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The Choice of Model Factors under Multiple Definitions of Risk

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Introduction

- Risk assessment in investment management is intrinsically multi-dimensional. Investors may be concerned in differing degrees with a multitude of risk measures such as tracking error, active risk, absolute volatility, VaR, CVaR, and "first passage" risk (drawdowns).
  - In recent years, the increasing regulatory focus on downside "tail risk" measures such as VaR and CVaR has extended the interest in this topic.
  - However, the financial literature is quite sparse on how the differing representations of risk impact the choice of model factors, particularly as we move from a simple single period model to a multi-period representation.

Presentation Outline

- Literature Review
  - Roll(1992), Wilcox(1994), Chow(1995), Kritzman/Rich (2002)
- Intuition on the choice of risk model factors
  - Alpha and risk factors are often chosen under single period assumptions
  - The fine print in "ceteris paribus"
- Formulation of joint objectives as a "dual benchmark"
  - Wang (1999), Pareto solution: Shah and Shectman (2000)
  - Numerical representation of a dual optimization problem
- Extensions to multi-period
  - Uncertainty of mean returns and factor exposure preferences
  - Example with Fama-French data

A Mean/Variance Analysis of Tracking Error

- Roll (1992)
  - The dichotomy of mean/variance efficiency and tracking error
  - Why should active managers worry about being not "too different" from a benchmark they are confident of beating
  - Obvious answer: managers care about their own risk of being fired as more than they care about the risks to the investors.
  - Consultants often focus on tracking error to justify emphasis on the manager evaluation to justify their existence
  - Quay (2013) argues asset owners condone this behavior in order to have someone to blame in periods of poor performance
- Consider two portfolios
  - The first has alpha 2, tracking error 2 and absolute volatility 15
  - The second has alpha 2, tracking error 2 absolute volatility 19

Why EAFE is for Wimps!

- Wilcox (1994)
  - Argues that cap weighted benchmarks like EAFE perform poorly because overvalued countries (e.g. Japan in the 1990s) become large weights in the index
  - Empirical tests showed that country weight schemes that were close to equal weight had better performance in terms of absolute return and risk
  - Proposes that managers prefer low tracking error to established benchmarks to avoid business risk
  - Wilcox cites a phone conversation with me as the basis of forming a joint benchmark that is part EAFE (preference for low tracking error) and part cash (preference for low absolute risk), and an efficient frontier established based on variation in country weightings.

Return, Risk and Relative Performance

- Chow (1995)
  - Formulates the active management problem as a three term objective: return, risk and relative performance
  - Since return is linear maximizing return also maximizes benchmark relative return (alpha)
  - The efficient frontier becomes a three dimensional "efficient surface" with return, absolute variance, and tracking variance
- As cash and any benchmark index have no covariance, mixing cash into the benchmark is mathematically equivalent to passing a plane through the efficient surface.
  - The intersection of the plane and the efficient surface is the efficient frontier to the "cash included' joint benchmark

The Mismeasurement of Risk

- Kritzman and Rich (2002)
  - Investors typically report on the probable distribution of terminal wealth at the end of some investment horizon. For example "If I have $100 million today, what is the probability that I will have less than $80 million five years from now"
  - A more realistic measure of investor risk might be to ask "If I have $100 million today, what is the probability that I will have less than $80 million at any one moment in the next five years?"
  - The "intra-horizon" risk calculation is a formalization of the practitioner concept of "drawdowns and other risk measures that purportedly include potential higher moments (VaR and CVaR)
  - The probability of an intra-horizon value below the floor is always greater than probability at the investment horizon since there is a non-zero probability of going below the floor and coming back above before the horizon date.

