Time Varying IC, Conditional Risk Model and Optimal Portfolio Turnover

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Main Insights

- A portfolio's optimal turnover and leverage are inversely related to IC volatility and should be much smaller than usually thought.
- The volatility of signal IC (strategy risk) is a dominant risk for a portfolio based on that signal.
- Conditional error covariance matrix that incorporates the strategy risk is the proper measure of risk to use for portfolio construction.
- Ignoring the strategy risk will result in suboptimal portfolios or over-leveraged portfolios.

Question

- Given an alpha signal with time varying payoffs or ICs, what is the optimal portfolio turnover and leverage ratio?

Optimal Portfolio Turnover
Qian, Hua and Tilney (2004)

- Qian, *et al*. derived the following interesting formula for the optimal portfolio turnover:

$$TO = \sqrt{\frac{N}{\pi}}\sigma_P\sqrt{1-\rho_z(1)}E_{cs}\left(\frac{1}{\sigma_i}\right)$$

Qian, Hua and Tilney (2004)

where:

- *N*: number of stocks in selection universe,
- σ_P: portfolio’s target tracking error,
- σ_i : stock i’s specific risk,
- $\rho_z(1)$: first order alpha signal correlation,
- No transaction cost is considered.

The turnover is higher

- The higher the tracking error
- The larger the number of stocks (proportional to the square root of N)
- The lower the forecast autocorrelation
- The lower the average stock-specific risk

- All very intuitive

Example

- For a portfolio with
  $\sigma_P = 5\%, N = 1000,\ \sigma_{\mathrm{i}} = 30\%$ and $\rho_{\mathrm{z}}(1) = 0.9$
  The expected monthly portfolio turnover from Qian *et al*. is about 94%

Optimal Portfolio Leverage
Qian, Hua and Tilney (2004)

- For the same portfolio, the optimal leverage ratio is about 420%

$$L = \sum_{i=1}^{N} | \Delta w_{i,t+1} |$$

$$= \sqrt{\frac{2N}{\pi}} \sigma_P E_{cs} \left(\frac{1}{\sigma_i} \right)$$

Turnover per Unit Leverage

- The turnover is directly proportional to the leverage. Their ratio is solely determined by the signal autocorrelation.

$$\frac{TO}{L} = \sqrt{\frac{1-\rho_z(1)}{2}}$$

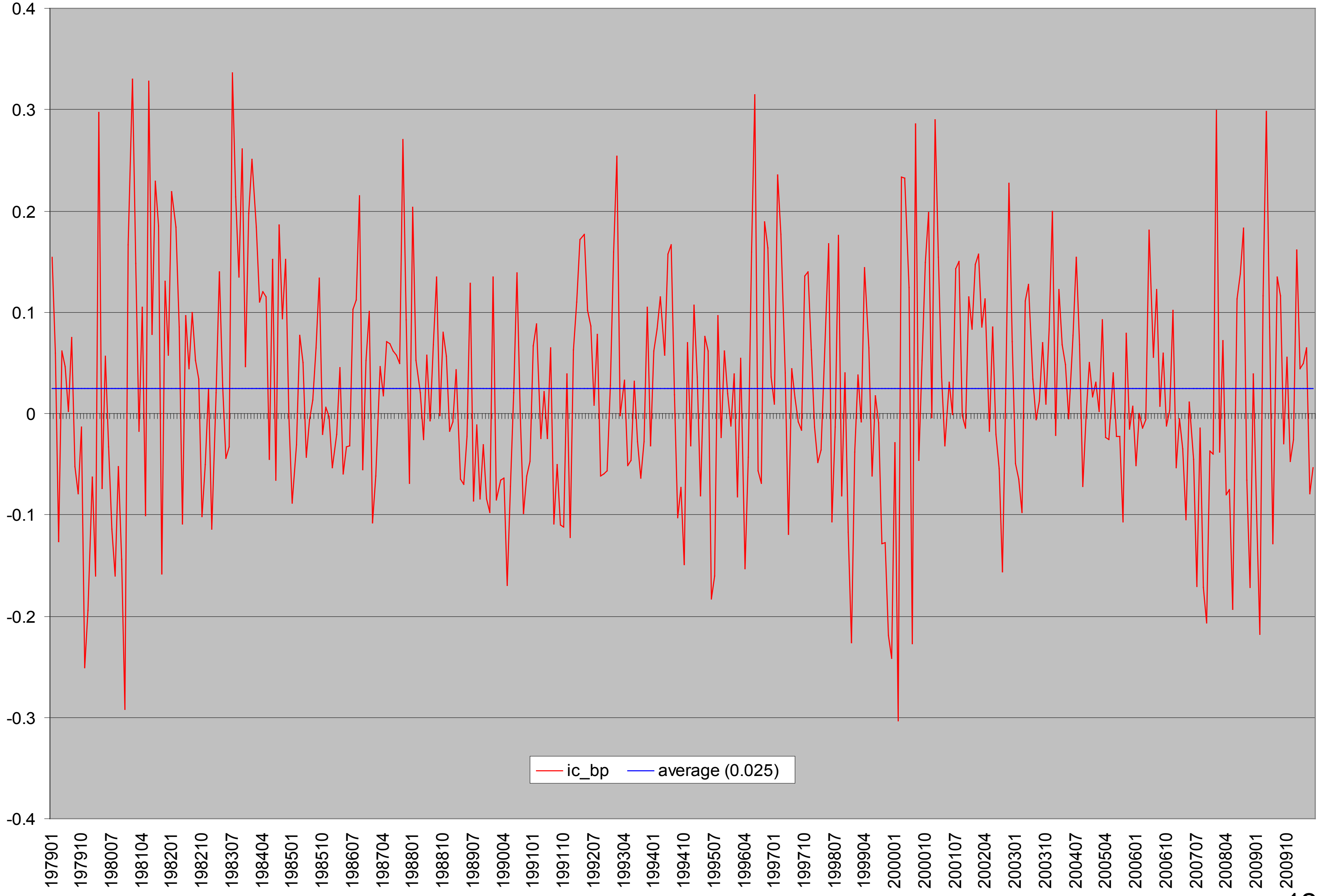
The Qian *et al*. Formula

- Is a very nice piece of research that puts various characteristics of a portfolio into a neat formula.
- The formula is derived using a diagonal covariance matrix which is equivalent to the assumption that the alpha signal has constant payoffs over time.

