I

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-6.7</b>	<b>9.9</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>-2.1</b>	<b>10.8</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>14.9</b>	<b>12.8</b>
<b>S&amp;P/BARRA Midcap 400 Growth</b>	<b>9.9</b>	<b>11.8</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>16.5</b>	<b>12.9</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>-28.8</b>	<b>15.0</b>
<b>Citi 3 Month Treasury Bill</b>	<b>96.3</b>	<b>3.1</b>
<b>T.E.</b>	<b>2.4</b>	
<b>Alpha</b>	<b>2.8</b>	

# Manager J

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-15.1</b>	<b>21.5</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>9.7</b>	<b>23.4</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>-18.2</b>	<b>27.6</b>
<b>S&amp;P/BARRA Midcap 400 Growt</b>	<b>0.8</b>	<b>25.5</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>18.5</b>	<b>27.8</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>8.3</b>	<b>32.5</b>
<b>Citi 3 Month Treasury Bill</b>	<b>96.0</b>	<b>6.8</b>
<b>T.E.</b>	<b>5.1</b>	
<b>Alpha</b>	<b>14.3</b>	

# Manager K

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error</b>
<b>S&amp;P/BARRA 500 Growth !</b>	<b>-12.3</b>	<b>10.2</b>
<b>S&amp;P/BARRA 500 Value !</b>	<b>20.4</b>	<b>11.1</b>
<b>S&amp;P/BARRA Midcap 400 Value</b>	<b>-19.9</b>	<b>13.0</b>
<b>S&amp;P/BARRA Midcap 400 Growth</b>	<b>12.8</b>	<b>12.0</b>
<b>S&amp;P/BARRA 600 SmCap Value</b>	<b>20.5</b>	<b>13.1</b>
<b>S&amp;P/BARRA 600 SmCap Growth</b>	<b>-24.9</b>	<b>15.3</b>
<b>Citi 3 Month Treasury Bill</b>	<b>103.5</b>	<b>3.2</b>
<b>T.E.</b>	<b>2.5</b>	
<b>Alpha</b>	<b>7.7</b>	

# Conclusions

- ◆ In order to help determine a manager's effective style mix, it is necessary to allow for short positions if the manager uses short positions.
- ◆ Confidence intervals and rolling period analysis are both necessary to determine if a manager is using tactical asset allocation.
- ◆ Removing indices that have style weights that are not statistically significant can help reduce the confidence interval magnitudes.

# CUSUM Analysis for Manager Evaluation and Monitoring

Dan diBartolomeo  
Northfield Asset Allocation Seminar  
September 2004

# Evaluating Investment Track Records

- ◆ Tradition in the investment industry is to evaluate active manager track records over a long period
  - At least 3 to 5 years
  - Some will argue over a market cycle is needed
- ◆ Typical measures such as information ratios may not be statistically significant for many decades for low risk strategies such as enhanced index funds

# Academic Evidence is the Reverse

- ◆ Academic studies refute the importance of evaluating long term track records
- ◆ Most studies show that if there is any persistence at all in manager performance, it has a short life of a year or less
  - What happened on average over the last five or ten years means nothing to the future
- ◆ Hendricks, Darryll, Jayendu Patel and Richard Zeckhauser. "Hot Hands In Mutual Funds: Short-Run Persistence Of Relative Performance, 1974-1988," Journal of Finance, 1993, v48(1), 93-130.

# The Key Question

- ◆ What time portion of a track record do we really need to evaluate?
- ◆ What we need is a procedure to draw the line between getting enough meaningful data within a manager's record and older, stale data that should be ignored
- ◆ Enter CUSUM

# A Robust Method Monitoring Manager

## Returns: CUSUM

- ◆ CUSUM is a technique developed in industrial operations theory to detect quality control problems
  - Philips, Thomas, Emmanuel Yashchin and David M Stein. "Using Statistical Process Control to Monitor Active Managers, Forthcoming in Journal of Portfolio Management, 2003
  - Blondell, David, Philip Hoang, John G. Powell and Jing Shi. "Detection Of Financial Time Series Turning Points: A New CUSUM Approach Applied To IPO Cycles," Review of Quantitative Finance and Accounting, 2002, v18(3,May), 293-315.
- ◆ CUSUM defines key turning points in the active return time series, and defines statistical significance of results subsequent to the key turning point

