Diversifying Risk Parity

Harald Lohre

Deka Investment GmbH

Northfield’s 25th Annual Research Conference
San Diego, August 7, 2012
Risk-Based Portfolio Construction

Given perfect foresight the Markowitz (1952) approach is the rationale of choice to generate efficient portfolios with an optimal risk-return trade-off.

Mean-variance optimization is confounded by estimation risk, especially the one in estimates of expected returns.

One solution: Refrain from estimating returns and resort to risk-based allocation techniques.

Minimum-variance portfolios are characterized by low volatility and still provide quite favorable return figures raising investors’ interest in risk-based concepts.
Diversification Pays...but How to Diversify?

- Minimum-variance portfolios typically load on low-volatility assets rendering them rather concentrated in few assets
- What about diversification? How to define diversification?

Literature review

- Number of assets (Evans and Archer, 1968)
- Herfindahl Index (Persson, 1993)
- Entropy of portfolio weights (Bera and Park, 2008)

Diversifying weights versus diversifying risk

Using a PCA of the portfolio assets Meucci (2009) extracts the uncorrelated risk sources and determines the effective number of uncorrelated bets
Diversified Risk Parity Strategies

Maximum diversification obtains for a risk parity strategy along the uncorrelated risk sources which we dub *diversified risk parity (DRP)*

Lohre, Opfer, and Ország (2012)

- Asset allocation study
- Demystifying uncorrelated risk sources
- Horse race: DRP vs. $1/N$, minimum-variance, risk parity, or the most-diversified portfolio

Lohre, Neugebauer, and Zimmer (2012)

- Equity portfolio selection within S&P 500
- Demystifying uncorrelated risk sources, horse race
- Link to equity factor portfolios
Managing Diversification, Meucci (2009)

• Consider a portfolio of $N$ assets with returns $\mathbf{R}$. Given weights $\mathbf{w}$ the resulting portfolio return is $R_w = \mathbf{w}' \mathbf{R}$

• Diversification pays when combining low-correlated assets: Construct uncorrelated risk sources by applying a principal components analysis (PCA) to the VCV $\Sigma$

• From the spectral decomposition theorem it follows that

$$\Sigma = \mathbf{E} \Lambda \mathbf{E}'$$

where $\Lambda = \text{diag}(\lambda_1, ..., \lambda_N)$ consists of $\Sigma$’s eigenvalues

• The columns of matrix $\mathbf{E}$ represent the eigenvectors of $\Sigma$ which define a set of $N$ uncorrelated principal portfolios with variance $\lambda_i$ for $i = 1, ..., N$ and returns $\tilde{\mathbf{R}} = \mathbf{E}' \mathbf{R}$
Multi-Asset Data

Standard multi-asset data ranging from Dec 1987 to Sep 2011:

- Government Bonds: Most favorable risk-adjusted return
- Remaining asset classes with similar return but higher volatility
- Correlations generally low but not zero

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>JPM Global Bond</td>
<td>6.9%</td>
<td>3.8%</td>
<td>0.96</td>
</tr>
<tr>
<td>MSCI World</td>
<td>5.9%</td>
<td>14.5%</td>
<td>0.18</td>
</tr>
<tr>
<td>MSCI Emerging Markets</td>
<td>8.3%</td>
<td>24.2%</td>
<td>0.21</td>
</tr>
<tr>
<td>DJ UBS Commodities</td>
<td>5.7%</td>
<td>15.5%</td>
<td>0.16</td>
</tr>
<tr>
<td>Barclays U.S. Aggr. Credit</td>
<td>6.9%</td>
<td>5.3%</td>
<td>0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation Matrix</th>
<th>Bonds</th>
<th>Equities Dev.</th>
<th>Equities Emg.</th>
<th>Comm.</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>1.00</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities Dev.</td>
<td>0.01</td>
<td>1.00</td>
<td>0.74</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Equities Emg.</td>
<td>-0.05</td>
<td>0.74</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comm.</td>
<td>-0.18</td>
<td>0.18</td>
<td>0.32</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Credit</td>
<td>0.53</td>
<td>0.25</td>
<td>0.20</td>
<td>0.09</td>
<td>1.00</td>
</tr>
</tbody>
</table>

- Apply PCA to generate uncorrelated principal portfolios
### Demystifying Principal Portfolios