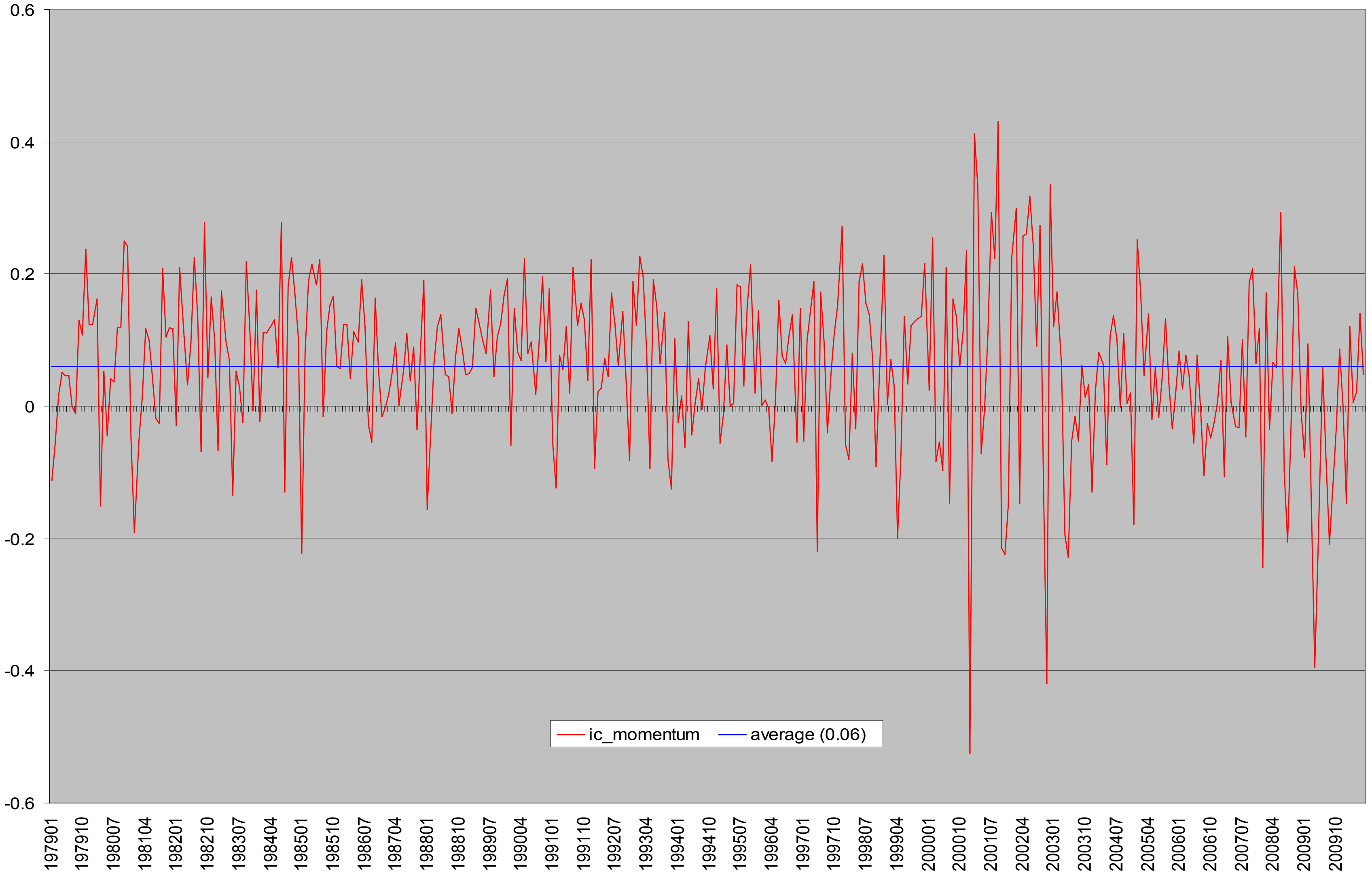
The Reality

- IC is time varying. Even though an alpha factor can have a positive average long term payoff, the payoff in a specific time period is random in nature.
- The strategy risk cannot be diversified away and becomes a dominant risk in portfolio level.
- The optimal portfolio turnover and leverage should be much smaller when the IC is time varying.