# Traditional Process Control

- ◆ Traditional process control focuses on process
  - Concentrate on the machines on production line
  - If they operate well, products should be good
  - Similar in spirit to performance measurement

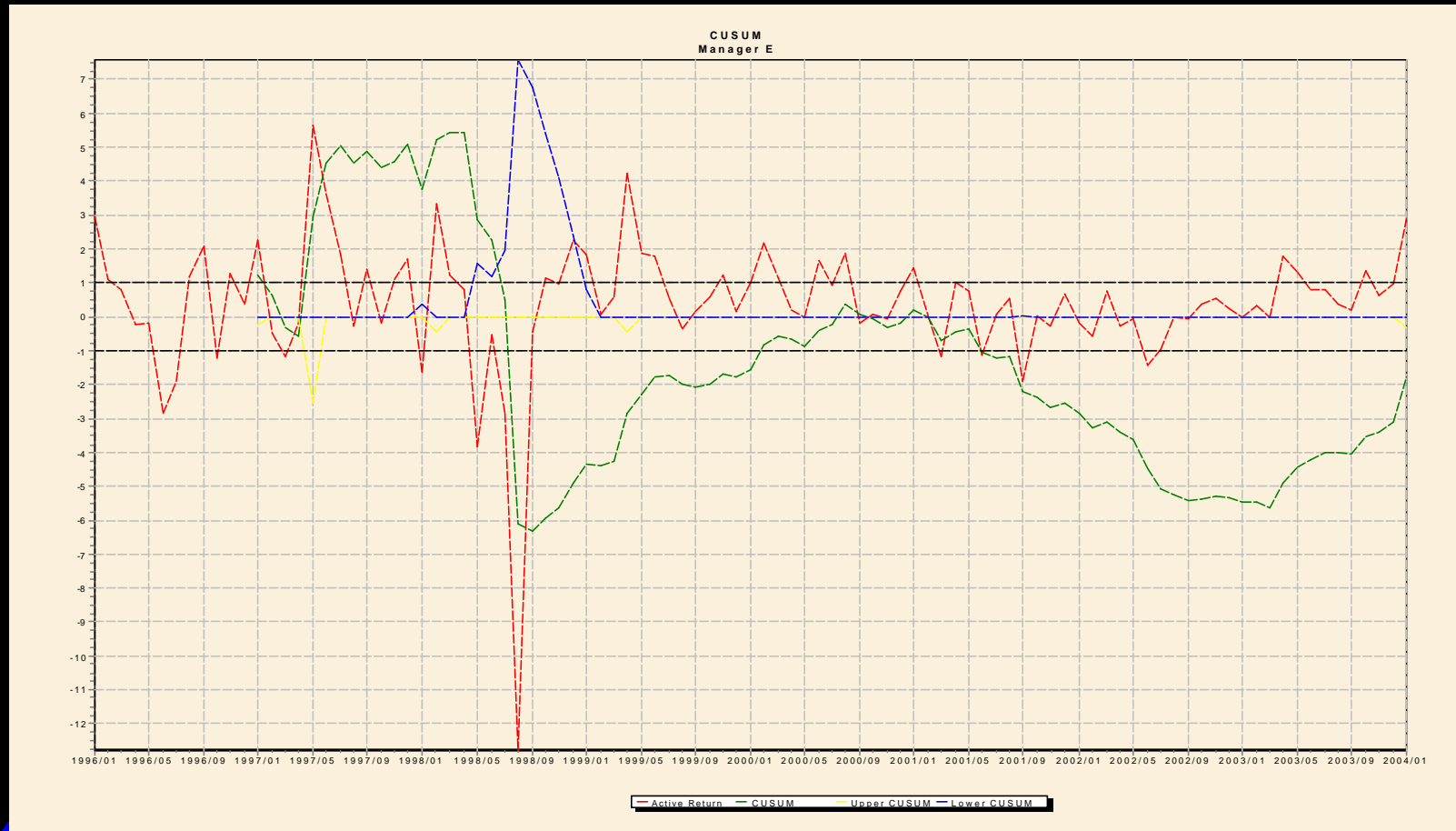
# Statistical Process Control

- ◆ Developed at Bell Labs in the 1930's by Walter Shewart, whose key insight was to focus on results.
  - The product is what counts
    - ❖ If it is good, the the process is good
    - ❖ If it us bad, then the process is bad
- ◆ Similar in spirit to performance monitoring
- ◆ Originally used to monitor Western Electric's telephone production lines
- ◆ Separate “signal” from noise

