

A Detailed Examination of Minimum Variance and Low Volatility Equity Strategies

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July 2011

Goals for This Presentation

- Review the literature on minimum Variance and low volatility equity strategies
- Assert that much of the apparent outperformance of these strategies relates to faulty expectations due to incorrect specification of the CAPM
- Describe our model for the “expected life of firms” as an metric for distinguishing between safe and risky stocks
- Present empirical evidence on both “expected life” strategies and minimum variance strategies across a variety of global markets
- Distinguish between the benefits of investing in particular sets of stocks as compared to using particular portfolio construction methods

Highlights of a Long Literature

- Haugen and Baker (1991)
 - Early evidence in support of low volatility equity strategies
- Lochoff (1998)
 - Leveraged short term bonds outperform long term bonds with comparable volatility
- Clarke, daSilva and Thorley (2006)
 - Using 25% weight to the minimum variance portfolio reduced volatility with no loss of return
- Blitz and VanVliet (2007)
 - Substantial return premium to low volatility across many markets from 1986 to 2006
- Buchner (2010)
 - Asserts specific risk not beta should be priced for illiquid assets
- Barro (2005) and Gabaix (2009)
 - Argue equity investors only worry about 1929 type crashes, so the equity premium over cash should be big but the premium of risky stocks versus not so risky stocks should be small

Abusing the CAPM

- CAPM as put forward by Sharpe (1962) assumes
 - Transaction costs and taxes are zero
 - All information is available to all investors
 - There are no limits on cross-border investing
 - The market clearing portfolio consists of all risky assets (including bonds, real estate etc.), not just a subset of equities that are capitalization weighted
 - The future consists of one long period of definable length and we know what the risk free rate is for that period
- **None of these hold true in the real world**
 - There is no reason to believe that a capitalization equity index should be mean-variance efficient
 - See Grinold (1992)
 - Fixes suggest a very flat security market line
- Empirical tests of return premiums to beta risk are joint tests of CAPM and our ability to estimate beta accurately
 - Easier said than done

Low Risk Investing: Buy stocks in companies that won't go bankrupt

- Merton (1974) poses the equity of a firm as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt
 - Alternatively, lenders are short a put on the firm assets
 - Default can occur only at debt maturity
- Black and Cox (1976) provide a "first passage" model
 - Default can occur before debt maturity
 - Firm extinction is assumed if asset values hit a boundary value (i.e. specified by bond covenants)
- Leland (1994) and Leland and Toft (1996)
 - Account for the tax deductibility of interest payments and costs of bankruptcy
 - Estimate boundary value as where equity value is maximized subject to bankruptcy

Default Correlations

- Hull and White (2001) and Overbeck and Schmidt (2005)
 - You can estimate default correlation if you knew the (unobservable) true interdependence between firms
- Estimate default correlation from asset correlation
 - Zhou (2001) derives default correlations from asset correlation
 - Frey, McNeil and Nyfeler (2005) use a factor model to describe asset correlations
- Include effect of correlation of changes in default boundary to asset correlations
 - Giesecke (2003, 2006)
- Take the easy way out: assume asset correlation is equal to equity return correlation
 - DeSerigny and Renault (2002) provide negative empirical results
 - CreditMetrics, Hull and White (2004)
 - Close if leverage levels are low and horizons are short

Equity Return Properties Help Out

- Defaults are usually rare events so it's impossible to directly observe default correlations over time
- The book value of firm assets is a very incomplete measure of firm assets, so observing asset volatility and asset correlations across firms are very weak estimates
- Equity return volatility and correlation are readily observable
- Zeng and Zhang (2002) shows asset correlations must arise from correlation of both equity and debt components
- Qi, Xie, Liu and Wu (2008) provide complex analytical derivation of asset correlations given equity return correlation

Bring on the Factor Models

- If you have an “equity only” factor model
 - Estimate pair-wise correlations for equity returns
 - See diBartolomeo 1998 for algebra
 - Convert to asset correlation using method of Qi, Xie, Liu and Wu
- If you have a “multi-asset class” factor model you can use the fundamental accounting identity to get a factor representation of asset volatility and equity
 - $\text{Assets} = \text{Liabilities} + \text{Equity}$
 - Asset volatility is just equity volatility de-levered, adjusted for covariance with the market value of debt
 - When interest rates rise equity values usually drop, but market value of debt definitely declines, reducing leverage
 - Convert to pair-wise asset correlation values

In Theory, We're Ready to Go

- With asset volatility and correlations estimated we can use our preferred structural model to estimate default probability of a firm
- Use method from Zhou to convert asset correlations to default correlations
- We can now produce joint default probabilities across firms
- However there are some pretty restrictive assumptions
 - Firm must have debt today
 - Firm must have positive book value today
 - **Balance sheet leverage must stay fixed in the future**

Reverse the Concept: Sustainability: The Expected Life of Firms

- Instead of trying to estimate how likely it is that firm goes bankrupt, let's reverse the logic
- We will actually estimate the "market implied expected life" of firms using contingent claims analysis
- Firms with no debt can now be included since it is possible that they get some debt in the future and default on that
- A quantitative measure of the fundamental and "social" concept of *sustainability*
- Published in diBartolomeo (*Journal of Investing*, 2010)
 - Related articles in Northfield newsletter June 2010 and March 2011

Our Basic Option Pricing Exercise

- Underlying is the firm's assets with asset volatility determined from the factor model as previously described
- Solve numerically for the "implied expiration date" of the option that equates the option value to the stock price
 - *Market implied expected life of the firm*
 - See Yaksick (1998) for computation of perpetual American call
- Include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately
- If you choose Black-Scholes as your option model, then you can solve BS for the implied time to expiration using a Taylor series approximation
- More complex option models allow for stochastic interest rates