Cross-Sectional IC of B/P for R2000



Cross-Sectional IC of Momentum for R2000

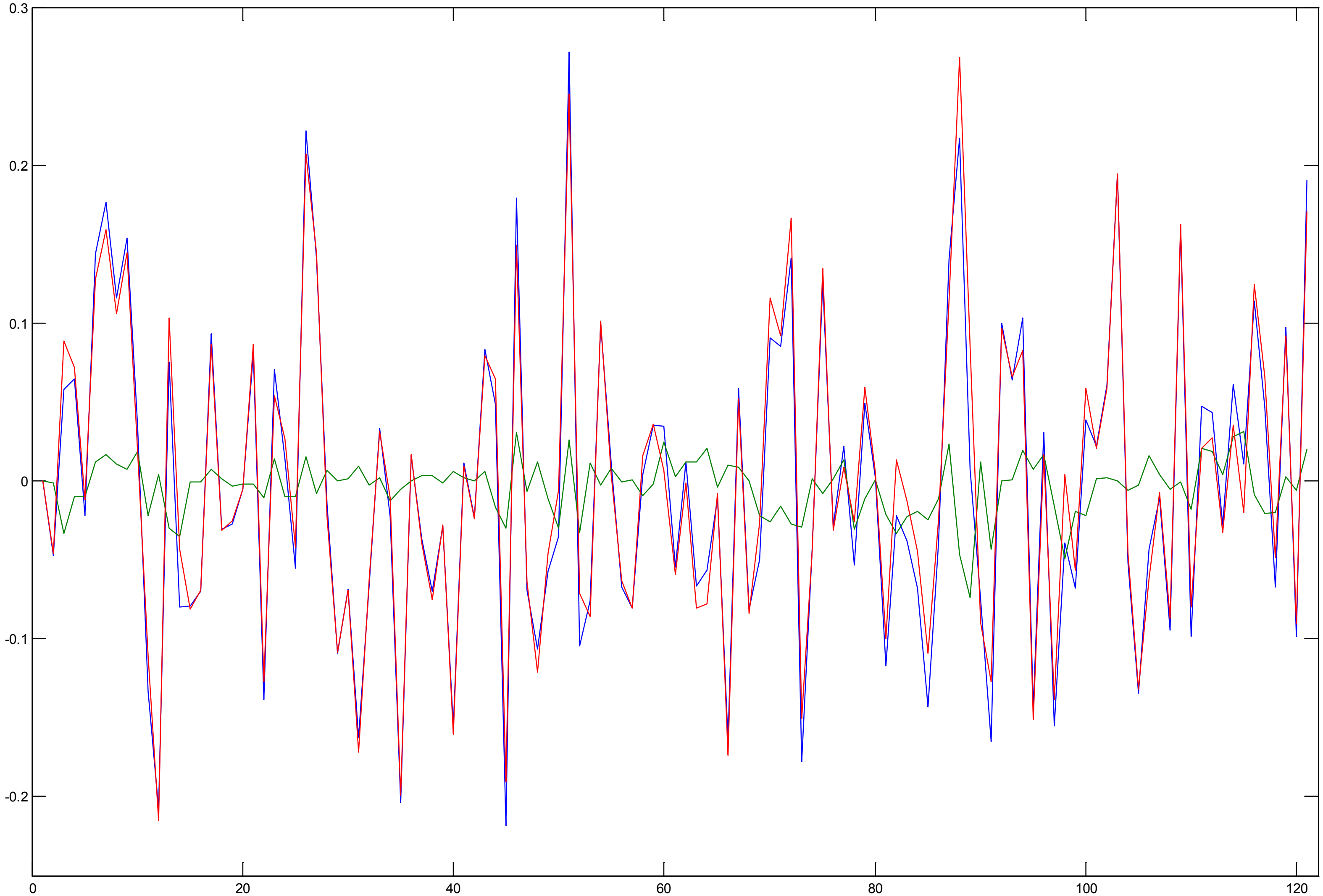


Risk Decomposition

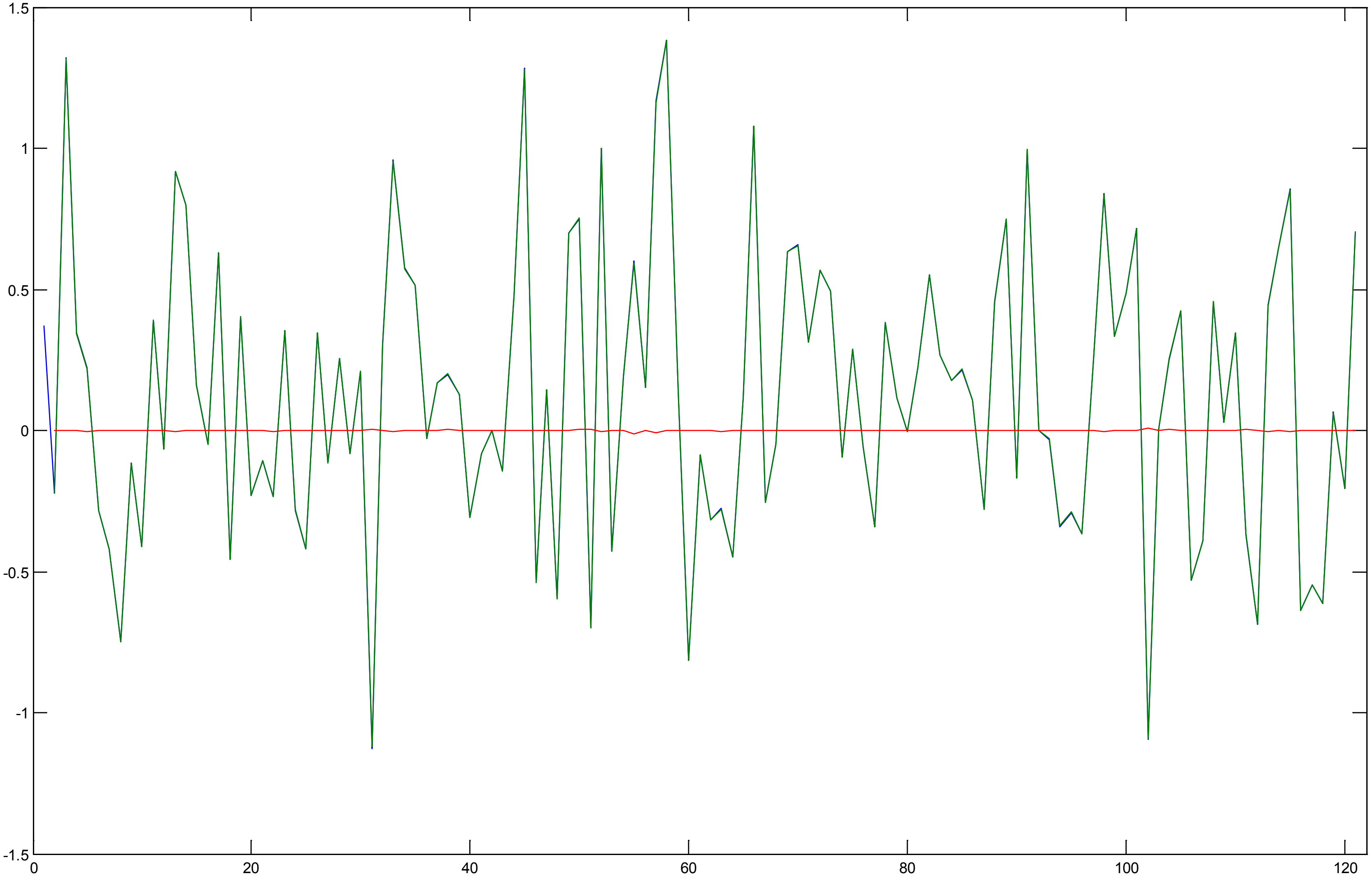
Total Risk = Strategy Risk + Residual Risk

- For individual stocks, the strategy risk is only about 1~5% of the total risk.
- In portfolio level, the strategy risk accounts for more than 95% of the total risk.

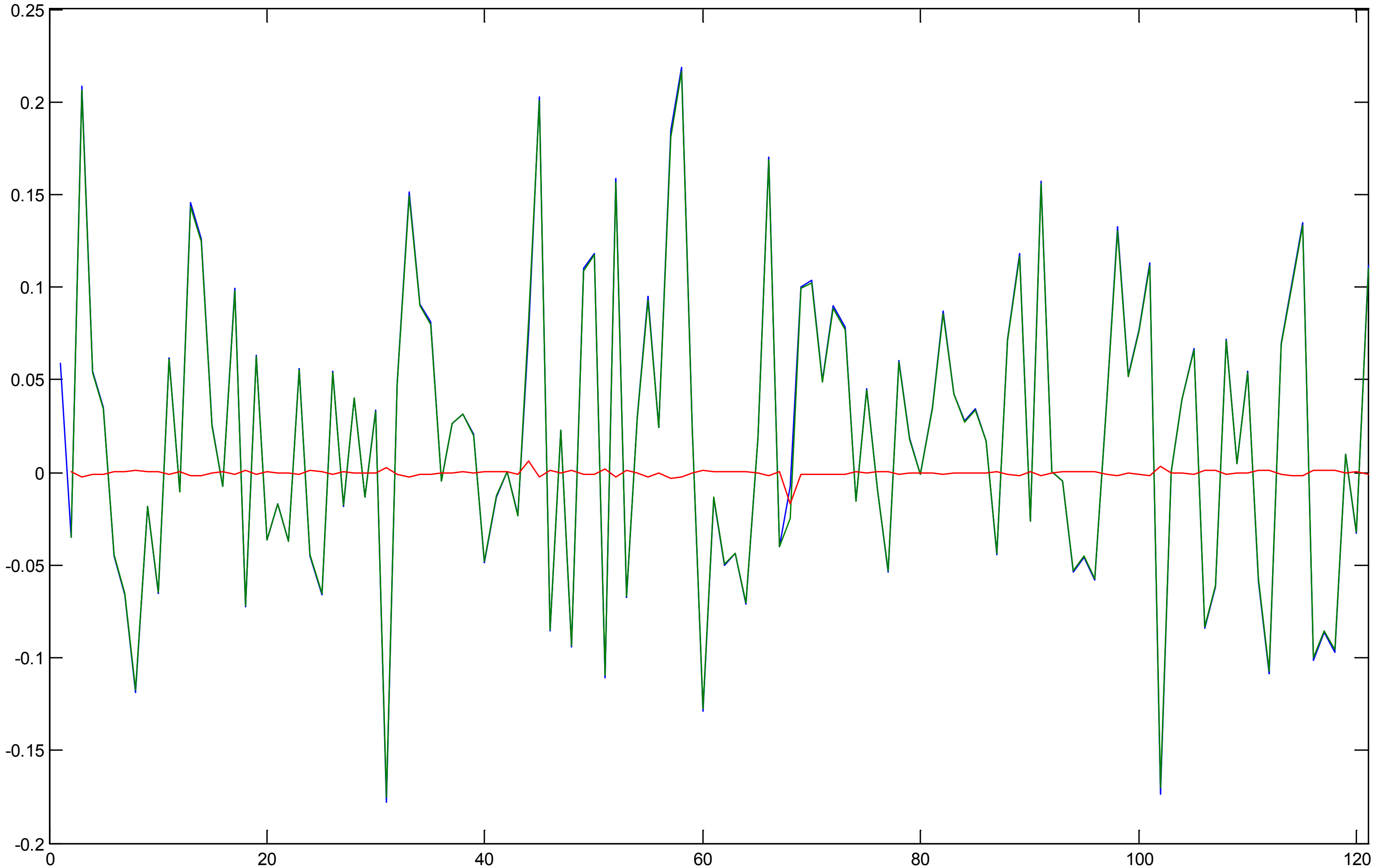
Stock Return Decomposition: Factor Exposure & Residual



Portfolio Return Decomposition: Risk Model w/o Strategy Risk



Portfolio Return Decomposition: Risk Model w Strategy Risk



Optimal Portfolio Turnover

- When IC is time varying with mean ρ and standard deviation of $\sigma_\rho \neq 0$, we have

$$TO = \frac{\sigma_P \sqrt{1-\rho_z(1)}}{\sqrt{\pi}\sqrt{\sigma_\varepsilon^2 / N + \sigma_\rho^2}} E_{cs}\left(\frac{1}{\sigma_i}\right)$$

where $\sigma_\varepsilon^2 = 1-\rho^2-\sigma_\rho^2$ is stock's idiosyncratic risk.

