Low Volatility Equity Investing:
Anomaly or Algebraic Artifact

Dan diBartolomeo

Northfield Asia Research Seminar

November 2013

Introduction

- *Since Haugen and Baker (1991), numerous papers have argued that low volatility equities strategies generate performance well above the expectations of equilibrium models such as CAPM.*

- *A few papers have advanced theories on manager behavior such as focus on tracking error and aversion to leverage as potential explanations. While we do not dispute these ideas, we instead will show that they are probably unnecessary to explain what has been observed.*

NORTHFIELD | modeling financial markets worldwide

My Main Assertion

- *The presentation will assert that over long periods, the algebraic differences between arithmetic, and geometric measures of return explain much of the effect.*

- *The remainder of the effect arises from investors failing to adjust their CAPM expectations for real world features of the hypothetical "market portfolio" such as skew, kurtosis, non-zero transaction costs and parameter estimation error.*

- *We will introduce a series of simple algebraic adjustments into CAPM for these defects, and suggest that their combined impact is substantial enough to probably explain the low volatility effect.*

A Bit of Literature Review

- Haugen and Baker (1991): Early evidence in support of low volatility equity strategies

- Lochoff (1998): Leveraged short term bonds outperform long term bonds with comparable volatility

- Clarke, daSilva and Thorley (2006): Using 25% weight to the minimum variance portfolio reduced volatility with no loss of return

- Blitz and VanVliet (2007): Substantial return premium to low volatility across many markets from 1986 to 2006

- Buchner (2010): Asserts specific risk not beta should be priced for illiquid assets

- Barro (2005) and Gabaix (2009): Argue equity investors only worry about 1929 type crashes, so the equity premium over cash should be big but the premium of risky stocks versus not so risky stocks should be small

Framing the Discussion

- What we observe is that low volatility strategies for equities seem to outperform persistently and across most countries

- There are two explanations that have been proposed:

  - Asset managers load up (overpaying in the process) on high risk stocks because they don't care about absolute risk, they just care about outperforming benchmarks (i.e. risk is tracking error). Biasing toward riskier stocks should improve benchmark relative returns. Managers may also "benchmark hug" avoiding low volatility stocks (Baker, Bradley and Wurgler, 2011 and Brennan, Cheng, Li 2012)

  - Many investors are constrained legally and otherwise from using leverage, so they again load up on higher risk stocks. This makes low risk stocks relatively cheap and you can outperform by leveraging a portfolio of low volatility stocks (Frazzini and Pedersen, 2011, Jacobs and Levy, 2011)

The Fly in the Ointment of Explanations

- The problem with both of these behavioral stories are that the effects are fairly obvious.
  - If low volatility stocks are always cheap, why wouldn't hedge funds that do use leverage dive in and buy them, thereby pushing up prices which should eliminate the advantage.
  - Benchmark hugging should also cause a "high volatility" effect which nobody has reported (until now maybe).

- We argue also that managers are not sub-optimally leverage averse
  - Anybody remember how much leverage there was in 2007?
  - We believe that leverage levels are actually excessively high once you incorporate the possibility of excessive transaction costs in a margin call liquidation

- *Even if these explanations are true they are not needed*

One Intriguing Result in a Prior Presentation

	Mean Monthly Return	Cumulative Return	Monthly Volatility	Annual Geometric Return
Q1 Equal	1.03	790.86	3.64	11.50
Q1 MinVar	1.07	840.43	2.96	12.34
Q5 Equal	1.33	713.77	9.15	10.90
Q5 MinVar	1.77	2940.15	6.80	**19.33**

This is a 1992 to 2010 back-test of our "high sustainability" (Q1) and "low sustainability" (Q5) portfolios as described in diBartolomeo (2010).

A minimum variance portfolio of high bankruptcy risk stocks provided by far the best return, providing a 7% per annum advantage over the minimum variance portfolio of "safe stocks".

What Does CAPM Actually Say?

- The CAPM is a single period model (*which precludes the possibility of compounding*) and therefore says there is *a* linear relationship between arithmetic average returns and systematic risk (beta).

