Equal opportunity currency investing

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DEFI

The principle of equal investment opportunity

- In principle all investors face the same investment opportunity, also in currency investing.

    - In particular, if two investors with different base perspectives bet on the same currency movement, their payoff will be identical.

    - On the same token, if the investors have identical return- and risk expectations, their mean-variance optimised currency portfolios must have the same expected payoff.

The principle of equal investment opportunity

- In order to respect the principle, consideration must be given to the way
  - return is defined,
  - risk is defined.
- We investigate what goes wrong if no consideration is given to either of the two.

The SIEGEL paradox

inconsistent returns

- Example : an American and a European exchange 100 000 dollars when the spot is at 1:1, and pay back when the spot is at 1.20 : 1 (dollars for a euro).

	inconsistent measure	consistent measure
American wins $ 20 000	$R^{arithmatic} = 1.20 - 1 = 20\%$	$R^{logarithmic} = \ln(1.20) = 18\%$
European loses $ 20 000	$R^{arithmatic} = 1/1.20 - 1 = -17\%$	$R^{logarithmic} = \ln(1/1.20) = -18\%$

The SIEGEL paradox

inconsistent return expectation

- Example of Black* : at t_0 an apple trades for an orange,

  at t_1 either one trades for two or two for one, with equal probability

- 1:1
  - P(2:1) = 0.5
  - P(1:2) = 0.5

- Inconsistent measure $$E[R] = 0.5 \cdot \left(\frac{2}{1} - 1\right) + 0.5 \cdot \left(\frac{1}{2} - 1\right) = 0.25$$

- Consistent measure $$E[R] = 0.5 \cdot \ln\left(\frac{2}{1}\right) + 0.5 \cdot \ln\left(\frac{1}{2}\right) = 0$$

* Black (1995) “Universal Hedging” *Financial Analysts Journal*

What goes wrong

undue opportunity

Siegel traders see performance opportunity in making pairs of mirrored deals.

08/05/09 Cours Spot Afficher Var % sur 1 jo Source CMPL

Composite (Londres

	USD	EUR	JPY	GBP	CHF	CAD	AUD	NZD	HKD	NOK	SEK
SEK	-.001	-.255	.343	.194	-.351	-.835	-.196	.720	-.001	.209	-
NOK	-.210	-.463	.134	-.015	-.559	-1.042	-.405	.510	-.210	-	-.209
HKD	.000	-.254	.344	.196	-.349	-.834	-.195	.721	-	.210	.001
NZD	-.716	-.968	-.374	-.522	-1.063	-1.544	-.910	-	-.716	-.507	-.715
AUD	.195	-.059	.540	.391	-.155	-.640	-	.918	.195	.406	.197
CAD	.841	.585	1.188	1.038	.489	-	.645	1.569	.841	1.053	.842
CHF	.351	.096	.696	.547	-	-.486	.155	1.075	.351	.562	.352
GBP	-.195	-.448	.149	-	-.544	-1.028	-.390	.525	-.195	.015	-.194
JPY	-.343	-.596	-	-.148	-.691	-1.174	-.537	.376	-.343	-.133	-.342
EUR	.254	-	.600	.450	-.096	-.582	.059	.978	.254	.465	.256
USD	-	-.254	.344	.196	-.349	-.834	-.195	.721	.000	.210	.001

What goes wrong

undue opportunity

- The existence of Siegel trading is one thing …

  … the anxiety it provokes is another !

  - Edlin (2002) accuses Siegel traders to be responsible for the reported forward discount bias.

  - Kemp and Sinn (2000) argue that Siegel traders lower welfare and should for this reason be banned from trading.

What goes wrong

undue beliefs

- Belief : the Uncovered Interest Parity can only hold in real terms.
- Engel (1984) : "Only expected real profits from forward market speculation can be strictly zero."
- His demonstration
  - resolves the Siegel paradox through a modified return measure,
  - proves nothing in terms of exchange rate behaviour.

What goes wrong

undue beliefs

- Black (1989,1995) ignores the inconsistency issue when
  - he mentions that there is mutual benefit in holding foreign currencies,
  - he derives the universal currency hedge ratio.
- In reality,
  - the expected gain from structurally holding foreign currencies is strictly zero in principle,
  - the optimal currency hedge ratio is 100% by default, yet may divert depending on the covariance structure with the assets held.