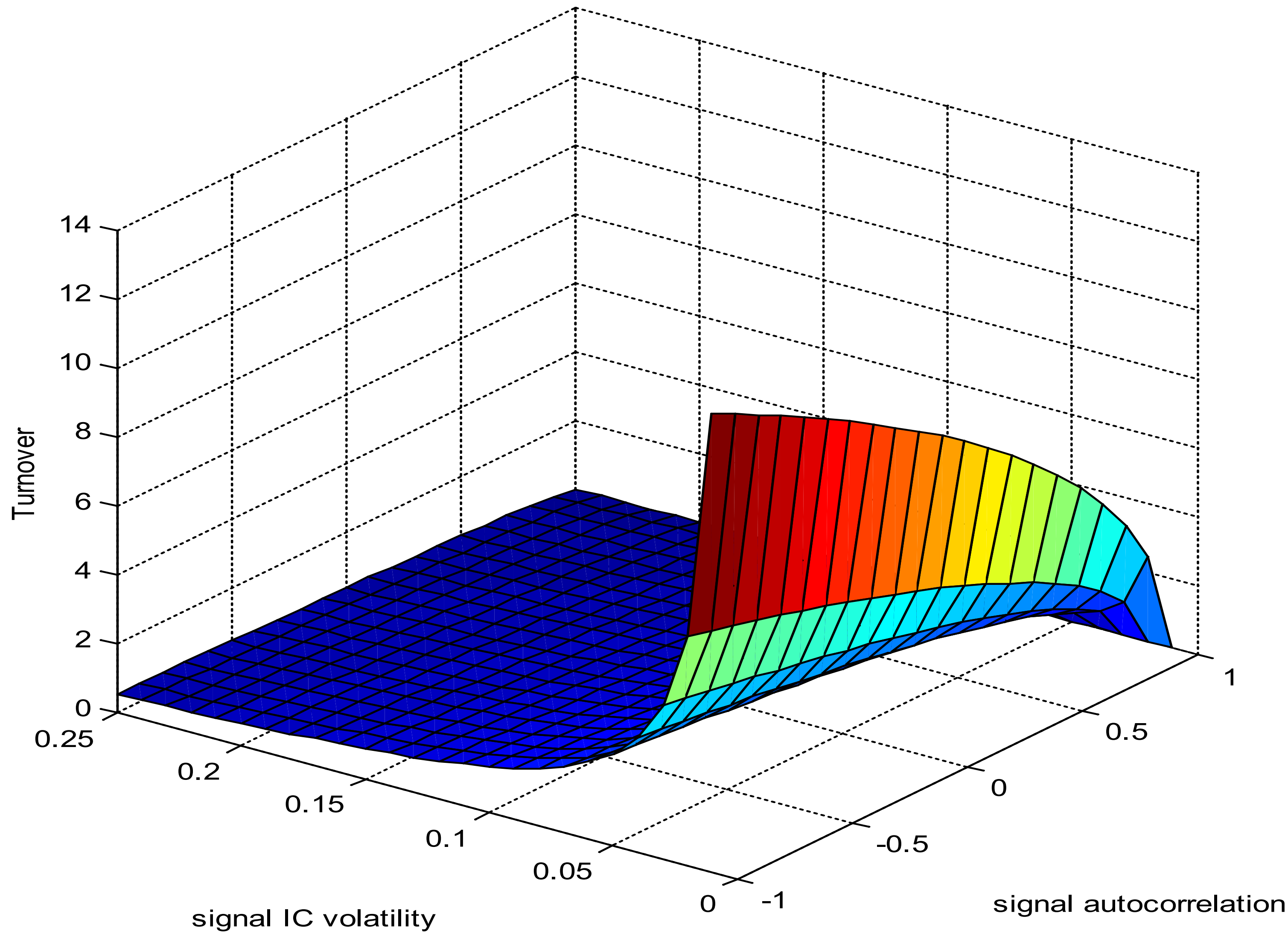
The turnover is higher

- The higher the tracking error
- The larger the number of stocks (~~proportional to the square root of *N*~~)
- The lower the forecast autocorrelation
- The lower the average stock-specific risk
- **The lower the time varying IC volatility**

In the Limit

- When the number of stocks *N* in the selection universe is large enough so that the idiosyncratic risks are being diversified away, the contribution of *N* to portfolio turnover and leverage will diminish, while the contribution of strategy risk will dominate.

Signal Autocorrelation, IC Volatility and Optimal Portfolio Turnover $\sigma_P = 5\%,\ \sigma_i = 30\%$



Optimal Portfolio Turnover

- For a portfolio with

  $\sigma_P = 5\%, N = 1000,\ \sigma_{\mathrm{i}} = 30\%,\ \rho_{\mathrm{z}}(1) = 0.9$ and $\sigma_\rho = 10\%$

  The optimal portfolio turnover is about 30% which is only one third of the optimal portfolio turnover when we assume the signal payoff is constant.

Optimal Portfolio Leverage

- For the same portfolio, the optimal leverage ratio is about 133% (instead of 420%).

$$L = \sqrt{\frac{2}{\pi}} \frac{\sigma_P}{\sqrt{\sigma_\varepsilon^2 / N + \sigma_\rho^2}} E_{cs}\left(\frac{1}{\sigma_i}\right)$$

Turnover per Unit Leverage

- The turnover per unit leverage is the same as before and is solely determined by the signal autocorrelation. The lowered turnover here is due to lowered leverage after the proper risk model is used.

$$\frac{TO}{L} = \sqrt{\frac{1-\rho_z(1)}{2}}$$

Strategy Risk Control

- Qian and Hua's "Strategy Risk", σ_ρ , gets into the portfolio play as a "control valve".
- When the perceived strategy risk is high then one would reduce the active bet to control the risk.
- When the perceived strategy risk is low then one would increase the active bet.

The Peril of Ignoring “The Strategy Risk”

- When one ignores the “strategy risk” in risk modeling, one will not be able to control risk effectively if no further constraints are imposed, and one will leverage too much and the portfolio will be wiped out when the bet goes badly wrong.

Over-Bet and Over-Leverage

- For a typical alpha signal with IC volatility of 15% and an investment universe of 1000 securities, one will usually over-bet and over-leverage by more than 5 times if no further constraints are imposed in optimization.
- The over-bet and over-leverage will cause the portfolio to be wiped out one day just as what happened to LTCM.

A Simulation Study

- Generate random factor return series with mean $\rho = 0.05$ and standard deviation of $\sigma_\rho = 0.2$;
- Generate a cross-section of 1000 normally distributed factor exposures Z for 120 months with signal autocorrelation $\rho_z(1) = 0.9$;
- Generate normally distributed idiosyncratic errors;
- Finally generate excess return using the above generated factor returns, factor exposures and idiosyncratic errors. The excess return volatility is generated by a uniform distribution in (0.05 0.40);

Given the true parameters, the true conditional covariance matrix, unconditional covariance matrix and diagonal covariance matrix are all known.