Filling in with “Distance to Run”

- For firm's with no debt or negative book value, we simply assume that non-survival will be coincident with stock price to zero, since a firm with a positive stock price should be able to sell shares to raise cash to pay debt
 - If you have a stock with 40% a year volatility you need a 2.5 standard deviation event to get a -100 return
 - Convert to probability under your distributional assumption for first passage risk
- We convert both measures to the median of the distribution of future survival in years
 - What is the number of years such that the probability of firm survival to this point in time is 50/50
 - Highly skewed distribution so we upper bound at 300 years
- Z-score the “median of life” for both measures and map the distance to run Z-scores into the “option method” distribution for firms with no debt

A Few Sample Results from March 31, 2010

- Current life expectations for all (5068) firms in years
 - Median 23, Mean 22.18, Cap Weighted 25.71
 - Revenue Weighted, 23.29
- Financials firms only (1132)
 - Median 24, Mean 21.69, Cap Weighted 18.95
 - Surprising (or maybe not) cap-weighted is a lot lower
 - **Revenue Weighted, 11.41**
- Non-Financials (3936)
 - Median 23, Mean 22.33, Cap Weighted 27.36
 - Revenue Weighted, 24.72
- Highlights:
 - AIG 7, Citicorp 6, GS 6
 - IBM 30, MSFT 32
 - RD 39, XOM 54

A Measure of Systemic Risk?

- Obviously, if the market thinks public companies are not going to be around very long, the economy is in a bad way
- Low equity valuations and high leverage equate to short life expectancy
 - Higher leverage can be sustained with higher growth rates that cause higher equity valuations
- We propose “revenue weighted” expected average life as a measure of systemic stress on an economy
 - By revenue weighting we capture the stress in the real economy
 - Avoids bias of cap weighting since failing firm’s have small market capitalization and don’t count as much

A Digression on “Too Big to Fail”

- For the full sample period of 1992 through March 31, 2010
- Non Financials:
 - Median 14.74, Cap Weighted 18.42
 - Revenue Weight 17.60
- Financials:
 - Median 22.28, Cap Weighted 17.06
 - Revenue Weight, 7.86
- “Too Big to Fail” is really real
 - Risk taking is heavily concentrated in the largest financial firms
 - Risk taking has been concentrated in the largest financial firms for at least 20 years

Quantifying "Sustainability"

- MSCI KLD DSI 400 index of US large cap firms considered socially responsible, 20 year history
 - Typically about 200 firms in common with the S&P 500
- July 31, 1995
 - DSI 400, Median 17, Average 17.91, Standard Deviation 9.93
 - S&P 500, Median 14, Average 15.40, Standard Deviation 9.28
 - Difference in Means is statistically significant at 95% level
- March 31, 2010
 - DSI 400, Median 30, Average 26.39, Standard Deviation 11.45
 - S&P 500, Median 30, Average 24.93, Standard Deviation, 10.92
 - Difference in Means is statistically significant at 90% but not 95%
- Testing on Disjoint Sets (DSI NOT S&P, S&P NOT DSI)
 - Statistically significant difference in means for every time period tested

Results to “Sustainability” Equity Investing (1992 through March 2010)

Table 1¹

	Mean Monthly Return	Cumulative Return	Monthly Standard Deviation	Annual Compound Return	Leveraged S&P Risk Equivalent Return
Q5 Equal	1.33	713.77	9.15	10.90	7.45
Q1 Equal	1.03	790.86	3.64	11.50	12.83
Q5 Cap	0.77	251.60	6.62	4.98	4.76
Q1 Cap	0.79	414.32	3.78	7.77	8.26
S&P 500 ²	0.75	347.74	4.32	6.78	6.78

MinVar Portfolios 2000-2010, 200 Max Positions, Northfield Risk Models

Table 2¹

	Mean Monthly Return	Cumulative Return	Monthly Standard Deviation	Annual Compound Return	Annual Sharpe Ratio
US Small Cap	.88	151.38	5.91	8.74	.294
US Large Cap	.76	164.75	1.95	9.25	.676
Europe	0.51	92.99	1.53	6.15	.385
Japan	0.22	31.16	1.53	2.50	.181
S&P 500 ²	0.16	6.16	4.72	.54	-.169

Combining Sustainability and MV(1992 through March 2010, 200 Max Position)

	Mean Monthly	Cumulative	Monthly Standard Deviation	Annual Compound Return	Annual Sharpe Ratio
Q1 MV	1.07	840.43	2.96	12.34	.81
Q5 MV	1.77	2901.15	6.80	19.33	.71

Empirical Key Points

- The high risk portfolios as measured by sustainability have higher arithmetic returns than low risk portfolios, but lower geometric returns
 - It may be possible that everybody is right. The CAPM predicts higher returns for higher risk over a single period, but does not address multi-period returns
- Equal weighted portfolios outperform capitalization weighted portfolios substantially before trading costs
- Sustainability portfolios are enhanced by MV portfolio construction but is of lesser benefit than the sustainability approach
 - An MV portfolio of “low sustainability” stocks is the winner with a compound return of over 19% per annum for nearly 20 years
 - An MV portfolio of “high sustainability” stocks produces the highest Sharpe ratio at over .8 for nearly 20 years

Conclusions

- There is substantial empirical evidence that passive investing in low risk securities produces superior compound rates of return
 - Our sustainability metric is the best measure I've seen for defining the relevant risk
- Careful consideration of the CAPM suggests that our expectation of a steep security market line is faulty
- Minimum variance portfolio construction is helpful, but has less impact than choosing securities on their sustainability