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>PP1</th>
<th>PP2</th>
<th>PP3</th>
<th>PP4</th>
<th>PP5</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPM Global Bond</td>
<td>-0.01</td>
<td>0.05</td>
<td>-0.04</td>
<td>0.51</td>
<td>0.86</td>
</tr>
<tr>
<td>MSCI World</td>
<td>0.43</td>
<td>0.23</td>
<td>-0.86</td>
<td>-0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>MSCI Emerging Markets</td>
<td>0.87</td>
<td>0.16</td>
<td>0.47</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>DJ UBS Commodities</td>
<td>0.24</td>
<td>-0.96</td>
<td>-0.14</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Barclays U.S. Aggr. Credit</td>
<td>0.04</td>
<td>0.01</td>
<td>-0.12</td>
<td>0.85</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

Panel A: December 1987 to September 2011

- Variance: 7.7% 2.2% 0.8% 0.3% 0.1%
- Percent Explained: 69.8% 19.9% 6.9% 2.8% 0.7%
- Cumulative: 69.8% 89.7% 96.5% 99.3% 100.0%
PP1 fairly dominant, accounts for at least 60% of variation
PP2 and PP3 represent some 20% and 10% of the variation, PP4 and PP5 account for a minor fraction
At the end PP1 accounts for 80% of overall variability indicating contagion effects emanating from the 2008 crisis
Decomposing Risk by Principal Portfolios

- $1/N$ is concentrated in the first principal portfolio
- $MV$ is highly concentrated in low-volatility assets
Effective Number of Uncorrelated Bets

Meucci (2009)

- A portfolio is well-diversified when the \( p_i \) are “approximately equal and the diversification distribution is close to uniform”
- Apply a dispersion metric to the diversification distribution:

\[
N_{Ent} = \exp \left( - \sum_{i=1}^{N} p_i \ln p_i \right)
\]  

(2)

- \( N_{Ent} \) intuitively is the effective number of uncorrelated bets:
  - \( N_{Ent} = 1 \) holds for a completely concentrated portfolio
  - \( N_{Ent} = N \) holds for a portfolio that is completely homogenous in terms of uncorrelated risk sources
Diversifying Risk Parity

- The maximum diversification portfolio is a risk parity strategy that is budgeting risk along the uncorrelated risk sources rather than the underlying portfolio assets.
- We dub this strategy *diversified risk parity (DRP)* and its weights $w_{DRP}$ obtain by solving

$$w_{DRP} = \arg\max_{w \in C} \mathcal{N}_{Ent}(w)$$

(3)

where the weights $w$ can be subject to a set of constraints $C$.
- Moreover, the framework allows for a litmus test of competing techniques like $1/N$, *minimum-variance*, *risk parity*, or the *most-diversified portfolio* of Choueifaty and Coignard (2008).
Risk-based Asset Allocation

- Constructing the diversified risk parity strategy we determine the principal portfolios via rolling window estimation.
- The first PCA consumes 60 months of data, thus, the strategy performance can be assessed from Jan 1993 to Sep 2011.
- For benchmarking the diversified risk parity strategy we consider four alternatives:
  1. $1/N$
  2. Minimum-variance (MV)
  3. Risk parity (RP)
  4. Most-diversified portfolio (Choueifaty/Coignard, 2008) (MDP)
- We enforce full investment and positivity constraints and rebalance all strategies at a monthly frequency.
Performance of Risk-Based Strategies

- $1/N$ with highest return and volatility gives lowest SR, MV with reasonable return at lowest volatility gives high SR
- RP: Middle-ground between $1/N$ and MV, MDP still ok
- DRP: Highest SR and convincing MDDs!
Diversified Risk Parity versus Risk Parity

Weights

Vola by Assets

Vola by PPs

• DRP reacts more timely to changes in risk structure
• Despite constraints DRP is well meeting its objective
• RP is rendered highly concentrated in terms of PPs at the end of the sample period
Diversification throughout Time

- $1/N$ dominated by the other strategies
- DRP maintains the highest number of bets throughout time
- MV and RP represent 3 bets, MDP close to 4, however, these strategies are losing ground at the end of the sample period
Diversified Risk Parity for Equity Portfolio Selection

- Diversifying *across* asset classes seems reasonable
- What about diversification *within* an asset class like equities?
- Significant exposure to a single (market) risk factor is a well-known issue which is usually addressed by means of diversification across sectors or styles
- The presented framework for achieving maximum diversification is highly appropriate
- We especially
  - determine the number of relevant risk sources
  - associate these principal portfolios to sectors and equity factors
  - document a dynamic DRP strategy to provide convincing performance and diversification characteristics
Extracting Principal Portfolios from the S&P 500

- Determine principal portfolios from Oct 1989 to Sep 2011
- In a given month, the PCA is restricted to the then active 500 constituents of the S&P 500
- In total, we deal with 1037 companies over the sample period
How many risk sources are embedded in the S&P 500?

