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Morgan Stanley Investment Management

Fairness in Trading
(joint work with Steve Satchell)

Dr. B. Scherer, June 2009

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"They've made these fund prospectuses much easier to read."

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An Example: The UK Regulatory Environment

FSA Principle #6: “A firm must pay due regard to its customers and treat them fairly”

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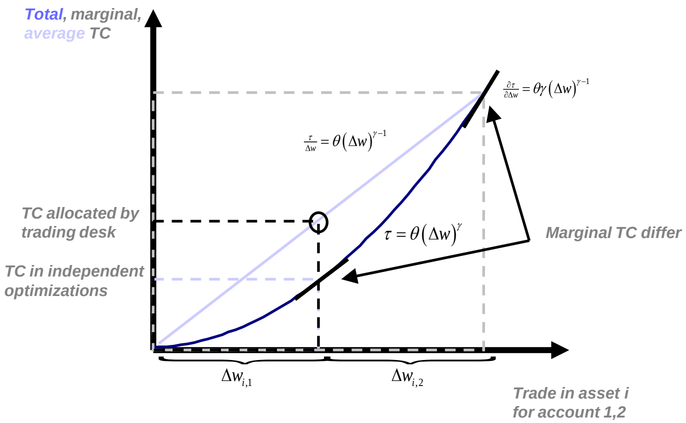
Optimization Across Multiple Accounts

Why is this a problem?

- Nonlinear transaction costs create an externality from one account one another



- Separate optimization of individual accounts will
    - not yield optimal positions
    - result in too much overall trading
    - underestimate alpha decay (capacity!) and transfer coefficient

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Optimization Across Multiple Accounts

Literature Review

- COASE (1960) Theorem: Market Solution For External Effects
- BRUNNERMEIER/PEDERSEN (2005) - Strategic Interactions among large traders
- Collusive, Pareto optimal solution (O'CINNEIDE /SCHERER/XU, 2006) versus COURNOT/NASH (CERIA, 2007)

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How Do Asset Management Firms Deal With This?

1. Ignorance (R^2 of 80%)
2. Randomization
   - Randomly select accounts that “trade first”. Disadvantages are cancelled out, Equal to random front-running
   - May take too long to establish fairness
   - Fair but inefficient
3. Trade accounts at different days of the month
   - Maximum # of accounts is 31?
   - Large dispersion
4. Optimize one representative “super account”
   - How? Replace the separate accounts by a single fund where fund size reflects different tracking errors / volumes
   - Only feasible if all clients have the same constraints

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Our Proposal

- We describe a procedure where separate optimizations can be combined and enhanced to reflect the interactions between the accounts due to trading.
- Given this aggregation of trading requirements, an optimization problem must be formulated that will allow each client to “see” the cost of trading based on the aggregate trading volume and not just on his own volume.
- Of course, the conundrum this presents is that we cannot determine one client’s trading needs until we know those of the others, so it is unclear how to get the process started.
- The result is “multi-account optimization” (O’Cinneide/Scherer/Xu, 2006)

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The General Case

- The utility function (RAPM for j-th the account) is given by

$$U_j\left(n_j, \sum_{i=1}^{m} n_i\right) = u(n_j) - t\left(\sum_{i=1}^{m} n_i\right)\frac{n_j}{\sum_{i=1}^{n} n_i}$$

- The trading cost function is convex

$$\frac{dt\left(\sum_{i=1}^{m} n_i\right)}{dn_j} > \frac{t\left(\sum_{i=1}^{m} n_i\right)}{\sum_{i=1}^{m} n_i}$$

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Nash Solution

- FOC (take derivative with other accounts held constant)

$$\frac{dU\left(n_j, \sum n_i\right)}{dn_j} = \frac{du\left(n_j\right)}{dn_j} - \frac{dt\left(\sum n_i\right)}{dn_j} \cdot \frac{n_j}{\sum n_i} + \frac{t\left(\sum n_i\right)}{\sum n_i} - \frac{n_j t\left(\sum n_i\right)}{\left(\sum n_i\right)^2} = 0$$