Portfolio Simulation Using Different Covariance Matrices

- Conditional Covariance Matrix with Strategy Risk incorporated.
- Diagonal Covariance Matrix without Strategy Risk .

Only tracking error constraint is imposed for the first three simulation. For the last simulation, I also impose leverage constraint.

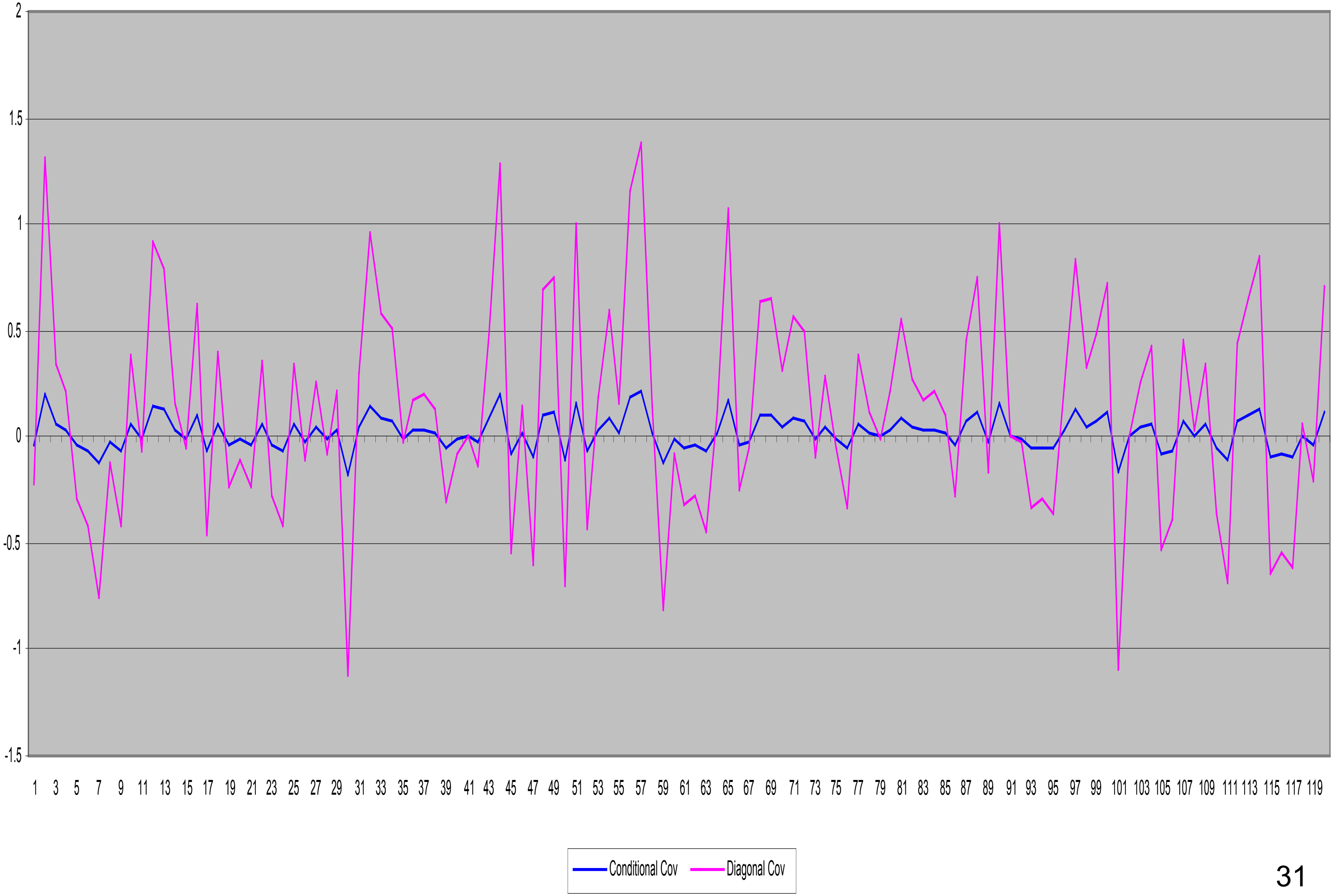
Portfolio Simulation I

$N = 1000$, $\mathrm{T} = 120$, $\rho_z(1) = 0.9$, Mean IC $= 0.05$, IC Standard Deviation $= 20\%$

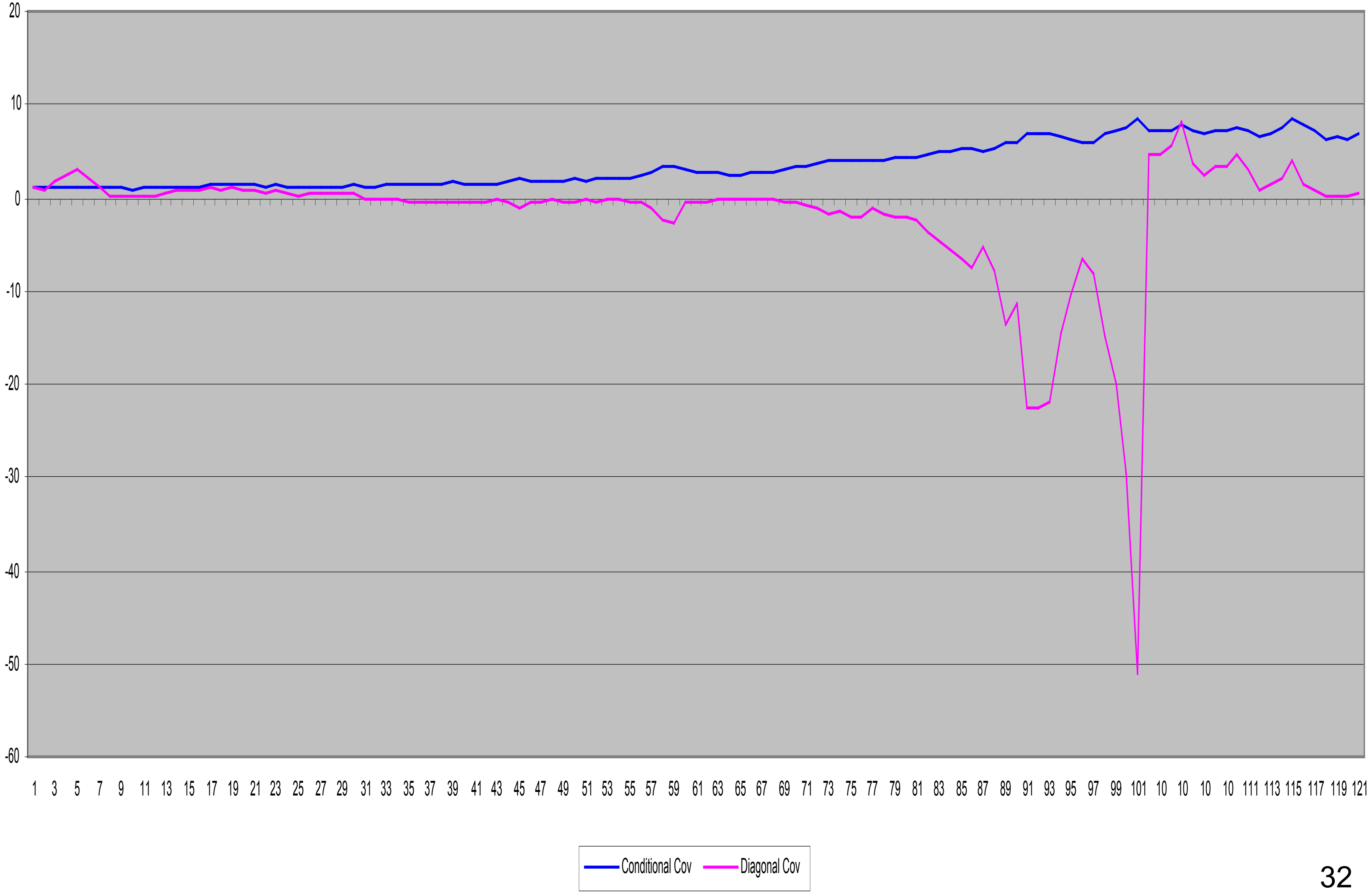
$\mathrm{Max}(\sigma_i) = 0.40$, $\mathrm{Min}(\sigma_i) = 0.05$, σ_i uniformly distributed, $E_{cs}(1/\sigma_i) = 5.94$

