

# **The Equity Market Premium Puzzle: CAPM and Minimum Variance Portfolios**

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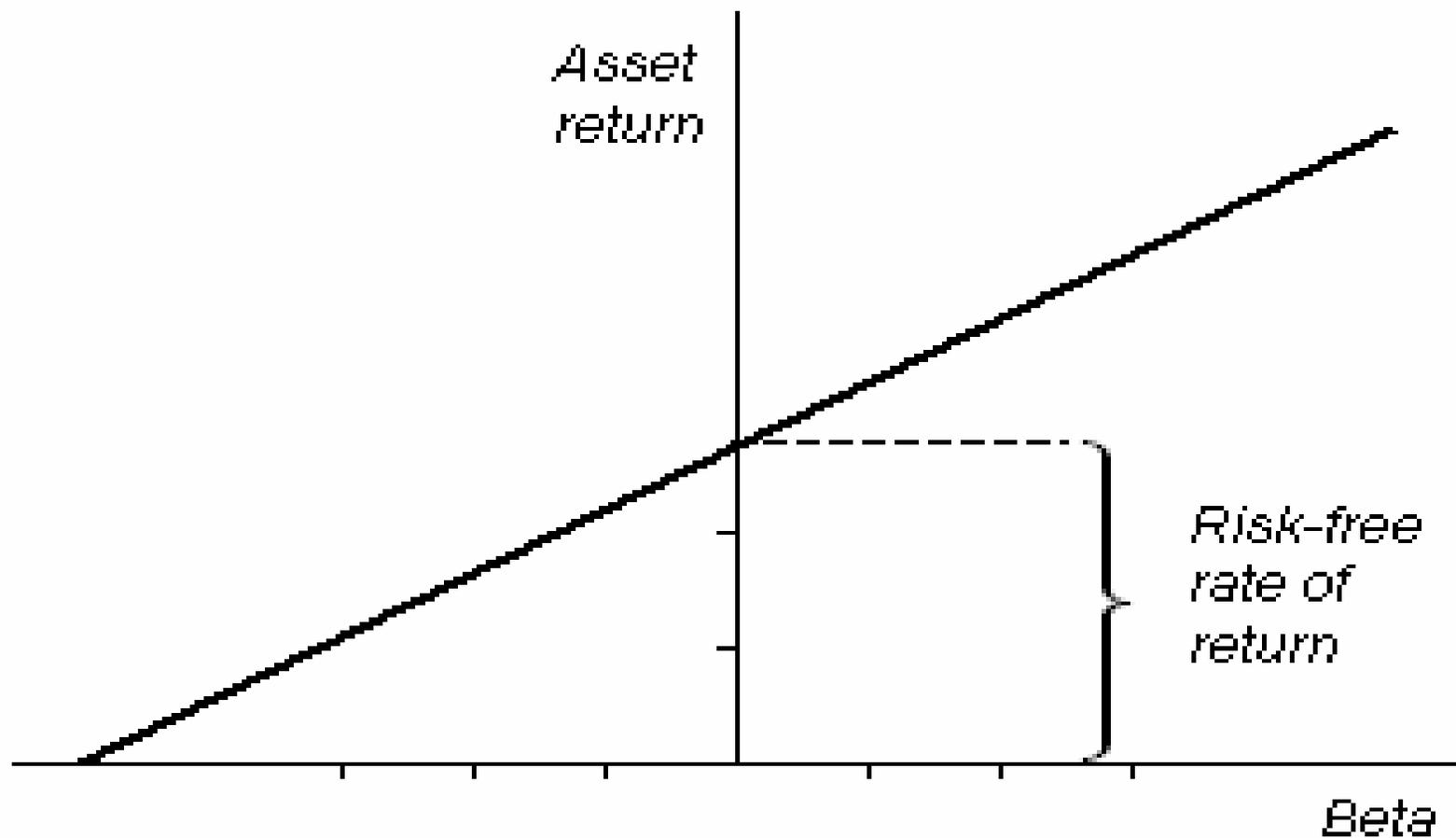
# Outline

- Part 1
  - CAPM: Linear relationship
  - Equity Risk Premium
  - Explanations
  - Summary
- Part 2
  - Empirical Study
  - Conclusion

