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A Note On Minimum Variance Investing

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Two Related Conjectures

- Risk minimization in itself is a meaningless exercise
  - The minimization of risk is on its own a meaningless objective. Risk needs to be traded off against return. The same applies to related concepts that try to maximize "diversity" as in FERNHOLZ (1999) or CHOUEIFATY/COIGNARD (2006) or to minimize concentration as in KING (2007).

- In a market without structure no portfolio allocation rule can outperform
  - I conjecture that the portfolio construction process behind minimum variance investing implicitly picks up risk based pricing anomalies. In other words the minimum variance portfolio leverages on widely published risk based mispricing. Effectively it tends to hold low beta and low residual risk stocks.

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Historical Evidence

- Inspired by early work from HAUGEN/BAKER (1991). For the period covering the years 1972 to 1989 the authors found that a minimum variance performance would outperform the Wilshire 5000 at lower risk.

- A vast number of studies followed their original paper. For the US stock market CHAN/KARCESKI/LAKONISHOK (1999), SCHWARTZ (2000) and JAGANNATHAN/MA (2003) and CLARKE/SILVA/THORLEY (2006) found both higher returns and lower realized risks for the minimum variance portfolio (MVP) versus a capitalization weighted benchmark.

- For global equity markets GEIGER/PLAGGE (2007), POULLAOUEC (2008) and NIELSEN/AYLURSUBRAMANIAN (2008) and all find qualitatively similar results.

Basic Portfolio Theory

- World with no risk free asset: Two fund separation, i.e. investors choose a combination of minimum risk portfolio and speculative demand depending on their risk aversion.

- World with risk free asset: Two fund separation, i.e. investors choose a combination of cash (minimum risk portfolio) and speculative demand depending on their risk aversion.
  - MVP is a dominate alternative. It offers less return per unit of risk as it deviates from efficiency.
  - Risk reduction is better achieved by adding cash
  - MVP becomes a mathematical artefact

- Does Sampling Error Change the Verdict? No! Rational (Bayesian) investors will still combine cash and market portfolio.
  - Investors will put more wealth into cash as cash is the only asset free of investment *and* estimation risk. Market portfolio remains unchanged

Optimal Weights in the MVP (1/3)

So far, I simply conjectured that the minimum variance portfolio is likely to invest into low residual risk and low beta stocks. Without this section the paper would be subject to suspicion of data mining

In a CAPM world with k assets, the $k \times k$ covariance matrix, $\mathbf{\Omega}$, can be decomposed into

(1) $$\mathbf{\Omega} = \boldsymbol{\beta}\boldsymbol{\beta}^T \sigma_m^2 + diag\left(\sigma_1^2, \sigma_2^2, ..., \sigma_k^2\right) = \boldsymbol{\beta}\boldsymbol{\beta}^T \sigma_m^2 + \mathbf{\Sigma}$$

Here, $\mathbf{\Sigma}$, denotes the $k \times k$ diagonal matrix of residual variances, $\sigma_1^2, \sigma_2^2, ..., \sigma_k^2$, on the main diagonal, $\boldsymbol{\beta}$ represents a $k \times 1$ vector of security betas, $\beta_1, \beta_2, ..., \beta_k$ and σ_m^2 represents the market variance. The inverse of $\mathbf{\Omega}$ is given by

(2) $$\mathbf{\Omega}^{-1} = \mathbf{\Sigma}^{-1} - \frac{\sigma_m^2}{1+\kappa}\mathbf{b}\mathbf{b}^T$$

I define $\kappa = \sigma_m^2 \sum_{i=1}^{k} b_i \beta_i$ and $b_i = \frac{\beta_i}{\sigma_i^2}$. The $k \times 1$ vector of optimal portfolio weights $\mathbf{w}$ for the minimum variance portfolio represents a characteristic portfolio