	Covariance w Strategy Risk		Diagonal Cov w/o Strategy Risk	
	Theory (Ding)	Realized	Theory (QHT)	Realized
TE	0.08	0.08	0.08	0.51
IR	0.25	0.24	0.25	0.25
Turnover	0.42	0.46	2.68	5.26
Leverage	1.90	1.87	11.99	11.93
TO/L	0.22	0.25	0.22	0.44
Avg Port Return		0.02		0.12
Max Port Return		0.22		1.38
Min Port Return		-0.18		-1.12

Simulated Portfolio Return I



Cumulative Portfolio Return I



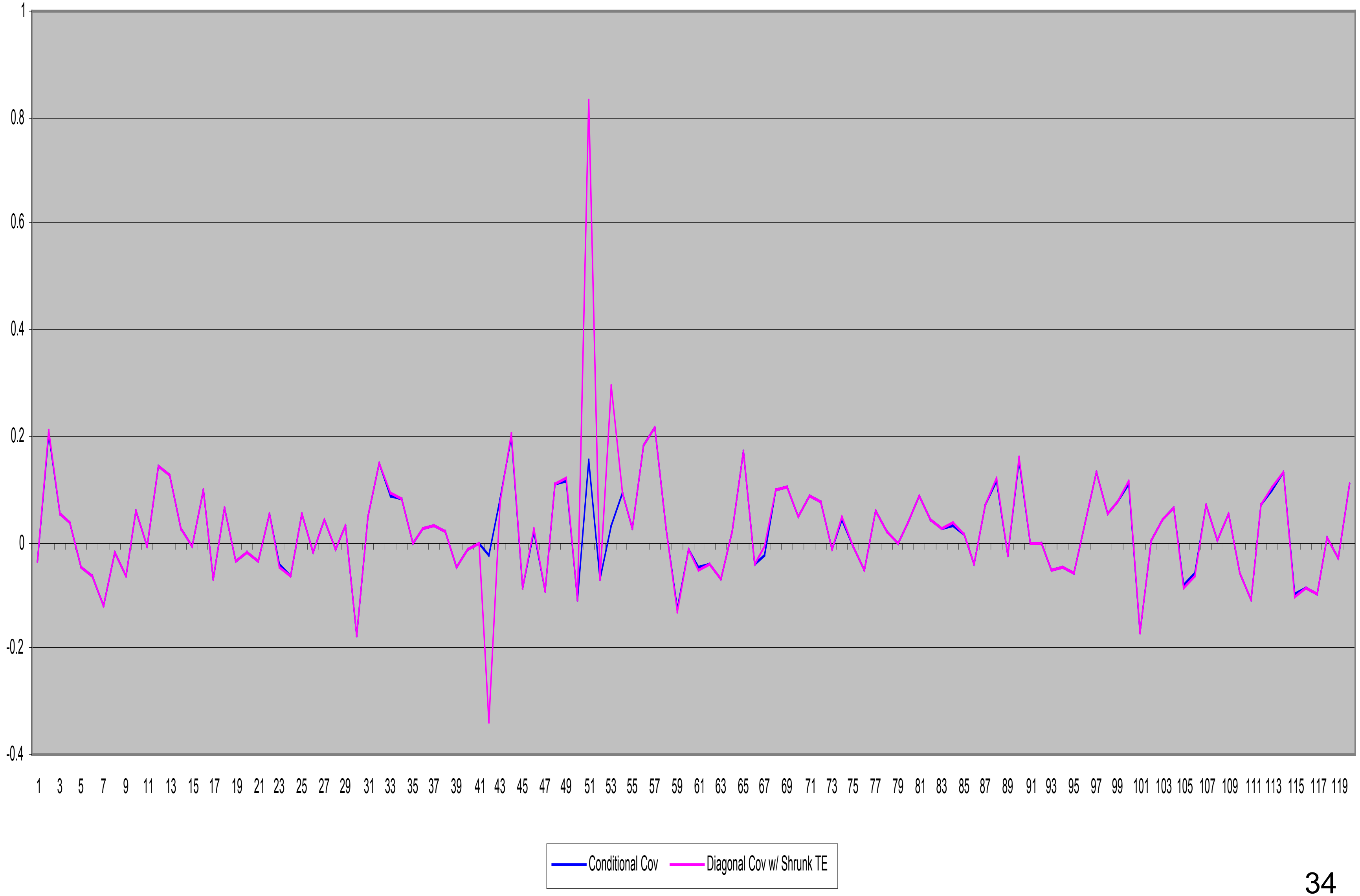
Portfolio Simulation II

$N = 1000,\ \mathrm{T} = 120,\ \rho_z(1) = 0.9,\ \text{Mean IC} = 0.05,\ \text{IC Standard Deviation} = 20\%$