- Determine a reasonable number of principal portfolios using the $PC_{p1}$ and $PC_{p2}$ criteria of Bai and Ng (2002)
- It is hardly reasonable to allocate any risk budget to higher principal portfolios
Demystifying Principal Portfolios: Sectors?

- PPs arising from a PCA over the most recent 60 months
- PP1 qualifies for a common market factor, PP2 is short IT and long most of the remaining sectors
- PP3 is long Energy and short in Financials, Consumer Discretionary/Staples; PP4 is long Utilities, Health Care, and Telecoms and short Materials and Industrials
Demystifying Principal Portfolios: Factors?

Characterize PPs in an extended Fama-French setting:

\[ R_{PPi,t} = \alpha + \beta_1 R_{M,t} + \beta_2 R_{Size,t} + \beta_3 R_{Value,t} + \beta_4 R_{Mom,t} + \beta_5 R_{Vola,t} + \beta_6 R_{Liqui,t} + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>PP1</th>
<th>PP2</th>
<th>PP3</th>
<th>PP4</th>
<th>PP5</th>
<th>PP6</th>
<th>PP7</th>
<th>PP8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>-0.07</td>
<td>-0.19</td>
<td>-0.08</td>
<td>0.06</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Market</td>
<td>25.95</td>
<td>9.59</td>
<td>-2.21</td>
<td>2.15</td>
<td>2.23</td>
<td>0.33</td>
<td>0.42</td>
<td>-1.03</td>
</tr>
<tr>
<td>Size</td>
<td>-20.75</td>
<td>-10.41</td>
<td>-14.02</td>
<td>-2.31</td>
<td>-2.58</td>
<td>6.50</td>
<td>4.86</td>
<td>7.00</td>
</tr>
<tr>
<td>Value</td>
<td>16.61</td>
<td>35.79</td>
<td>20.62</td>
<td>-16.23</td>
<td>5.36</td>
<td>3.65</td>
<td>-8.87</td>
<td>-10.10</td>
</tr>
<tr>
<td>Mom</td>
<td>-4.05</td>
<td>-6.60</td>
<td>9.15</td>
<td>7.80</td>
<td>-0.35</td>
<td>-2.70</td>
<td>-1.49</td>
<td>-0.07</td>
</tr>
<tr>
<td>Vola</td>
<td>5.16</td>
<td>-30.63</td>
<td>8.36</td>
<td>-3.31</td>
<td>5.66</td>
<td>0.91</td>
<td>-0.95</td>
<td>2.48</td>
</tr>
<tr>
<td>Liqui</td>
<td>4.21</td>
<td>-23.96</td>
<td>2.07</td>
<td>4.89</td>
<td>-17.35</td>
<td>-19.04</td>
<td>4.43</td>
<td>7.61</td>
</tr>
<tr>
<td>Adj. R^2</td>
<td>94.0%</td>
<td>57.6%</td>
<td>13.0%</td>
<td>22.2%</td>
<td>34.6%</td>
<td>9.6%</td>
<td>8.3%</td>
<td>8.0%</td>
</tr>
</tbody>
</table>
Risk-based Equity Strategies

- Constructing the diversified risk parity strategy we obtain PPs by PCA estimation over a rolling 60 months window
- Strategy performance from Oct 1989 to Sep 2011
- For benchmarking the diversified risk parity strategy we consider four risk-based alternatives next to the S&P 500:
  1. $1/N$
  2. Minimum-variance (MV)
  3. Risk parity (RP)
  4. Most-diversified portfolio (Choueifaty/Coignard, 2008) (MDP)
- We enforce full investment and positivity constraints together with maximum stock weights of 5% and rebalance all strategies at a monthly frequency
## Performance of Risk-Based Strategies