- The optimal solution trades off marginal utility from investment performance versus transaction costs which in turn are a trading weighted combination of marginal and average transaction cost.

$$\frac{du\left(n_j\right)}{dn_j} = \left[\frac{n_j}{\sum n_i}\right] \cdot \frac{dt\left(\sum n_i\right)}{dn_j} + \left(1 - \left[\frac{n_j}{\sum n_i}\right]\right) \cdot \frac{t\left(\sum n_i\right)}{\sum n_i}$$

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What Would Happen If Everybody Traded a bit Less?

- Note: No **INDIVIDUAL** incentive to deviate as FOC for NASH is satisfied
- If we would however start trading slightly less for account k …

$$\frac{dU\left(n_j, \sum n_i\right)}{dn_k} = -\left[\frac{n_j}{\sum n_i}\right]\left[\frac{dt\left(\sum n_i\right)}{dn_k} - \frac{t\left(\sum n_i\right)}{\sum n_i}\right] < 0$$

- … the accounts could do collectively better as

$$\frac{dU\left(n_j, \sum n_i\right)}{dn_k} < 0$$

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The Intuition

- We can make **ALL** clients better off in a collusive equilibrium
- COURNOT/NASH equilibrium is unacceptable for an asset manager that is required to achieve the best for his clients
- In a COURNOT/NASH equilibrium each client pays the average costs of trading but creates higher (convex cost function) marginal costs on the “community”. Ignoring these costs hurts all.

Multiple Account Optimization – An Example

Model Set Up (joint work with Steve Satchell)

- Two accounts of different size, *s*, that trade one asset *n*.
- Quadratic transaction cost function

$$t = \frac{q}{2}\left(n_1 s_1 + n_2 s_2\right)^2$$

$$t_{total} = \underbrace{t \times \frac{(n_1 s_1)}{(n_1 s_1 + n_2 s_2)}}_{account\ 1} + \underbrace{t \times \frac{(n_2 s_2)}{(n_1 s_1 + n_2 s_2)}}_{account\ 2}$$

- Standard preferences for each account (note that the cost term reflects cost sharing, i.e. both accounts trade simultaneously

$$VA_i = n_i s_i m - \frac{l}{2}(n_i s_i)^2 s^2 - \frac{q}{2}(n_i s_i + n_j s_j)(n_i s_i)$$

- Note: utility equals risk adjusted performance measure

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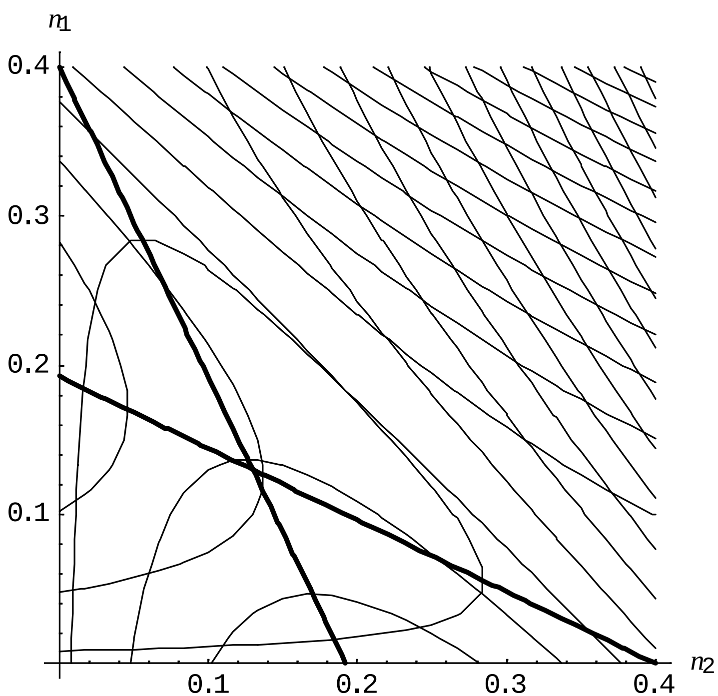
Stand Alone Solution