What goes wrong

undue beliefs

The partial hedge of the Market Portfolio containing equity and bond positions in seven major markets, which Black & Litterman (1992) bring forward, is justified by the covariance structure in their example :

Equity

Currency / corr	DEM	FRF	JPY	GBP	USD	CAD	AUD
DEM	2%	3%	5%	-1%	3%	5%	0%
FRF	3%	8%	4%	4%	6%	9%	5%
JPY	1%	10%	18%	4%	-2%	4%	12%
GBP	2%	5%	6%	6%	-2%	11%	15%
CAD	5%	10%	12%	14%	24%	32%	18%
AUD	-1%	7%	5%	6%	7%	19%	27%

Bonds

Currency / corr	DEM	FRF	JPY	GBP	USD	CAD	AUD
DEM	36%	15%	27%	9%	26%	24%	9%
FRF	33%	15%	21%	9%	22%	21%	8%
JPY	21%	19%	45%	19%	18%	22%	9%
GBP	22%	5%	24%	27%	18%	25%	9%
CAD	14%	4%	5%	13%	15%	24%	13%
AUD	5%	-3%	10%	5%	0%	4%	20%

Resolving the return inconsistency issue

- Option 1 : All currency investors adopt the logarithmic return measure.
- Option 2 : All currency investors adopt the return measure from one standard perspective, as suggests Chu (2005).
- Option 3 : Be aware of the measurement problem when comparing between currency perspectives.

➔ First step towards respecting the principle of equal investment opportunity

Risk definition

inconsistent returns lead to inconsistent risk

European view	*dollar-euro*	**American view**
Return	*SPOT*	Return
	1.25	
4.17%	*1.20*	-4.00%
-7.69%	*1.30*	8.33%
2.36%	*1.25*	-2.31%
Volatility		Volatility
5.23%		5.50%

Consistent risk definition

using log-returns

- In log-returns the covariance matrices measured from different base perspectives are exact rotations of each other.

- Example between dollar, euro and yen

$	€	¥
€	0.82%	0.32%
¥	0.32%	0.83%

€	$	¥
$	0.82%	0.49%
¥	0.49%	1.00%

¥	€	$
€	1.00%	0.51%
$	0.51%	0.83%

- $V_{\$} = L^{T} V_{€}\, L$ where the conversion operator L is

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Formally,

- The conventional return measure doesn't respect the Euclidean properties, when applied on currencies

- Let $D: \Re^n \rightarrow \Re$ be a Euclidean distance function. Then

$$
\begin{array}{lll}
(i) & \text{positive semi-definiteness} & D(x, y) \geq 0 \quad \text{and } D(x, y) = 0 \text{ if } x = y \\
(ii) & \text{symmetry} & D(x, y) = D(y, x) \\
(iii) & \text{triangular inequality} & D(x, y) \leq D(x, z) + D(z, y)
\end{array}
$$

- The return volatility is not a Euclidean distance function for it doesn't respect the property of symmetry (*ii*).

- In non-Euclidean space the laws of algebra don't apply.

Currency risk structure

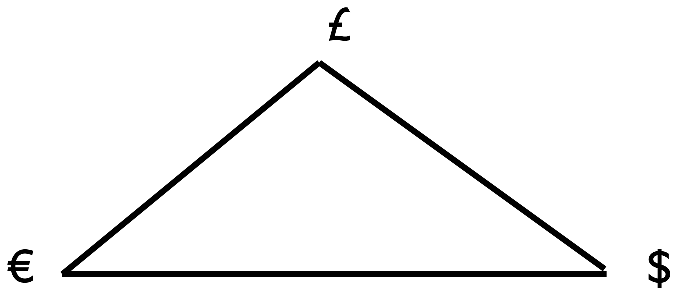
geometrical representation

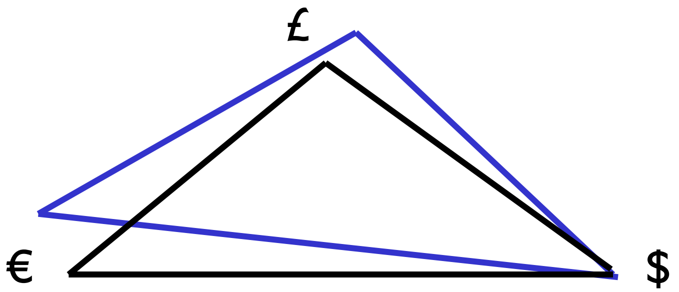
Euclidean space

using the logarithmic norm

non-Euclidean space

using the Laspeyres norm





The arctangents of the angles correspond to the correlation between the two currencies on the legs measured from the currency in the corner.