(3) $$\mathbf{w} = \frac{\mathbf{\Omega}^{-1}\mathbf{I}}{\mathbf{I}^T\mathbf{\Omega}^{-1}\mathbf{I}}$$

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Optimal Weights in the MVP (2/3)

where the $k \times 1$ vector $\mathbf{I}$ of ones, denotes the characteristics. Minimum variance portfolio risk amounts to $\sigma_{mv}^2 = \frac{1}{\mathbf{I}^T \mathbf{\Omega}^{-1} \mathbf{I}}$, where $\mathbf{I}$ represents a vector of ones. Substitute (2) in (3) to arrive at

(4) $$\mathbf{w} = \sigma_{mv}^2 \left(\mathbf{\Sigma}^{-1} \mathbf{I} - \frac{\sigma_m^2}{1+\kappa} \mathbf{b}\mathbf{b}^T \mathbf{I} \right)$$

We can now calculate the optimal holding w_j as an element of the right hand side $k \times 1$ vector in (4).

(5) $$\begin{aligned} w_j &= \sigma_{mv}^2 \left(\frac{1}{\sigma_j^2} - \frac{\sigma_m^2}{1+\kappa} b_j \sum_{i=1}^k b_i \right) \\ &= \frac{\sigma_{mv}^2}{\sigma_j^2} \left(1 - \frac{\sigma_m^2}{1+\kappa} b_j \sigma_j^2 \sum_{i=1}^k b_i \right) \\ &= \frac{\sigma_{mv}^2}{\sigma_j^2} \left(1 - \frac{\sigma_m^2}{1+\kappa} \beta_j \sum_{i=1}^k b_i \right) \\ &= \frac{\sigma_{mv}^2}{\sigma_j^2} \left(1 - \beta_j \left[\frac{\sigma_m^2}{1+\kappa} \sum_{i=1}^k b_i \right] \right) \end{aligned}$$

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Optimal Weights in the MVP (3/3)

The term $\frac{\sigma_m^2}{1+\kappa}\sum_{i=1}^{k} b_i$ is difficult to interpret We could also give the term an average beta meaning realizing that $\frac{\sigma_m^2}{1+\kappa}\sum_{i=1}^{k} b_i = \frac{\sigma_m^2 \sum_{i=1}^{k} b_i}{1+\sigma_m^2 \sum_{i=1}^{k} b_i \beta_i}$, but close to one for realistic parameterizations. We then arrive at an expression for the optimal weight of stock j in the MVP:

(6) $$\boxed{w_j \approx \frac{\sigma_{mv}^2}{\sigma_j^2}(1-\beta_j)}$$

The optimal weight on asset j is high if residual risk σ_j^2 is small. Low residual risk assets will ceteris paribus obtain a positive weight in the minimum variance portfolio. At the same time a low β (below one) will also create a positive portfolio weight. This proofs the earlier conjecture that the minimum variance portfolio is likely to pick up low beta and low residual risk stocks.

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MSCI BARRA Minimum Volatility Index

- Regime dependent performance

	All Periods		Bull Markets		Bear Market	
	Minimum Variance	MSCI US	Minimum Variance	MSCI US	Minimum Variance	MSCI US
Mean	0.027	0.013	1.019	1.342	-2.027	-2.738
Volatility	3.560	4.773	2.712	3.436	4.209	5.903
t-value	0.088	0.031	3.547	3.685	-3.157	-3.041
Sharpe-ratio	0.008	0.003	0.376	0.391	-0.481	-0.464
Skewness	-0.736	-0.641	0.301	0.146	-0.621	-0.107
Kurtosis	1.936	0.784	0.214	-0.287	0.717	-0.387
Minimum	-0.146	-0.177	-0.055	-0.058	-0.146	-0.177
JB-Test (p-value)	0.000	0.002	0.469	0.733	0.159	0.839

Table 1. Descriptive Statistics. The table shows basic statistics for the MVP and the MWP for all periods (1999:1 to 2009:12), bull markets (1999:1 – 2000:10 & 2003:2 – 2007:10 & 2009:3 – 2009:12).