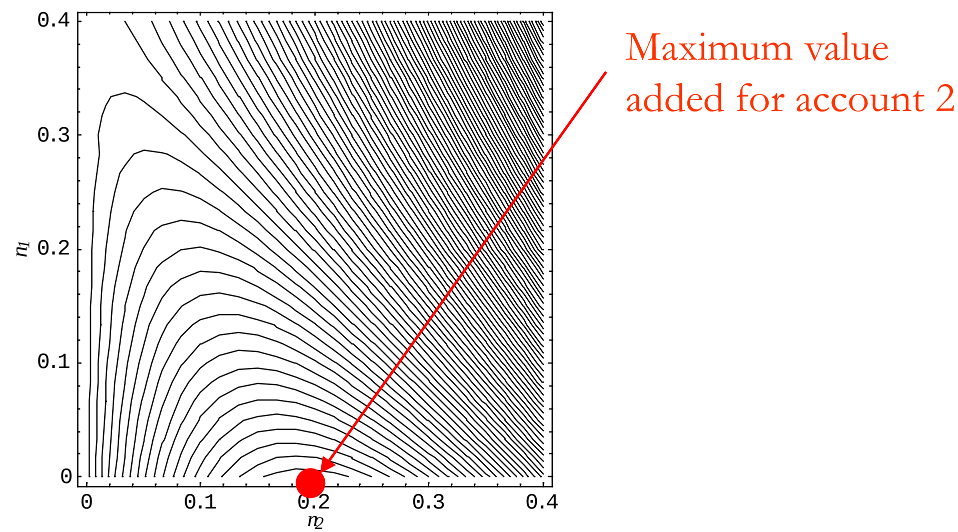
Optimize accounts separately without taking interactions into account (batch job)

- "Optimal" trading

$$n_i^{SA} = \arg\max_{n_i} \left[n_i s_i \mu - \frac{\lambda}{2}(n_i s_i)^2 \sigma^2 - \frac{\theta}{2}(n_i s_i)^2 \right] = \frac{\mu}{s_i(\theta + \lambda \sigma^2)}$$

$$VA_i^{SA} + VA_j^{SA} = \frac{\lambda \mu^2 \sigma^2}{(\theta + \lambda \sigma^2)^2}$$





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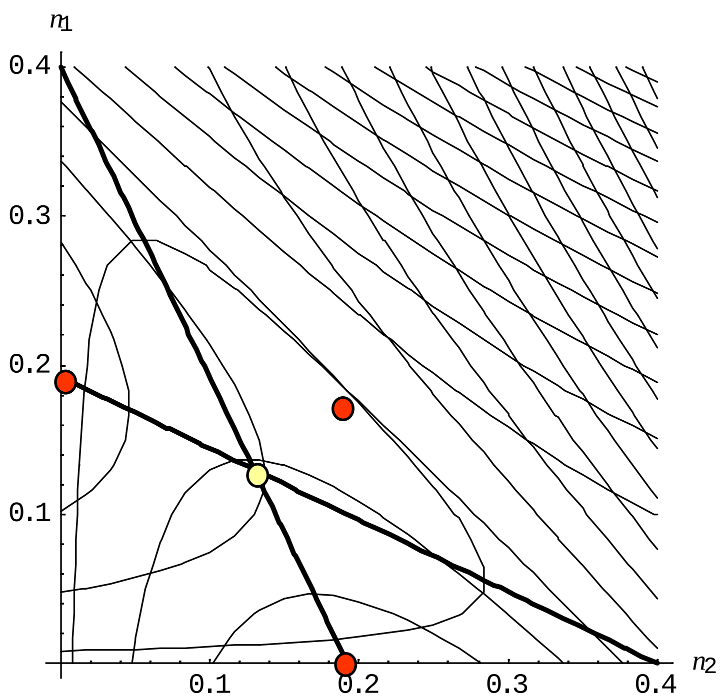
COURNOT/NASH-Solution