- The relationship between geometric (with compounding) returns and risk (beta) **must be a convex function if the relationship of arithmetic returns and risk is linear**

  - If returns are a random walk, the compound rate of return will be the arithmetic average minus half the variance,

  - Messmore (1995),

  - Wilcox (2003) extends this to include higher moments

NORTHFIELD | modeling financial markets worldwide

Forming Expectations by Abusing the CAPM

CAPM as put forward by Sharpe (1962) has a lot of important assumptions

- Transaction costs and taxes are zero
- All information is available to all investors
- There are no limits on cross-border investing
- The market clearing portfolio consists of all risky assets (including bonds, real estate etc.), not just a subset of equities that are capitalization weighted
- **The future consists of one long period** and we know what the risk free rate is for that period
- All investors can borrow at the risk free rate
- Beta values for securities are known (not estimated)

General Implications of the CAPM Assumptions

None of these assumptions hold true in the real world so there is no reason to believe that a capitalization weighted equity index should be mean-variance efficient.

See Grinold (1992) for a good summary

A variety of papers have tried to better model CAPM by making one or more of the assumptions more realistic (e.g. borrowing costs above the risk-free rate). All the fixes suggest a flatter flat security market line

Empirical tests of return premiums to beta risk are joint tests of CAPM and our ability to estimate beta accurately.

Our Basic Algebra

$$E[A_i] = R_f + B_i (R_m - R_f) \qquad\qquad (1)$$

$$B_i = ((S_i / S_m) * P_{im}) \qquad\qquad (2)$$

$$E[A_i] = R_f + ((S_i / S_m) * P_{im}) (R_m - R_f) \qquad\qquad (3)$$

$$E[G_i] = E[A_i] - S_i^2/2 \qquad\qquad (4)$$

$$E[G_i] = R_f + S_i * ([P_{im}*(R_m - R_f) / S_m] - S_i/2) \qquad\qquad (7)$$

NORTHFIELD | modeling financial markets worldwide

More Algebra

For the special case $P_{im} = 1$

$$S_i = B_i S_m \tag{8}$$

$$E[G_i] = R_f + B_i S_m * ([(R_m - R_f) / S_m] - B_i S_m / 2) \tag{9}$$

We can multiply all this out and take the first derivative of this expression with respect to B_i to calculate where the critical value of B_i^* where G_i will be expected to peak (for algebra fans we could also use the "product rule").

The higher the Sharpe ratio of the market portfolio, the higher the value of B_i^*. See Klepfish (2013)

The General Case

- For any values such that $(S_i/2) > (P_{im} * (R_m - R_f)/S_m)$, the expected value of G_i will be a decreasing function of S_i and the expected geometric return will be *below the risk-free rate.*

- For any values such that $(S_i/2) < (P_{im} * (R_m - R_f)/S_m)$ the expected value of G_i will be an increasing function of S_i

- This assumes S_m and R_f are always positive.
  - If markets are rational, the expected value of R_m will always be positive and larger than R_f.
  - CAPM says the P_{im} for any reasonable portfolio should be positive and is **bounded at 0 and 1**
  - You can solve for the peak geometric return as function of beta

NORTHFIELD | modeling financial markets worldwide

"Low Vol" and the Market Portfolio

- CAPM defines a "market portfolio" that consists of all risky assets (not just stocks)
  - This would include all of the bond market that is not risk free for whatever you are defining as the single period
  - There is the messy problem of double counting because of securitization of lots of fixed income securities
  - The "multi-asset class" market portfolio will include some stuff that is illiquid

- A typical stock portfolio would look quite high in volatility compared to the revised market portfolio.
  - Depending on the correlation you assume across asset classes, it is likely that typical stock portfolios would have a pretty high beta **(i.e. probably higher than B_i*)**