# The CUSUM Technique

- ◆ Created by E.S. Page in 1954
  - Reliably detects small process shifts
  - Insensitive to probability distribution
  - Provably optimal: detects process shifts faster than any other method.
  - Extremely robust, good under almost any definition of optimality
  - Much better than exponentially weighted moving average.
- ◆ Mathematically its very simple and tractable
- ◆ Easily analyzed algebraically or graphically

# CUSUM (Green) Plot Shows Regimes of Over and Under Performance



# Our Implementation of CUSUM

- ◆ Calculate excess returns for a manager, either over a known benchmark, or a benchmark inferred from returns-based style analysis
- ◆ Hold out a short sample period at the beginning to get an initial estimate of the mean and standard deviation of excess return
- ◆ Standardize each excess return by subtracting prior mean and dividing by prior standard deviation
- ◆ Calculate the cumulative sum of the standardized excess returns
- ◆ Apply backward looking likelihood test to find where CUSUM value is most significant
- ◆ Throw away prior return data, and concentrate your analysis on the period from the critical point until now

# Conclusions

- ◆ CUSUM has a wide variety of applications in operations research, and economic forecasting
- ◆ It is not a timing tool for hiring and firing managers
- ◆ It is a robust, and under certain assumptions, mathematically optimal way to detect shifts in the quality of a process
- ◆ Provides an excellent tool for deciding how much of a manager's track record is relevant to current operations and conditions

# Estimating Returns for Asset Allocation

Northfield Asset Allocation Seminar  
September 2004

# Expected Returns

- ◆ Optimum portfolio weights are very sensitive to return expectations.
  - Small changes in expected returns → large weight changes.
- ◆ Historic returns are often used, but they are not usually good predictors of future returns.
  - Domestic Equity returns are upwardly biased by an expansion of P/E ratios.
  - Bond returns are upwardly biased by decreases in interest rates.
  - Active returns are upwardly biased because of survivorship bias, particularly with hedge funds.

## Literature: Historic and Expected Returns

- ◆ Bradford Cornell, *The Equity Risk Premium*, 1999
- ◆ William Bernstein, *The Birth of Plenty*, 2004
- ◆ Jeremy Siegel, *Stocks for the Long Run*
- ◆ Dimson, Marsh, & Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns*
- ◆ Arnott and Bernstein, “What Risk Premium is Normal,” *Financial Analyst Journal*, Mar/Apr 1992
- ◆ Dempsey, “The Nature of Market Growth, Risk and Return (DOGMA),” *Financial Analyst Journal*, May/June 2002
- ◆ Ibbotson & Chen, “The Supply of Stock Market Returns,” [http://icf.som.yale.edu/pdf/Supply\(v5\).pdf](http://icf.som.yale.edu/pdf/Supply(v5).pdf)
- ◆ Harvard Business School Case Study: “The Harvard Management Company (2001)”

# Inflation Adjusted U.S. Stock Market Returns

## Jeremy Siegel: Stocks for the Long Haul

Time Period	Dividend Yield	Compound Total	Standard Deviation
		Yearly Return	Annual Returns
1802-1870	6.4%	7.0%	16.9%
1871-1925	5.2%	6.6%	16.8%
1926-1997	4.6%	7.2%	20.4%

# Historic Returns

- ◆ When calculating historic returns, even long histories show returns that are much higher than what probably can be expected in the future.
- ◆ The US is the “surviving economy” and was an emerging market in 1802, the point at which real per capita GDP growth began to grow at 2% for what is now the “developed world.”

# Capital Asset Pricing Model

- ◆ Expected Return is:

Expected Return = Risk Free Rate +  $\beta$  \* Market Return

- ◆ What is the Risk Free Return for a reasonable investment horizon:

- Risk Free Return = Inflation + Embedded Productivity
- Market Return: The “equity risk premium”
- Beta = Systematic risk, proportional to covariance between asset and market portfolio.

# What is the Market Portfolio

- ◆ It should include the entire US stock market. The Russell 3000 or Wilshire 5000 are reasonable proxies. The MSCI World index does not include small cap stocks.
- ◆ It should include corporate bonds, since the market should be (mostly) independent of the market's capital structure. Analogous to Modigliani Miller.
- ◆ It should include foreign stocks since they are part of a well diversified portfolio.
- ◆ It should *not* include government or agency bonds, since they are related to the supply and demand for current vs. future consumption. See Robert Fuhrman, Northfield Newport Conference 2004.