Interaction is accounted for but treated as given

- First order condition (solving leads to reaction functions below)

$$\frac{dVA_i}{dn_i} = m s_i - l\, n_i s_i^2 s^2 - q n_i s_i^2 - \tfrac{1}{2} q n_j s_i s_j = 0$$



$$n_j^{CN} - n_j^{SA} = \frac{2m}{s_j\left(3q + 2l\, s^2\right)} - \frac{m}{s_j\left(q + l\, s^2\right)}$$

$$= -\frac{qm}{s_j\left(q + l\, s^2\right)\left(3q + 2l\, s^2\right)} < 0$$

$$VA_i^{CN} - VA_i^{SA} = \frac{2m^2\left(q + l\, s^2\right)}{\left(3q + 2l\, s^2\right)^2} - \frac{l\, m^2 s^2}{2\left(q + l\, s^2\right)^2}$$

$$= \frac{m^2 q^2\left(4q + 3l\, s^2\right)}{2\left(q + l\, s^2\right)^2\left(3q + 2l\, s^2\right)^2} > 0$$

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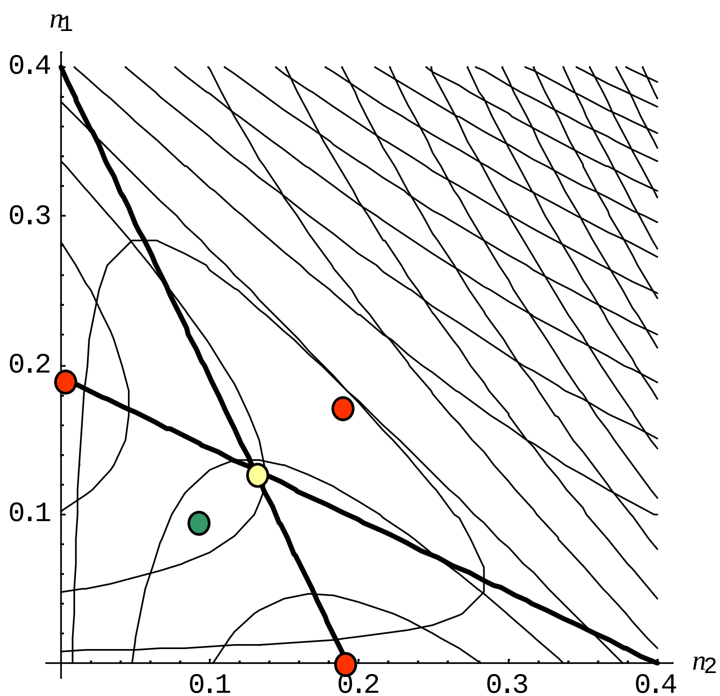
Collusive Solution

Full interaction is accounted for

- Combined objective function (monopoly)

$$VA = VA_i + VA_j = n_i s_i m - \tfrac{l}{2} s_i^2 n_i^2 s^2 - \tfrac{q}{2}(n_i s_i + n_j s_j)(n_i s_i) + n_j s_j m - \tfrac{l}{2} s_j^2 n_j^2 s^2 - \tfrac{q}{2}(n_i s_i + n_j s_j)(n_j s_j)$$

- Leads to less trading and higher value added (risk adjusted client



$$n_i^C / n_i^{CN} = 1 - \frac{q}{4q + 2l\,s^2} < 1$$

$$VA_i^C - VA_i^{CN} = \frac{1}{2}\frac{q^2 m^2}{(2q + l\,s^2)(3q + 2l\,s^2)^2} > 0$$

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Does Account Size Matter?