MSCI BARRA Minimum Volatility Index

- Regime dependent performance

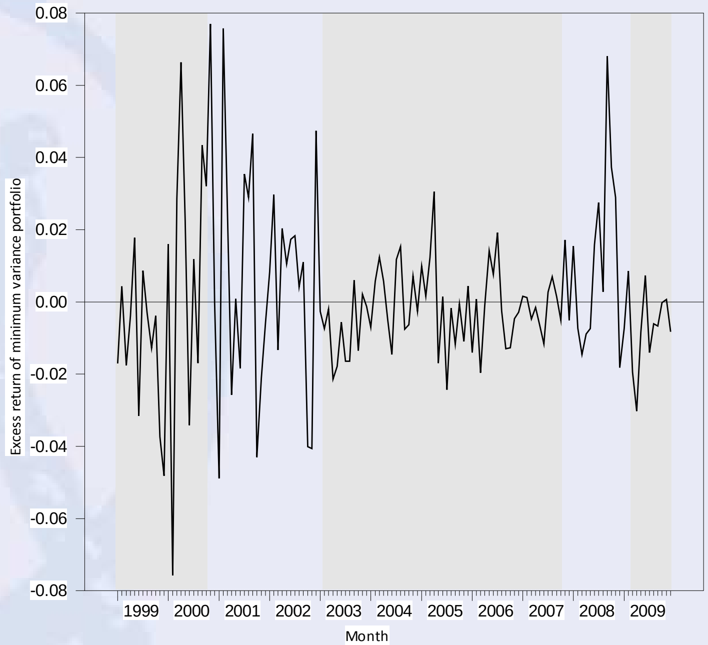

Figure 1: Monthly Excess returns of MSCI BARRA US minimum volatility portfolio versus MSCI US Equity portfolio for January 1999 to December 2009. The shaded areas shows the bull markets (1999:1 – 2000:10 & 2003:2 – 2007:10 & 2009:3 – 2009:12)

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Factor Portfolios – What Should Drive The MVP Excess Returns?

- Market
  - MVP has an ex ante beta smaller than one
  - Expected loading: negative

- Value
  - Value stocks tend to have lower volatility (long tail / credit risk)
  - Small PV of growth opportunities
  - Expected loading: positive

- Size
  - Large stocks are better diversified (both by business units as well as geographical reach)
  - Expected loading:

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Factor Portfolios – What Should Drive The MVP Excess Returns?

- Beta anomaly
  - Low beta stocks on the other side tend to earn more than their beta implies.
  - In other words the cross sectional relation between beta and return is too flat as described in FAMA/MCBETH (1973)
  - Expected loading: positive

- Residual Risk anomaly
  - ANG et al (2006) find that stocks with high residual risk exhibit too low CAPM-adjusted returns
  - BLITZ/VLIET (2007) demonstrates that investors tend to overpay for volatility - possibly because of leverage restrictions – which leaves low volatility stocks unattractive
  - Expected loading: positive

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Factor Portfolios

• Basic Statistics

Type	Characteristic Portfolio		Factor Return		
Name	**Beta**	**Residual Volatility**	**HML**	**SMB**	**Market**
Symbol	R_{β}	R_{σ}	R_{HML}	R_{SMB}	R_{MKT}
Description	Beta and cash neutral Portfolio of long small and short large beta Stocks	Long/short portfolio of long small residual volatility and short large residual volatility stocks	Value factor	Size factor	Market factor
Source	Own Calculation	Own Calculation	FAMA/FRENCH	FAMA/FRENCH	FAMA/FRENCH
Mean	1.691	1.224	0.321	0.578	0.022
Volatility	9.678	10.042	4.975	3.617	4.754
t-value	2.007	1.401	0.742	1.836	0.053
Sharpe-ratio	0.175	0.122	0.065	0.160	0.005
Skewness	-0.065	-0.715	-0.169	0.466	-0.607
Kurtosis	1.173	4.007	4.552	1.978	0.622
Minimum	-0.242	-0.418	-0.208	-0.116	-0.172
JB-Test (p-value)	0.02	0.00	0.321	0.578	0.022