# Remedial Finance 101

- Sharpe (1964):  $R_{i,t} = \alpha_i + \beta \times R_{m,t} + \varepsilon_{i,t}$
- In practice, often simplified to:  $R_{i,t} = \beta \times R_{m,t} + \alpha_{it}$
- CAPM assumes linear relationship between beta and asset return
  - The return on a stock goes up and down proportionality to its exposure to the market

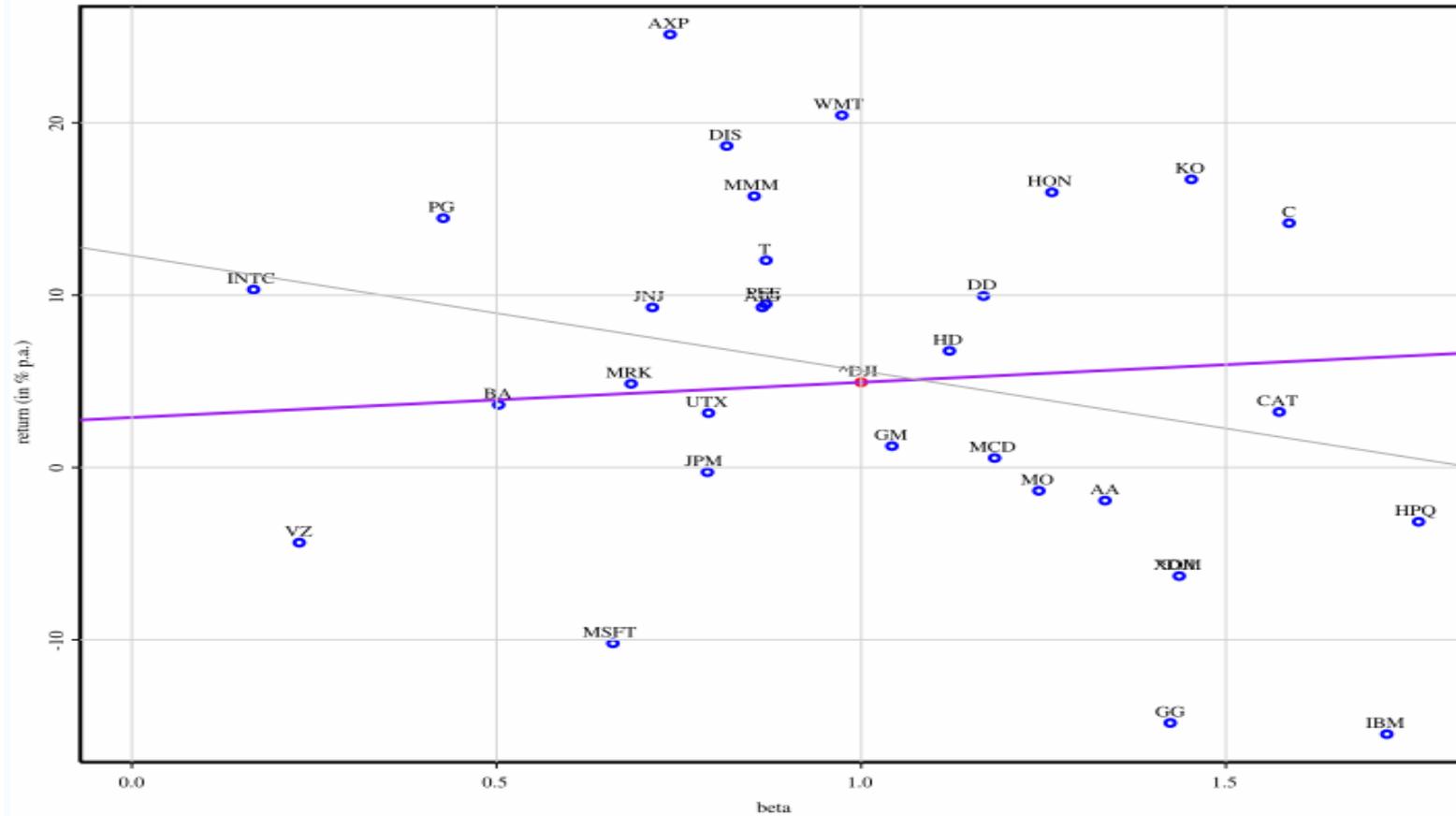
# Security Market Line



# SML Suggest

- The SML suggest that a portfolio with beta = 0.5 yields the same return as a portfolio that is 50% of the market portfolio and 50% cash
- Real market data indicates a flatter SML with smaller incremental increases in return for additional market exposure than CAPM would suggest