# Expected Returns Using CAPM Model

$$R_f = 4\%, R_m = 3.5\%$$

Asset Class	Beta	Expected Return
US Bonds	0.12	4.42
Non US Bonds	0.19	4.67
US High Yield Bonds	0.34	5.19
US Value Stocks	0.99	7.47
Non US Stocks, Developed	1.23	8.31
Emerging Market Stocks	1.26	8.41
US Growth Stocks	1.39	8.87

# Bayes-Stein Adjustment

- ◆ The reduction in errors arising from Bayesian shrinkage estimators is a mathematically provable result.
- ◆ Bayesian shrinkage jointly minimizes the errors in the return expectations over the portfolio, rather than trying to minimize the errors in each asset class return expectation separately.
- ◆ The “inadmissibility” of sample mean as an estimator for multivariate (portfolio) problems was proven by Stein in 1955.

# Bayes-Stein Methods

- ◆ We need to distinguish between using these techniques:
  - To explicitly compensate for “error maximization” in mean-variance optimization.
  - For improving the quality of return forecasts that we make. This technique provides better numbers for what we actually believe, but requires providing more information than just historical data.

# Literature: Bayesian Adjustment

## ◆ Compensating for Error Maximization

- <http://www-gsb.uchicago.edu/fac/arnold.zellner/more/CURRENT-PAPERS/bayshrin.pdf>
- Jorion, “International Portfolio Diversification with Estimation Risk,” *Journal of Business*, July 1985
- Jorion, P., “Bayes-Stein Estimation for Portfolio Analysis,” *Journal of Financial and Quantitative Analysis*, September 1986
- Jorion, P., “Bayesian and CAPM Estimators of the Means: Implications for Portfolio Selection,” *Journal of Banking and Finance*, June 1991

## ◆ Requires Additional Data:

- Black and Litterman, “Global Portfolio Optimization,” *Financial Analyst Journal*, 1992
- He and Litterman, “The Intuition Behind Black-Litterman Model Portfolios,” Goldman Sachs

# Bayesian Priors

- ◆ There are really two dimensions of priors.
  1. Whether the prior is diffuse or centralized.
  2. Whether the prior is “uninformative” or “informative” or somewhere in between: “semi-informative.”
- ◆ Imagine we have some historical data for 3 asset classes: stocks, bonds, cash.
- ◆ The sample period return for stocks is -3%, bonds 4%, cash 9%.

# Diffuse Priors

- ◆ A diffuse prior would be one where the prior could take on a variety of values.
- ◆ Imagine we just picked some random number between -3 and 9. This would be a diffuse and uninformative prior.
- ◆ This is the technique Markowitz and Ussman used in their recent paper that compared Bayesian methods and re-sampling.

# Semi Informative Prior

- ◆ A “slightly informative” prior would be to assume that the prior was random, but normally distributed between 0 and 9 (implying mean 4.5)
- ◆ It is economically irrational for any investor to invest in a risky asset with negative expected (excess) returns.

# Centralized Prior

- ◆ Let's say the government regulated interest rates on bonds to always be 6%.
- ◆ In such a case, it might be sensible to always use 6% as a prior. The prior would have a single value rather than a distribution.
- ◆ Grinold's alpha scaling rule of thumb is very similar to a Bayesian shrinkage where  $W = IC$  and  $U_p = 0$  (a single fixed value).

# Time Horizon

- ◆ While we normally talk about “long term” returns in asset allocation, real world investors usually have intermediate horizons over which they view things.
- ◆ For example, a 30 year zero coupon Treasury bond has wildly volatile annual returns but the return over 30 years is known with a high degree of certainty if we hold one bond all the way.