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Index</th>
<th>1/N</th>
<th>MV</th>
<th>RP</th>
<th>MDP</th>
<th>DRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return p.a.</td>
<td>7.5%</td>
<td>9.9%</td>
<td>8.1%</td>
<td>9.2%</td>
<td>7.9%</td>
<td>11.0%</td>
</tr>
<tr>
<td>Vola p.a.</td>
<td>13.8%</td>
<td>17.2%</td>
<td>11.8%</td>
<td>14.0%</td>
<td>13.1%</td>
<td>15.1%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.28</td>
<td>0.36</td>
<td>0.38</td>
<td>0.39</td>
<td>0.33</td>
<td>0.49</td>
</tr>
<tr>
<td>Max DD</td>
<td>-47.5%</td>
<td>-55.9%</td>
<td>-38.2%</td>
<td>-47.6%</td>
<td>-39.6%</td>
<td>-35.8%</td>
</tr>
<tr>
<td># Assets</td>
<td>500.0</td>
<td>500.0</td>
<td>36.2</td>
<td>500.0</td>
<td>38.1</td>
<td>43.4</td>
</tr>
<tr>
<td>Turnover</td>
<td>0.4%</td>
<td>2.2%</td>
<td>14.7%</td>
<td>3.7%</td>
<td>16.2%</td>
<td>25.3%</td>
</tr>
</tbody>
</table>

- **1/N**: High return at the highest volatility gives medium SR
- **MV**: Reasonable return at the lowest volatility gives higher SR
- **RP**: In between 1/N and MV with large MDD
- **MDP**: MDP lags in terms of returns and SR
- **DRP**: Highest SR and smallest MDD!
Risk Decomposition by Sectors or Principal Portfolios

- **MV**: Defensive sectors, concentrated in PP1 and PP2
- **RP**: Close to $1/N$, highly concentrated
- **DRP**: Active sector allocation, tracks the number of relevant bets, balanced risk decomposition across PPs
• $1/N$ dominated by the other strategies
• DRP maintains the highest number of bets throughout time
• MDP and RP represent around 2 bets, MV slightly better, however, these strategies are losing ground at the end of the sample period
Explaining the Performance of Risk-Based Strategies

\[ R_{RBS,t} = \alpha + \beta_1 R_{M,t} + \beta_2 R_{Size,t} + \beta_3 R_{Value,t} + \beta_4 R_{Mom,t} + \beta_5 R_{Vola,t} + \beta_6 R_{Liqui,t} + \varepsilon_t \]

<table>
<thead>
<tr>
<th>Index</th>
<th>Risk-Based Allocations</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>1/N</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.09%</td>
</tr>
<tr>
<td>Market</td>
<td>-</td>
</tr>
<tr>
<td>Size</td>
<td>1.55</td>
</tr>
<tr>
<td>Value</td>
<td>1.08</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.25</td>
</tr>
<tr>
<td>Volatility</td>
<td>1.72</td>
</tr>
<tr>
<td>Liquidity</td>
<td>1.02</td>
</tr>
</tbody>
</table>

**Adjusted R^2**
- S&P 500: 71.7%
- 1/N: 47.2%
- MV: 43.7%
- RP: 42.1%
- MDP: 25.8%
- DRP: 17.0%

- MV, RP, and MDP load on the low-volatility anomaly
- DRP small adj. \( R^2 \), large value tilt
• RP with value and small cap tilt. Constantly loading on volatility factor

• DRP with sizable value tilt that is diminishing over time, no volatility factor exposure, time-varying momentum exposure
Conclusion

- We have introduced the diversified risk parity strategy that achieves maximum diversification by equally budgeting risk to each of the uncorrelated risk sources.
- Besides providing convincing risk-adjusted performance, DRP is meeting its diversification objective well:
  - across asset classes
  - within equities
- The competing alternatives tend to be rather concentrated in a few bets.
- DRP has a built-in mechanism for tracking the prevailing risk structure, thus providing a more robust way for achieving maximum diversification throughout time.
Principal Risk Parity for Asset Allocation

- Principal Risk Parity (PRP): Budget risk across principal portfolios proportional to their contribution to total variance
- PRP strategy is tracking closely the principal portfolio’s variance decomposition over time
- Higher return at higher risk!

![Volatility Contributions by Principal Portfolios in %](chart1.png)

![Rolling Window](chart2.png)
Principal Risk Parity for Equities

- Principal Risk Parity (PRP): Budget risk across principal portfolios proportional to their contribution to total variance
- PRP strategy is tracking closely the principal portfolio’s variance decomposition over time
- High return at higher risk!
Risk-Based Commodity Investing

- Bernardi, Leippold, and Lohre (2012) support alternative risk parity strategies for commodities as well
- Commodities are characterized by high heterogeneity—translating into 8 relevant PPs
- DRP and PRP both provide superior performance