Consistent risk forecasts

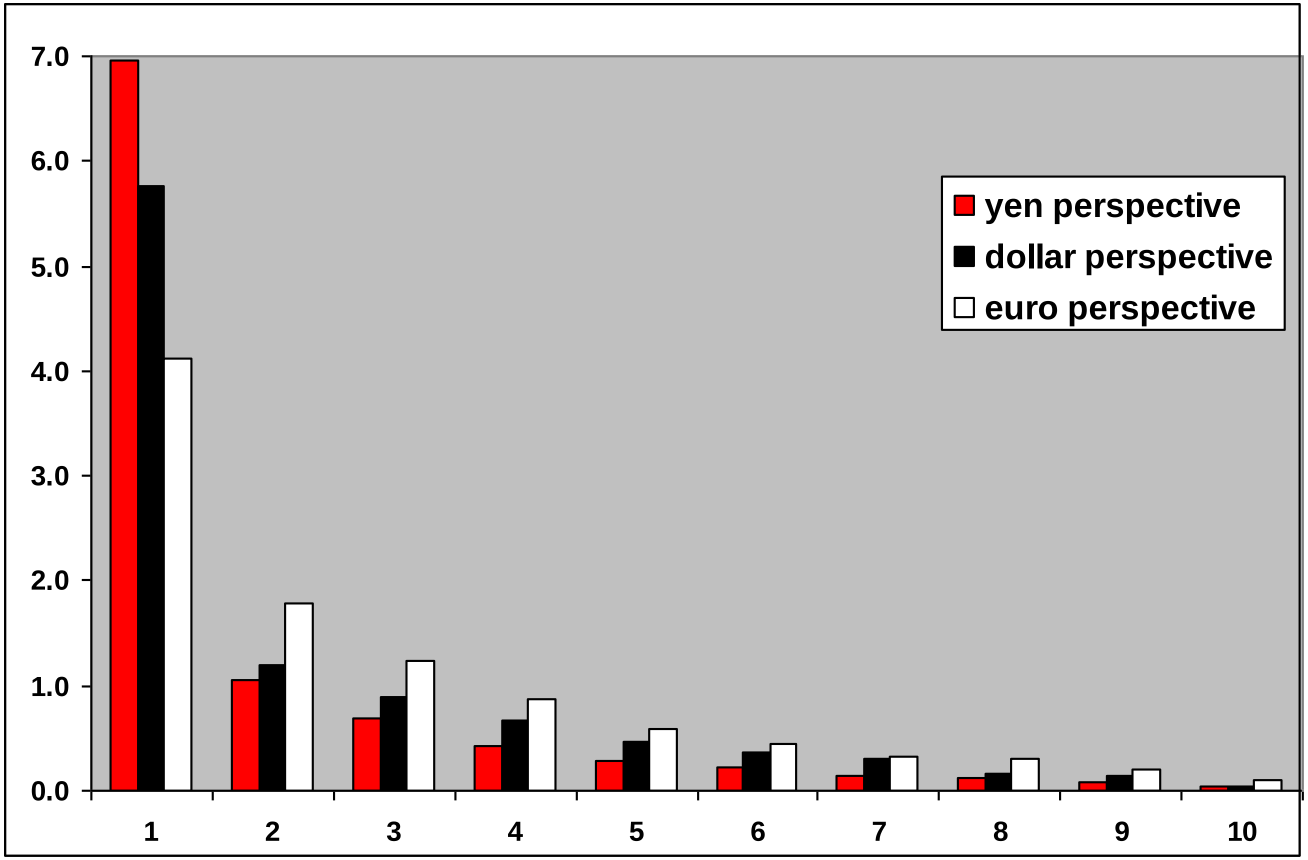
- Modelled risk is not necessarily consistent between perspectives, since typically a restrained number of risk factors is retained in a model.

- The structural covariance matrices estimated from a dollar, euro and yen perspective are not automatically exact rotations of each other, even if the log-norm is applied.

➔ Model risk is not automatically consistent

Model risk is not automatically consistent …

… for the spectral decomposition of currency risk differs.



***measured over 1999 to 2009 among the ten developed world currencies.**

Model risk is not automatically consistent

- If all investors would build a one-factor risk model from their home perspective,
  - a Japanese would have 70% of the variance explained,
  - an American would have 58% explained,
  - and a European 41% explained.
- Those models don't give an equal perception of risk.