Adjusting for Lack of Liquidity

- Lo, Getmansky and Makarov (2004) argues that for financial markets to operate rationally, they should not be predictable.
  - To the extent that trading costs inhibit trading, the changes in observed market prices will lag their true economic values so the volatility we calculate from the return data will be understated
  - In 2008, Anish Shah (Northfield) created an algebraic correlation for this downward bias. For example, to correct for the serial correlation in the monthly returns of the Barclay's High Yield bond index, you need increase the perceived volatility by 27%
  - For real estate, the correction can *triple* conventional risk values
  - If multi-asset class market portfolio includes some illiquid assets, we would need to upward adjust the expected volatility of the market portfolio, **again pushing B$_i$* to the left**

NORTHFIELD | modeling financial markets worldwide

Higher Moments of Returns

- Almost all empirical studies show that financial market returns have negative skew and positive excess kurtosis

  – We could also consider the impact of skew and kurtosis on the difference between the geometric and arithmetic means of returns as in Wilcox (2003). Given the standard deviation, skew and kurtosis values (in decimal forms) we can calculate the value of the volatility that would provide the same degree of arithmetic difference between the two forms of the return mean.

$$V \sim (S_m^2 - (2/3)^* \, MS_m^3 + (1/2)^* \, KS_m^4)^{.5}$$

- V = adjusted volatility, S_m = market volatility, M = skew, K = excess kurtosis

- Since $M < 0$ and $K > 0$, $V > S_m$, **so B_i^* moves to the left**

NORTHFIELD | modeling financial markets worldwide

A Bit More on Higher Moments

- The calculation on the prior page assumes that investors are growth optimal and are all trying to maximize their geometric return

- The composition of the market portfolio suggests that as a whole investors are more risk averse than growth optimal, so we may need to adjust more strongly for higher moments so **B_i* moves downward more**

- $V \sim (S_m^2 - (200/RAP) [(2/3)* MS_m^3 + (1/2)* KS_m^4])^{.5}$

  In the absence of explicitly stated risk aversion we approximate
  $RAP = 6 * S_m$

Estimation Error in Beta and Market Volatility

- CAPM assumes beta and market volatility values are known, not estimated
  - To the extent that real world beta values are just estimates, the overall return dispersion associated with a given beta value increases
  - Michaud (1998) argues that near the top of the efficient frontier, this additional estimation risk can dominate the problem turning the efficient frontier downward.

- Consider a beta value of 1 with a standard error of .2 and $S_m = 20$ and $P_{im} = 1$

  If we have a 50% chance that the true beta = .8 and a 50% chance that the true beta = 1.2, we get an effective $S_m = 20.4$ not 20, so **B_i* moves left again**

NORTHFIELD | modeling financial markets worldwide

Implications for Low Volatility Managers

- If we are correct that the unexpectedly good performance of "low vol" equity investing is the result of poorly formed expectations rather than a market anomaly there is good and bad news for asset managers
  - The bad news is that it is hard to claim managerial skill for achieving a result that is apparently consistent with markets being efficient
  - The good news is that since there is no anomaly being exploited, there is no reason to believe that the non-existent anomaly will be arbitraged away in the future.

Conclusions

- While we do not dispute the behavioral explanations of the observed performance of "low vol" equity strategies, we assert that no such explanation is probably necessary.

- We suggest that *very careless interpretation of the CAPM* has lead investors to form incorrect expectations, which has caused the observed performance of low volatility equity portfolios to be perceived as outperforming what should be expected in an efficient market.

- We have shown that the relationship between geometric returns and beta should be convex, and that the value of B_i^* is a function of the Sharpe ratio of the market portfolio.

- We have also illustrated some reasonable algebraic adjustments to compensate for some of the unrealistic assumptions of the CAPM

Parting Apologies

If we shadows have offended, think but this and all is mended,
That you have slumb'red here while these visions did appear.
And this weak and idle theme, No more yielding but a dream.

Gentles, do not reprehend. If you pardon, we will mend.
And, as I am an honest Puck, if we have unearned luck
Now to escape the serpent's tongue, We will make amends ere long;

Else the Puck a liar call. So, good night unto you all.
Give me your hands, if we be friends,
And Robin shall restore amends.

Puck, (closing verse)
A Midsummer Night's Dream
William Shakespeare