# Beta vs. Return for Dow



**Description** dow jones index and its 30 stocks; the CAPM and the security market line (purple), the grey line is the linear model, mean yield is assumed to be 2.9%

**Source** data by quote.yahoo, automatically retrieved with GNU R, see graph source below

**Date** 8 nov. 2006

**Author** Thomas Steiner

# “Equity Premium Puzzle?”

- Is this the phenomena coined by Mehra / Prescott (1986, 2003) describing the improbably high risk aversion one must have to own bonds given the immense equity return premium?
- Narrow the problem down to Equity Markets only, no bonds in the universe, only a Risk Free Rate.
- Does a Minimum Variance Portfolio (MVP) produce greater return per unit of risk?

# Define the “Equity Market Premium Puzzle”

- The very small marginal return per unit of risk in equity markets, demonstrated by a “flat capital market line”
  - According to Benartzi & Thaler (1993) an equity investor is indifferent between even odds of getting \$100,000/\$50,000 and a certain payoff of \$51,200;
  - Clarke, DeSilva & Thorley (2006) document a reduction in volatility of 25% holding MVPs while maintaining comparable returns for US markets between 1968 & 2005;
  - Blitz & Van Vliet (2007) document low volatility outperformance of 12%.
    - Lowest decile minus the highest volatility decile on a Global Universe on data between 1986 - 2006...
    - 6% if we exclude tech bubble

# How is this useful?

- In Equity markets, Minimum Variance Portfolios, leveraged up to the appropriate level of risk, offer better returns than Market Portfolios:
  - The addition of MVP to an asset allocation *Increases* equity contribution to both risk & return; (Arai, 2007)
  - Domination of cap weighted indices by MVP highlights the difference between Beta and Equity premium. (Arai, 2007).

# Behavioral

- Behavioral Finance:
  - Loss aversion / Prospect theory (Benartzi & Thaler - 1993)
    - Asymmetric utilities to gain & loss based on “reference point”
  - Short term risk indifference / Crash Aversion (Barro 2005), (Gabaix 2007)
    - Long term capital appreciation trumps short term volatility
    - Risk of major crash is the only one that matters

# Style

- Style effects (diBartolomeo):
  - Low volatility portfolios tend to be “value” biased
  - Value portfolios have negative skew, therefore investors expect higher than CAPM returns (CAPM assumes skew of 0) to compensate:
    - Think of momentum (buy on up, sell on down) as being analogous to CPPI, or being long a put option...
    - Then Value would be like being short a put option hence the negative skew...

# CAPM Assumptions

- Which Risk Free rate do we use?
  - CAPM assumes same rate for borrowing & lending
    - If there's a spread between borrowing & lending rates... investors would demand a higher return than predicted by CAPM to borrow money to leverage an MVP up to Market Portfolio Risk
  - Perhaps investors don't care about short term rates, but actually use longer term STRIPs
    - This would further flatten the SML...
    - Only applicable to normal, not inverted yield curve scenarios

# Real World Hindrances

- Haugen + Baker (1991) argue that Cap weighted market indices are inefficient due to increased transaction costs, taxes and restrictions on short selling.
  - They built a portfolio with of 1000 stocks with minimum variance over the trailing 24 months, then rebalanced quarterly;
  - The resultant portfolio had higher returns & lower variance than the Wilshire 5000 during the sample period: 1972 -1989.