Risk modelling approach

- The perspective in which a currency risk model is estimated matters.

General case : ***a multiple of perspectives***

- A *financially sound* model - providing equal opportunity universally - should fit best in all perspectives.

Particular case : ***one common perspective***

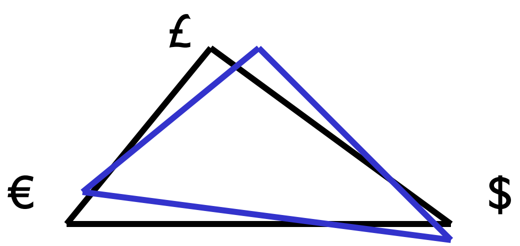
- In a business situation where all clients have the same currency perspective, estimating within this perspective serves best.

What goes wrong

in case of multiple perspectives

define several currency spaces

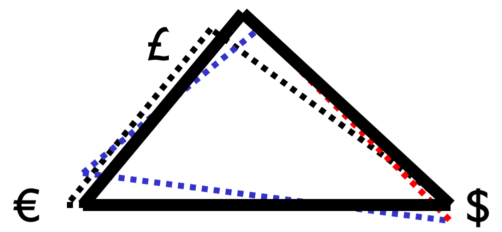
fitted one by one



- No universal risk perception
  - no equal opportunity
  - no portable alpha

define a common currency space

finding the best simultaneous fit



- Universal risk perception
  - equal opportunity
  - portable alpha

An *equal opportunity* risk model

in case of multiple perspectives

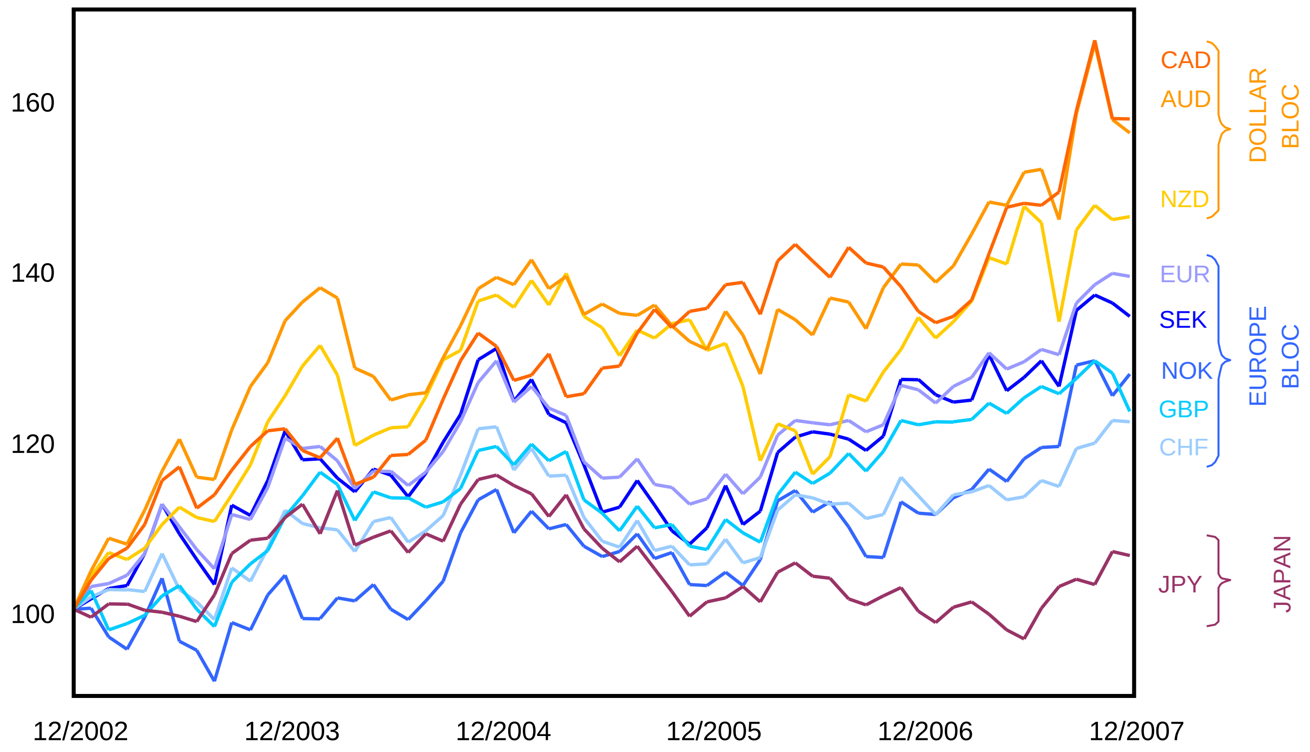
- A model that defines the risk structure effectively in all perspectives.