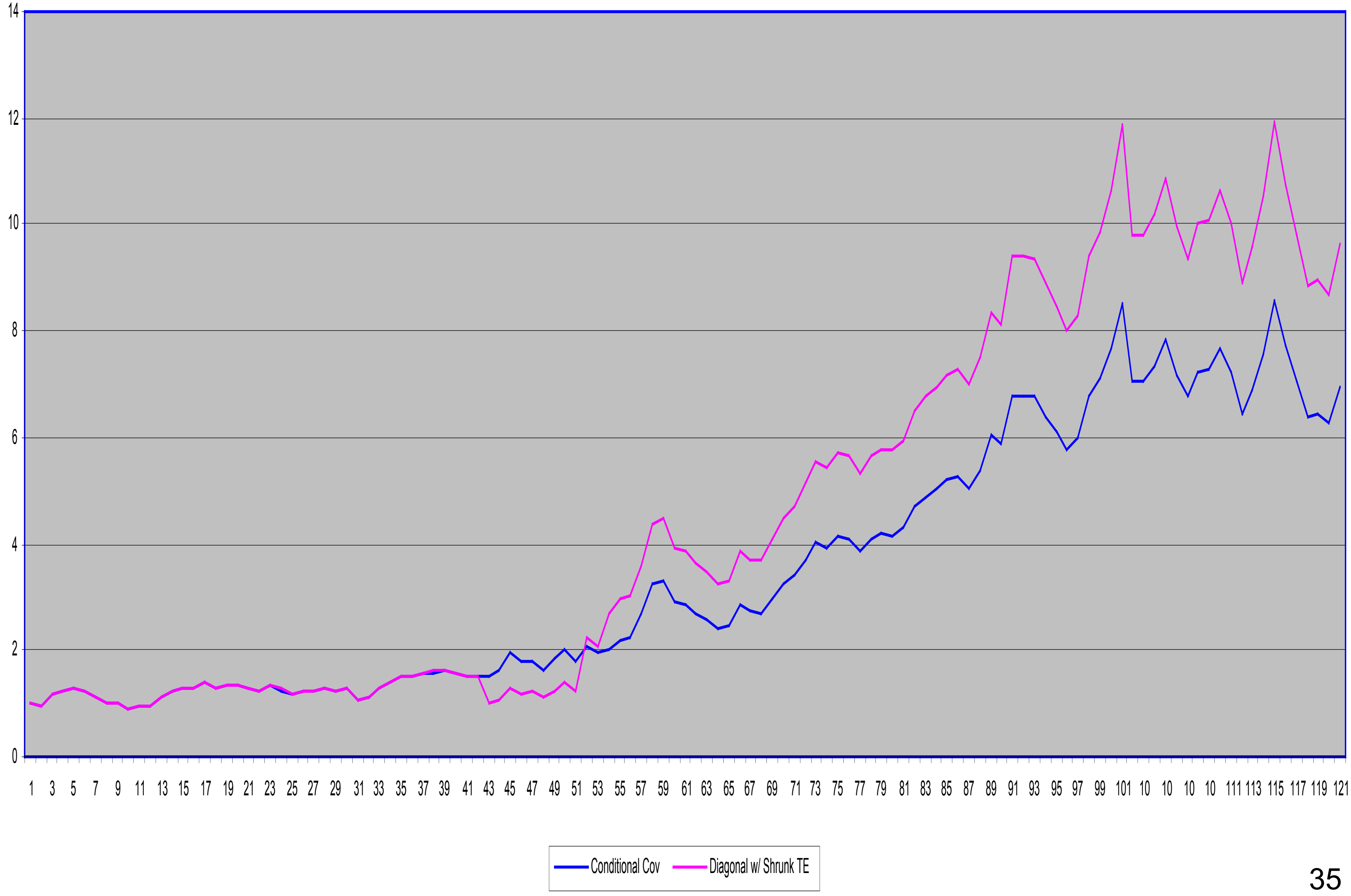
$\mathrm{Max}(\sigma_i) = 0.40,\ \mathrm{Min}(\sigma_i) = 0.05,\ \sigma_i \text{ uniformly distributed},\ E_{cs}(1/\sigma_i) = 5.94$

	Covariance w Strategy Risk		Diagonal Cov w/o Strategy Risk but Shrunk TE	
	Theory (Ding)	Realized	Theory (QHT)	Realized
TE	0.08	0.08	0.013	0.12
IR	0.25	0.24	0.25	0.21
Turnover	0.42	0.46	0.42	0.69
Leverage	1.90	1.87	1.90	2.13
TO/L	0.22	0.25	0.22	0.33
Avg Port Return		0.02		0.02
Max Port Return		0.22		0.83
Min Port Return		-0.18		-0.34

Simulated Portfolio Return II



Cumulative Portfolio Return II



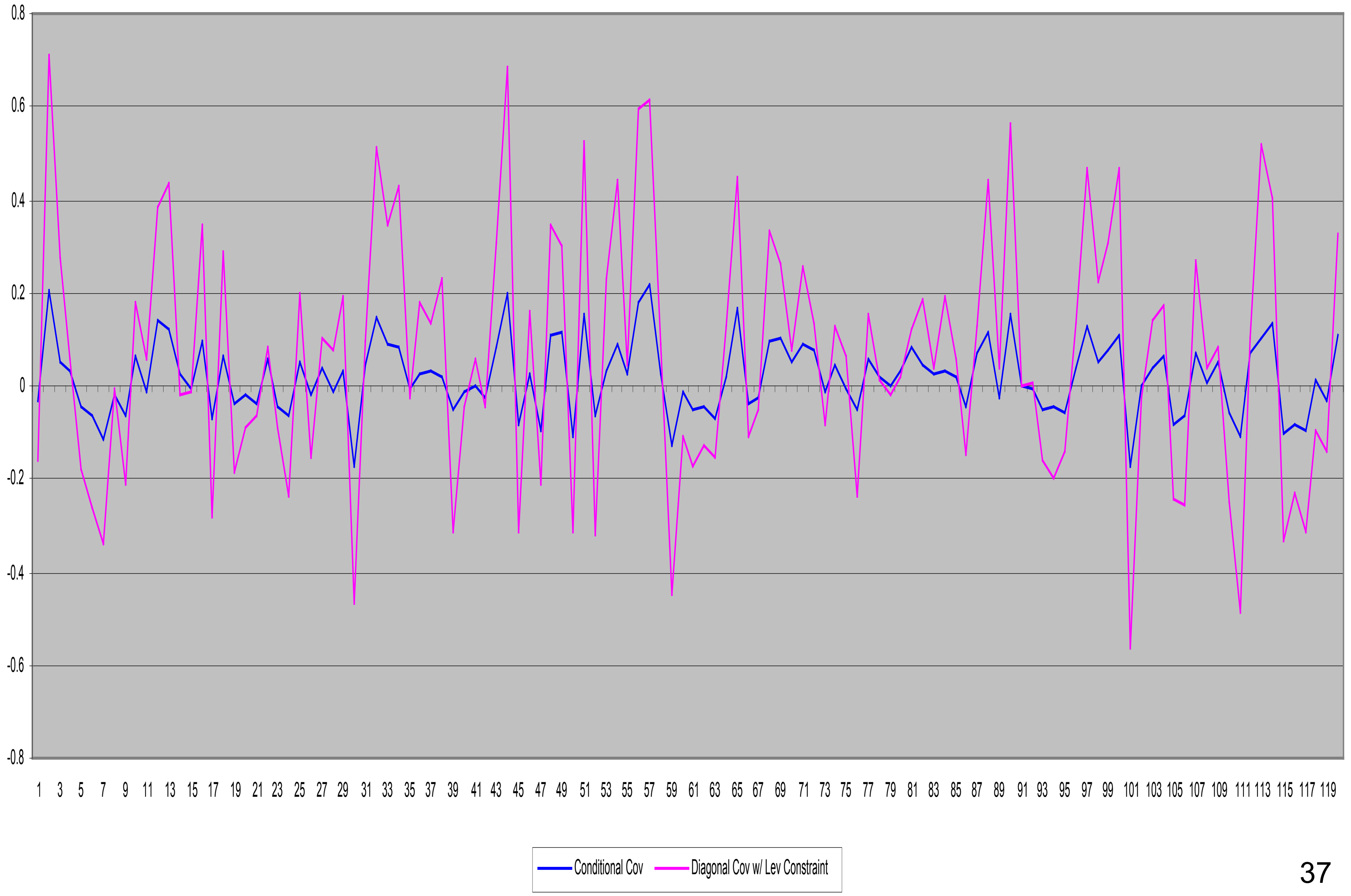
Portfolio Simulation III

$N = 1000,\ \mathrm{T} = 120,\ \rho_z(1) = 0.9,\ \text{Mean IC} = 0.05,\ \text{IC Standard Deviation} = 20\%$