# “Equity Yield Curve”

## Wisdom from Fixed Income Markets:

- Lochoff (1998) argues that buying at the short end of the yield curve & leveraging up to desired risk level yields higher returns due to greater marginal return per unit of risk at the short end of the curve;
- Applying this logic to equity markets doesn't take a leap of faith:
  - Applying a PV model of future cash flows puts low volatility stocks at the short end of the curve as they pay out in the short term
  - Conversely high volatility stocks are expected to pay cash flows further in the future and are thus at the long end of the “equity yield curve” (not my term...)
  - Risk is composed of duration + interest rate volatility, which decreases over time, making an even flatter frontier
- Bernstein & Tew “The Equity Yield Curve” add in a duration term to the CAPM to try to explain the higher beta of small cap stocks that have lower dividend yields.

# Mis-specified CAPM

- If the assumption that residuals of market returns are uncorrelated were true there would be no Northfield as we know it.
- Clearly if the model is not fully specified, the shape of the Efficient Frontier will reflect this:
  - If the model were to contain a low vs. high volatility factor, (Blitz & Van Vliet: difference between top & bottom decile by volatility) perhaps the Equity Premium Puzzle would be less puzzling.

# Extended CAPM Framework

- Explicitly accounting for the difference in returns between High & Low volatility Stocks could lead to a model like:

$$R_i = \beta R_{m,t} + \delta R_{volFMP} + \alpha_{i,t}$$

- Where  $R_{volFMP}$  is a factor modeling portfolio consisting of the top decile by volatility minus the bottom decile by volatility

# Mis-specified Market Portfolio

- If there are only Equities in the model:
  - The MVP will have significantly higher risk & return than than a corresponding MVP with Bonds.
  - The frontier will be much flatter.
- Introducing bonds to the universe, conversely will have the effect of increasing the marginal return per unit of risk.
- CAPM was never intended for use with just a single asset class

# Including Bonds

- Fuhrman (2004) breaks bonds into:
  - Those that should be counted as different maturity risk-free assets;
  - Those that should be part of the market portfolio (corporate bonds);
  - And those that should not be double counted (e.g. securitized bonds, C\_Os of all kinds).

# Fuhrman's Model

$$\beta = \frac{Cov(EqMkt, S)}{Var(EqMkt)}$$

$$\beta^* = \frac{w^* Cov(EqMkt, S) + (1 - w^*) Cov(r, S)}{w^* Var(EqMkt) + (1 - w^*) Cov(r, EqMkt)}$$

- $\omega^*$  = proportional weight of equity market in total market portfolio
- $S$  = given equity security
- $r$  = bond market
- $\beta^*$  = equity beta adjusted to reflect whole market portfolio
- $\beta$  = beta relative to the equity market only
- $EqMkt$  = equity market only

## Results –Financial Beta Up, Technology Down

<b>Sector</b>	<b>Equity <math>\beta</math></b>	<b>Market <math>\beta</math></b>	<b>% Diff</b>
Technology	1.452	1.350	-7.55%
Discretionary	0.915	0.883	-3.66%
Materials	0.925	0.896	-3.20%
Health	1.193	1.186	-0.61%
Industrial	0.988	0.983	-0.49%
Energy	0.700	0.713	1.81%
Telecom	0.846	0.872	2.96%
Consumer Staples	0.759	0.784	3.16%
Financial	1.107	1.161	4.69%

# Fuhrman Summarizes Fuhrman

- “In general, interest sensitive stocks will have total-market betas that are higher than their betas estimated by the equity markets alone.”
- Now that credit risk has been shown to be a big risk for financial and home building sector, is it possible that bonds and other credit factors should be included in the market proxy?

# Summary – Part 1

- CAPM frame provides a linear relationships between risk and return
- In practice the equity risk premium does not increase proportionality with risk
- Alternative theory attempt to further our understanding of this behavior
  - Specification to the CAPM
  - Application of fixed income model
  - Equity style
  - Investor behavior

# Empirical Research Using Northfield Risk Models

1. Reconfirm Arai's Results using the Northfield Fundamental Model Universe – including ADR and small (but not micro) cap stocks.
2. Reduce turnover by  $\frac{1}{2}$  to simulate a reasonable passive strategy.
3. If low beta is so great, what about 130/30 to get an even lower beta?