- Thanks to a stylised fact it is feasible to build such model, i.e.
  - there are two currency clusters : Europe and the dollar zone,
  - their adverse movements explain ± 15% of the variance in **all** perspectives.

- A financially sound model can be specified with two factors:
  - a dollar factor : separating the US dollar from the rest,
  - an inter-zone factor : separating the two clusters.

Stylised fact

two currency clusters



Sources: Bloomberg. Sinopia Calculations

Model specification

in US dollar perspective

- A two-factor linear model : $$R_{it} = \beta_i \cdot F_t^{\$} + \gamma_i \cdot F_t^{zone} + \varepsilon_{it}$$

where $\boldsymbol{R_{it}}$ is the (centred) exchange rate return of currency *i* with respect to the dollar over month *t*, $\boldsymbol{\beta_i}$ and γ_i are the sensitivities of currency *i* to the dollar factor $\boldsymbol{F^{\$}}$ and the zone factor $\boldsymbol{F^{zone}}$ respectively, and ε_{it} are i.i.d. residuals.

- Schematically :

currency	β	γ
Canadian dollar Australian dollar New Zealand dollar	≈ 1	positive (τ)
Euro Swedish crown Norwegian crown Sterling Swiss franc	≈ 1	negative ($-\tau$)
Yen	≈ 0	≈ 0

Model specification

in US dollar perspective

- The covariance matrix $V_\$$ is specified as

$$V_\$ = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \cdot F \cdot \begin{bmatrix} \beta \\ \gamma \end{bmatrix}^T + \Sigma$$

Σ being a diagonal matrix of residual variances and ***F*** a diagonal matrix of factor variances.

- Schematically, for the three currency categories (*dollar zone, Europe, Yen*)

$$V_\$ = \begin{bmatrix} 1 & \tau \\ 1 & -\tau \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_\$^2 & 0 \\ 0 & \sigma_{zone}^2 \end{bmatrix} \cdot \begin{bmatrix} 1 & \tau \\ 1 & -\tau \\ 0 & 0 \end{bmatrix}^T + \Sigma$$

Model rotation

Consistency check

The model is converted into perspective *x* through the rotation operator *L*, resulting in :

Europe

$$\begin{bmatrix} 0 & -2\tau \\ 1 & -\tau \\ 1 & -\tau \end{bmatrix}$$

US dollar

$$\begin{bmatrix} 1 & \tau \\ 1 & -\tau \\ 0 & 0 \end{bmatrix}$$

Japan

$$\begin{bmatrix} -1 & -\tau \\ -1 & \tau \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \tau \\ 0 & 2\tau \\ 1 & \tau \end{bmatrix}$$

dollar zone

Model adaptation

- American and Japanese have a similar perception

$	EUR	CHF	NZD	AUD	JPY
EUR		0.91	0.61	0.63	0.30
CHF			0.50	0.44	0.43
NZD				0.82	0.11
AUD					0.09
JPY					

¥	EUR	CHF	NZD	AUD	USD
EUR		0.93	0.73	0.75	0.57
CHF			0.63	0.59	0.47
NZD				0.88	0.55
AUD					0.55
USD					

- European and Australian have different perceptions

€	USD	CHF	NZD	AUD	JPY
USD		0.10	0.23	0.20	0.62
CHF			-0.14	-0.39	0.37
NZD				0.72	0.07
AUD					0.03
JPY					

AUD	EUR	CHF	NZD	USD	JPY
EUR		0.95	0.35	0.63	0.64
CHF			0.41	0.65	0.71
NZD				0.32	0.28
USD					0.78
JPY					

* **Tables show correlations measured over 1999-2008 on monthly data**

Conclusion

- In order to respect the principle of equal opportunity in currency investing, one needs to
    - adopt log-returns
    - build a multi-fit risk model

- The issue is directly relevant for
    - asset managers serving clients with different home perspectives,
        - to harmonise their portfolio optimisation procedures
        - deliver portable alpha
    - risk model providers …