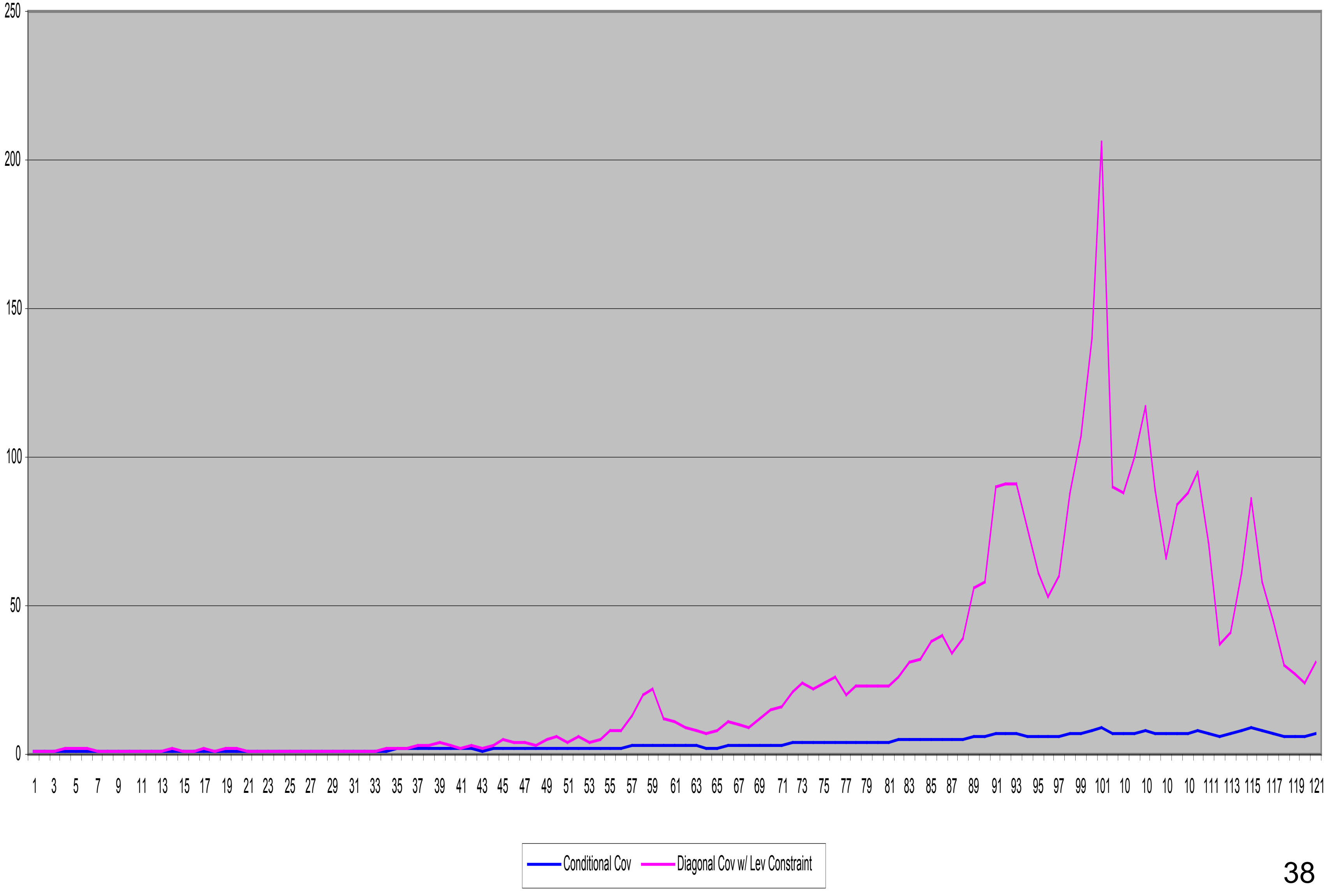
$\mathrm{Max}(\sigma_i) = 0.40,\ \mathrm{Min}(\sigma_i) = 0.05,\ \sigma_i \text{ uniformly distributed, } E_{cs}(1/\sigma_i) = 5.94$

	Cov w Strategy Risk		Diagonal Cov w/o Strategy Risk but Leverage Control	
	Theory (Ding)	Realized	Theory (QHT)	Realized
TE	0.08	0.08	0.08	0.27
IR	0.25	0.24	0.25	0.24
Turnover	0.42	0.46		0.82
Leverage	1.90	1.87	1.90	1.88
TO/L	0.22	0.25		0.44
Avg Port Return		0.02		0.06
Max Port Return		0.22		0.71
Min Port Return		-0.18		-0.56

Simulated Portfolio Return III



Cumulative Portfolio Return III



Observations

- The theoretical and realized portfolio characteristics match quite well when we use the risk model that incorporates strategy risk (conditional covariance).
- The portfolio generated by diagonal covariance matrices was wiped out twice during the 120 time periods.
- The portfolio generated by diagonal covariance matrices with shrunk TE is close to the portfolio generated by conditional covariance matrix with a higher realized TE and lower realized IR.
- The portfolio generated by diagonal covariance matrices with leverage constraints has a much higher realized return and risk.

Conclusions

- When the cross-sectional information coefficient is time varying, the optimal turnover and leverage ratio are usually much smaller than those derived when IC is a constant.
- The conditional covariance matrix that incorporates the "Strategy Risk" is the proper measure of risk for portfolio construction purposes.
- The diagonal covariance matrix alone cannot control portfolio risk effectively.
- For the one factor model, it seems the diagonal covariance matrix with an adjusted target TE using

$$\text{Adjusted TE} = \frac{\text{Target TE}}{\sqrt{\text{N}}\sigma_{\rho}}$$

as suggested by Qian and Hua (2004) can help in controlling risk.

References

- Ding, Zhuanxin. "The Fundamental Law of Active Management: Time Series Dynamics and Cross-Sectional Properties." (June 16, 2010). Available at SSRN: http://ssrn.com/abstract=1625834 or http://dx.doi.org/10.2139/ssrn.1625834.
- Ding, Zhuanxin, "The Statistics of Cross-Sectional Information Coefficient. " (July 5, 2011). Available at SSRN: http://ssrn.com/abstract=1826303 or http://dx.doi.org/10.2139/ssrn.1826303.
- Ding, Z. and C. W. J. Granger. 1996. "Modelling Volatility Persistence of Speculative Returns: A New Approach." *Journal of Econometrics*, Vol. 73, Issue 1, 185-215.
- Grinold, Richard C. 1989. "The Fundamental Law of Active Management." *The Journal of Portfolio Management*, vol. 15, no. 3 (Spring): 30–38.
- Grinold, Richard C. 1994. "Alpha is Volatility Times IC times Score." *The Journal of Portfolio Management*, vol. 20, no. 4 (Summer): 9–16.
- Grinold, R. C. and R. Kahn. 2000. *Active Portfolio Management*. McGraw-Hill.
- Qian, E., R. Hua and J. Tilney. "Portfolio turnover of quantitatively managed portfolios." *Proceeding of the 2nd IASTED International Conference*, Financial Engineering and Applications, Cambridge, MA, 2004.
- Qian, Edward, and Ronald Hua. "Active Risk and Information Ratio." *The Journal of Investment Management*, vol 2, no. 3 (2004), pp. 20-34.
- Qian, E., Hua, R., and Sorensen, E.H. (2007). *Quantitative Equity Portfolio Management: Modern Techniques and Applications,* London: CRC Press.
- Rothschild, M. "Asset Pricing Theories." *Uncertainty, Information and Communication, Essays in honor of Kenneth J. Arrow*, Vol III, 1986.
- Ye, Jia. "How Variation in Signal Quality Affects Performance." *Financial Analysts Journal*, vol. 64, no. 4 (2008), 48-61.