# Compare the NIS Fundamental and a Global Universe

## Comparing the Style Coefficients: Northfield Factor Model Estimation Universe S&P Broad Market Global Index

What's Missing in the NIS Universe: **Non US Value Stocks**  
**Non US Small Cap Stocks**

What's Missing in the S&P GBMI: **½ Domestic Small Cap Stock Weight**

Fund Name	S&P Citi Global Broad Market	Error*	Northfield Fundamental Universe	Error*
Russell 1000 Value	22.42	0.65	24.64	1.50
Russell 1000 Growth	23.93	0.58	36.43	1.33
Russell 2000	<b>5.35</b>	<b>0.53</b>	<b>12.27</b>	<b>1.23</b>
S&P/Citi Primary Growth World xU.S.	17.72	1.26	22.82	2.91
S&P/Citi Primary Value World xU.S.	<b>14.94</b>	1.44	<b>0.00</b>	3.34
S&P/Citi Emerging Markets	3.71	0.39	3.84	0.91
S&P/Citi Extended Markets World xU.S.	<b>11.94</b>	1.16	<b>0.00</b>	2.69
Tracking Error:	1.38		0.60	

# NIS Fundamental Universe vs. World Proxy

**What Weights Do You Need in Complementary Funds  
to best fit the S&P Broad Market Global Index?**

## Using S&P/Citi Indices

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error*</b>
<b>Northfield Fundamental Universe</b>	66.50	1.30
<b>S&amp;P/Citi Primary Value World xU.S.</b>	24.15	1.98
<b>S&amp;P/Citi Extended Market World xU.S.</b>	9.35	1.80
<b>Tracking Error:</b>	1.05	

## Using DFA Funds

<b>Fund Name</b>	<b>Style Wt.%</b>	<b>Error*</b>
<b>Northfield Fundamental Universe</b>	74.74	1.40
<b>DFA Tax Managed International Value</b>	19.87	2.19
<b>DFA International Small Company</b>	5.38	1.82
<b>Tracking Error:</b>	1.30	

# Empirical Results

<b>Variable</b>	<b>Bench- mark</b>	<b>Base Case</b>	<b>100% T/O</b>	<b>130/ 30</b>
<b>Systematic RAP</b>	N/A	1	1	1
<b>UnSystematic RAP</b>	N/A	3	3	5
<b>Average Ex-Ante Risk</b>	18.78	8.28	8.97	8.72
<b>Annualized Realized Risk</b>	15.55	12.33	12.02	11.57
<b>Average Ex-Ante Alpha</b>	0.00	0.00	0.00	0.00
<b>Annualized Realized Alpha</b>	0.00	1.68	1.80	-1.44
<b>Average Annualized T/O</b>	N/A	346	92	90
<b>Average Beta</b>	1.04	0.34	0.40	0.25

# Estimation Error – A Problem for MVP

- Minimum Variance Portfolios are Particularly sensitive to estimation error:
  - Cap weighted estimation universe has an ex-ante total risk of 18.78.
  - Cap weighted estimation universe has a realized total risk of 15.55 – in other words the risk model **overestimated** risk of a portfolio that approximates a market portfolio.
  - MVP has an ex-ante total risk of 8.28
  - MVP has a realized total risk of 12.33. In other words, the risk model **underestimated** risk of a portfolio designed to have a the minimum total risk.

# Research to be performed by you

1. Is this a portfolio that can be used to add alpha, or is the tilt to low beta enough?
2. What about other risk models?
  1. APT
  2. Single Country
  3. Global on a true global portfolio

# Topics for Empirical Study

- Test the extent to which the Equity Market Risk Premium persists when:
  - volatility is explicitly accounted for in the risk model;
  - Bonds are included in the investment universe.

# Conclusions

- There are many reasons, both Behavioral & Rational for the disproportionate risk adjusted returns of Equity Market Minimum Variance Portfolios
- Two ways one could try to rationalize this behavior in an extended CAPM framework are:
  - Expanding the universe of securities, including international stocks trading on domestic markets.
  - Explicitly accounting for systematic risk due to volatility in the risk model
- Minimum Variance Portfolios are particularly sensitive to risk model estimation error, since they favor stocks that will have low market, fundamental, industry and stock specific risks that are most likely to be underestimated.

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