Table 1: Summary for explanatory data. The table shows basic descriptive statistics for monthly data from January 1999 to December 2009. HML (Value), Size (SMB) and Market risk premium are taken from K. FRENCH`s website while the long/short portfolio for beta and residual risk are calculated as described above.

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Results

- Multifactor regressions

Dependent Variable	MVP excess return								
Regression Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
α	0.001	0.001	0.000	0.003	-0.001	-0.001	-0.001	-0.000	-0.000
	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
β_{MKT}		-0.349					-0.312	-0.349	-0.329
		(0.039)***					(0.029)***	(0.037)***	(0.029)***
β_{HML}			0.143				0.141		0.075
			(0.069)**				(0.030)***		(0.020)***
β_{SMB}				-0.370			-0.209		-0.113
				(0.047)***			(0.038)***		(0.045)**
β_{β}					0.105			0.064	0.042
					(0.030)***			(0.014)***	(0.026)**
β_{σ}						0.231		0.126	0.055
						(0.01)***		(0.023)***	(0.019)***
$\overline{R}^2$	0.00	0.51	0.08	0.32	0.18	0.29	0.73	0.79	0.83
DW	1.85	2.01	1.81	1.86	1.64	1.69	1.99	1.73	1.82

Table 1: Results from various linear regressions with excess returns (returns MVP minus MWP) as dependent variable against a variety of explanatory variables. The table also reports adjusted R^2 and DURBIN/WATSON test statistic. Variable significant at the 1%, 5% and 10% level are marked by ***,** and *.

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Better Portfolios

- SCHERER (2004) and LEDOIT/WOLF (2008) use bootstrapping rather than the JOBSON/KORKIE (1981) test. It is well known that the later is not valid if we deviate from iid normal returns.

	Confidence Interval					
	99%		**95%**		**90%**	
Expected block size	Lower	upper	Lower	upper	lower	Upper
1	0.004	0.287	0.026	0.264	0.043	0.243
2	0.005	0.288	0.027	0.264	0.043	0.243
4	0.028	0.229	0.043	0.248	0.058	0.229
8	0.036	0.266	0.049	0.246	0.062	0.267
12	0.038	0.266	0.053	0.245	0.064	0.230

Table 1: Bootstrapped difference in Sharpe-ratios. The table shows the upper and lower values for the 99%, 95% and 90% confidence interval. As an example, the 95% confidence interval for an expected block size of 12 covers a SHARPE-ratio difference from 0.011 to 0.374. In other words the difference in SHARPE-ratio is statistically significant.

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Summary (1/2)

- Investing in the minimum variance portfolio instead of investing into a cap weighted market portfolio is not a clever forecast free strategy. It simply picks up risk based pricing anomalies

- Investors in the MVP need some conviction on the persistence of these risk based pricing anomalies. Most importantly I show that 83% of the variation of the minimum variance portfolio excess returns can be attributed to the FAMA/FRENCH factors as well as the returns on two characteristic anomaly portfolios.

- These risk based characteristic portfolios are particularly successful in explaining the performance of the MVP. On their own, they almost crowd out the FAMA/FRENCH factors. All regression coefficients (factor exposures) are highly significant, stable over the estimation period and correspond remarkably well with our economic intuition.

Summary (2/2)

- The paper also shows that a combination of MWP and risk based characteristic portfolios provides investors a statistically significant improvement over the indirect pickup via the minimum variance portfolio.
- This also puts some concern on results by JAGANNATHAN/MA (2003) that link the superior performance of a long only constrained minimum variance portfolio with Bayesian shrinkage.