# Jorion's Bayes-Stein Adjustment

All Bayesian estimates have the form:

$$U_{bs} = W (U) + (1-W) U_p$$

where:

$U_{bs}$  = the Bayes adjusted expectation of return

$W$  = is the weight given to the return based on the data we observe

$U$  = return based on the data we observe (history)

$U_p$  = the return "prior"

# Stein Shrinkage Estimator

Jorion's Technique estimates  $W$ :

$$W = 1 - [(n-2) / (T-n-2)] - 1 / [(U-U_p)^T Q^{-1}(U-U_p)]$$

where:

$T$  = number of time periods of data

$n$  = number of asset return time series

$Q$  = covariance matrix of return time series

# Single Value Prior

- ◆ Jorion's method that has a single value prior for all asset classes addresses estimation risk in optimization.
- ◆ Our actual expectations have not changed based on the Bayesian adjustments
- ◆ This is the way our beliefs ought be presented to an optimizer. Rationale:
  - When our beliefs about returns are uncertain, the uncertainty makes the asset classes less distinct from one another.
  - Efficient return estimators shrink the return spreads amongst the asset classes.

# Jorion's Bayes-Stein Adjustment

- ◆ The prior is the historical return on minimum variance portfolio formed of all the assets, excluding a risk free asset.
- ◆ If you have risky assets, the minimum return you would expect from any one of them individually would be the return that could be achieved by investing in the portfolio that yields the lowest risk.

# Bayesian Adjustment: Risks

- ◆ Once we've adjusted the means away from the sample means, the standard deviations will increase slightly
- ◆ The adjusted correlations *may* also differ *slightly* from the sample values. In practice, correlations do not change when using two significant digits.
- ◆ From an optimization standpoint, we are biasing the covariance matrix to a slightly more conservative posture.

# Jorion's Adjustment 1979-2004

Asset Class	Historic Return	Bayesian Return	Risk
Cash	4.94	4.93	0.58
US Bonds	8.62	7.60	4.54
Non US Bonds	10.60	9.04	10.46
US Equity	13.18	10.91	15.83
Non US Equity	10.08	10.08	17.76

# Currency Hedge Non US Bonds

Asset Class	Historic Return	Bayesian Return	Risk
Cash	4.93	4.93	0.58
US Bonds	8.62	7.61	4.54
Non US Bonds	<i>8.10</i>	<i>7.23</i>	<i>3.66</i>
US Equity	13.18	10.92	15.83
Non US Equity	10.08	10.09	17.76

# Bayesian Adjustment

- ◆ Bayesian Adjustment reduces expected return for bonds by about 1%.
- ◆ Bayesian Adjustment reduces expected return for stocks by about 3%.
- ◆ US \$ depreciation caused Non US Bonds.
  - 2% extra return
  - Increase in volatility 3.6 to 10.5

# Informative Priors

- ◆ Black-Litterman uses an informative prior, where we are bringing a lot of new information into the problem: in this case the CAPM.
- ◆ We estimate what we think the values of the returns should be, given each asset's weight in the global market portfolio

# Black-Litterman Model

- ◆ This model is used for active asset allocation at Goldman-Sachs: as described in He and Litterman, “The Intuition Behind Black-Litterman Model Portfolios,”
- ◆ The model’s key action is the estimation of the vector of implied returns based on estimations of the:
  - Market’s risk tolerance
  - The weights of each asset in the market portfolio
  - The covariance between the assets in the market

# Black-Litterman Model

- ◆ An asset's expected returns should vary from the implied returns depending on the relative outlook for that asset and the confidence in the active prediction.
- ◆ The implied returns are calculated:
  - $\Pi = S w / \lambda$
  - $\Pi$  is the vector of implied returns
  - $w$  is the portfolio weight vector
  - $S$  is the covariance matrix
  - $\lambda$  is the market's risk tolerance parameter (RAP)

# Black-Litterman Return Estimation

Asset Class	Weight in Market Portfolio	Implied Return
Cash	0	4.56
US Bond	20	5.00
Non US Bond	30	5.11
US Stock	50	8.02
Non US Stock	20	7.89

# What About Actively Managed Funds?

- ◆ Use style analysis vs. benchmarks for which you have “good” expected return estimates.
- ◆ Use the style factor weighted average of the asset class returns to establish a baseline expected return.
- ◆ Adjust the alpha for selection (you are not likely to pick something that hasn’t been good) and survivorship bias.
- ◆ Add the alpha to the expected returns due to beta.

# Conclusions

- ◆ Historic returns, even over long time histories, are poor predictors of future (and hence expected) returns.
- ◆ Bayesian adjustment accounts for the uncertainty of return forecasts. Jorion's Bayes Stein adjustment and Black-Litterman are both examples of Bayesian adjustment.
- ◆ CAPM and Black-Litterman are equilibrium models to estimate expected returns from historic covariance data.