Changing the model setup

- So far total trading (weight times account size) has been the same for both accounts under the collusive solution.

- This is not “super-realistic”. In reality total trading is higher for large accounts.

- We model risk aversion the inverse of account size, i.e. we trade more for large accounts as they exhibit lower risk aversion.

$$l_i = \frac{1}{s_i}$$

- Now: portfolio weights are the same, but total trading differs

$$n_1^C = n_2^C = \frac{m}{q(s_1 + s_2) + s^2}$$

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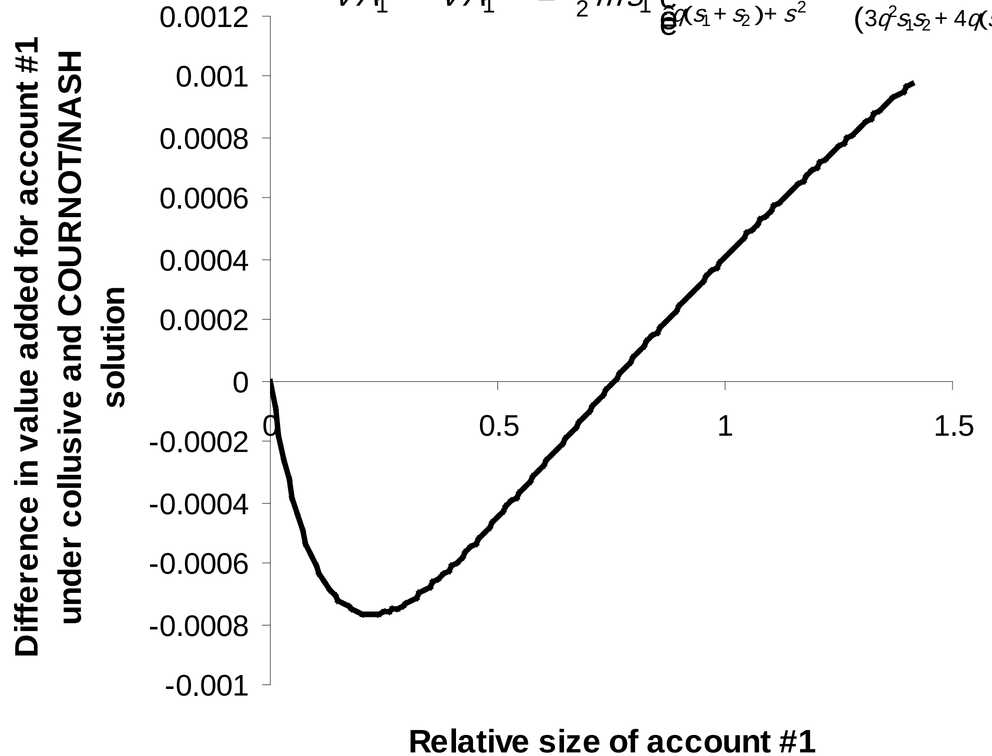
COURNOT/NASH versus Collusive Solution

The Impact of Account Size

$$VA_1^C - VA_1^{CN} = \tfrac{1}{2} m^2 s_1 \left[\frac{1}{q(s_1+s_2)+s^2} - \frac{4(qs_1+s^2)(qs_2+2s^2)^2}{(3q^2 s_1 s_2 + 4q(s_1+s_2)s^2 + 4s^4)^2} \right]$$



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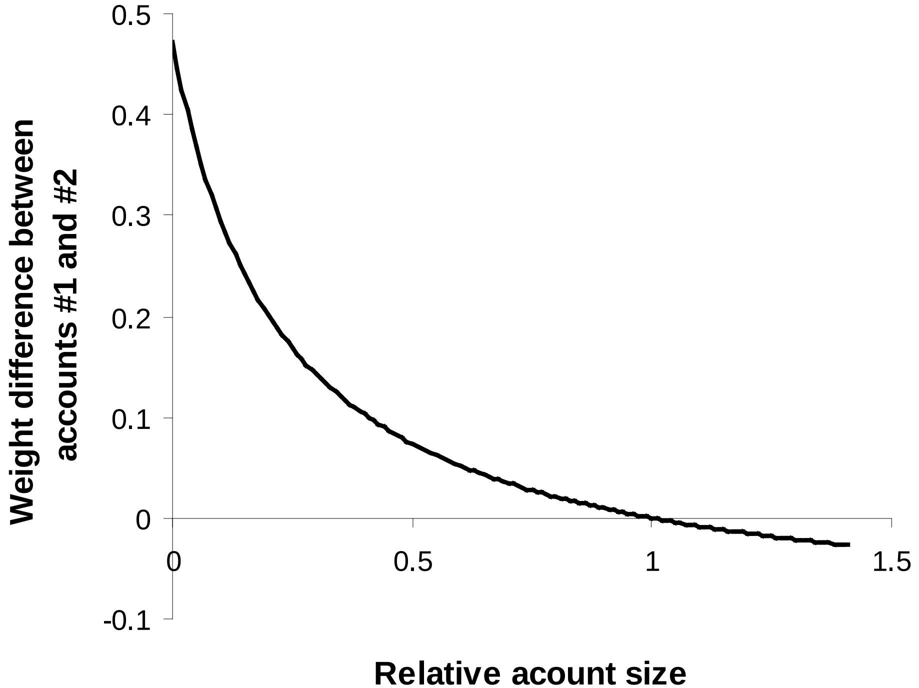
Why Do Small Accounts Prefer COURNOT/NASH?

Excess Trading in COURNOT/NASH equilibrium benefits the small account



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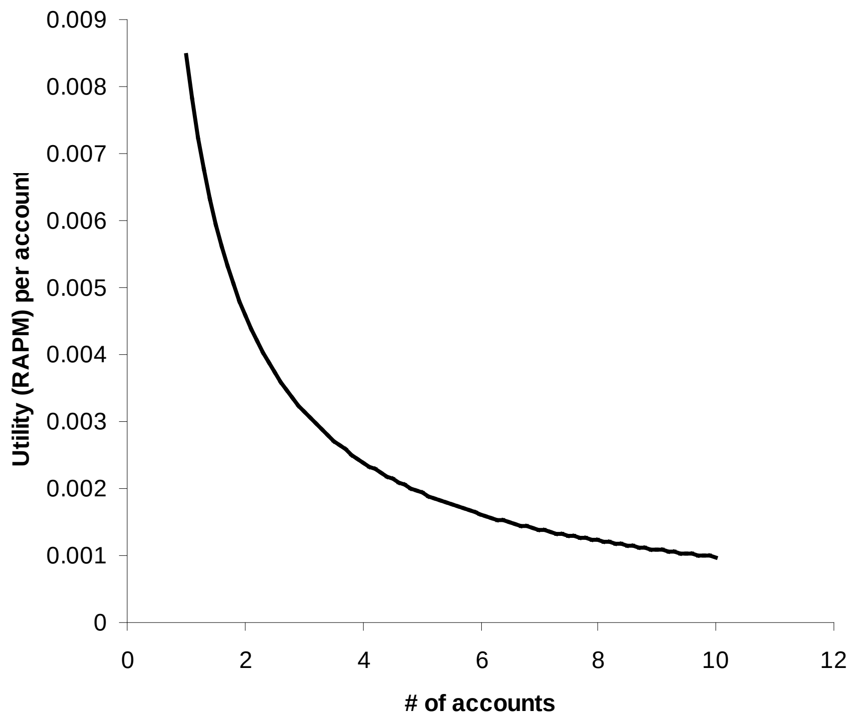
Implications For Asset Management: CAPACITY!



- Performance falls as the number of (equal sized accounts) increases. For a given critical value (y-axis) we can infer capacity (from the x-axis).

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