Math Representation of Dual Benchmarks

- For the absolute/relative problem we have

$$U = R - \lambda_a \sigma_a^2 - \lambda_{te} \sigma_{te}^2$$

  - But what if we have two index benchmarks (e.g. the S&P 500 and Russell 1000) for which we are jointly concerned about tracking variance?

$$U = R - \lambda_{sp} \sigma_{sp}^2 - \lambda_{R1} \sigma_{R1}^2$$

This optimization problem can be solved easily by noting that the Russell 1000 can be disaggregated into the S&P 500 and a long/short portfolio of differences in the two indices. We already know λ_{sp}

Representation of Dual Benchmark Optimization

- $U = R - \lambda_{sp}\sigma_{sp}^2 - \lambda_{R1}\sigma_{sp\text{-}R1}^2$

  Where "sp-R1" is the long/short portfolio of weight differences

- Let $K = \lambda_{R1} / \lambda_{sp}$

- $U = R - \lambda_{sp}\sigma_{sp}^2 - K\lambda_{sp}\sigma_{sp\text{-}R1}^2$

- $U = R - \lambda_{sp}\sigma_{sp}^2 - \lambda_{sp}(K^{.5}\sigma_{sp\text{-}R1})^2$

The term in red can be represented using "composite assets" in the Northfield Optimizer. The problem collapses to the usual two term problem, much like the absolute/relative case

Risk Measures and the Choice of Model Factors

- Different definitions of risk might influence our choice of which factors include in both alpha and risk models.
  - If I hold a benchmark index fund and add a low beta stock to it, tracking error will increase but absolute risk decreases.
  - Some factors such as balance sheet gearing (debt/equity or similar ratio) would have comparable behavior. If my portfolio has an average gearing level greater than X, this will increase tracking error and increase absolute risk, while portfolio gearing below X will increase tracking error but must reduce absolute "tail risk" (a company with no debt cannot go bankrupt).
  - However, for many factors (e.g. momentum, book/price) there is no immediate intuition as to which way factor bets normally contribute to absolute risk, potentially making factors bets harder to interpret.

Fun with Fama and French Factors

- FF five factor model
  - Mkt-RF (market risk premium)
  - SML (small minus large)
  - HML (high book/price minus low book/price)
  - RMW (robust minus weak profitability)
  - CMA (conservative minus aggressive investment)
- Annual Data from 1964 through 2014
- All style factors returns are calculated “raw” and are not orthogonal to the market return

Factor Data Summary Statistics

	RP	SMB	HML	RMW	CMA	RF
Mean	6.62	3.96	4.87	3.08	4.17	4.27
St Dev	17.92	14.25	13.64	9.36	9.74	2.61
Skew	-0.65	0.53	-0.24	-0.37	0.36	-0.14
Kurtosis	-0.16	1.18	0.64	0.99	0.14	-0.70
Pearson	1.00	0.24	-0.29	-0.27	-0.36	-0.22
Geometric	5.01	2.95	3.94	2.64	3.70	4.24

Getting Alpha from Zero Alpha Factors

- We make each of the FF factors less interesting by demeaning the series so that the mean factor return is zero. Each also has a high volatility so looks really bad as an alpha factor

- But CMA has a large negative covariance with market risk premium which reduces portfolio volatility and increases geometric mean returns

	RP	RP+SMB	RP+HML	RP+RMW	RP + CMA
Mean	6.62	6.62	6.62	6.62	6.62
St Dev	17.92	20.01	19.15	17.86	17.05
Skew	-0.65	-0.30	-0.54	-0.56	-0.36
Kurtosis	-0.16	-0.01	-0.18	0.60	-0.10
Geometric	5.01	4.62	4.79	5.03	5.17

The Two Dimensions of Long Term Returns

- When we estimate the future volatility of a portfolio, we are using units of standard deviation which represents the dispersion of a distribution around a known mean.
  - Similar to a gambler playing roulette. You can lose but the odds and payoffs are known

- In the real world, investors don't know with certainty what the future mean return will be. *We have both volatility around the mean, and uncertainty of the mean return.*
  - Similar to a gambler playing poker in a casino. We can lose because of the random draws of cards, but we don't know the true odds and payoffs because we don't know how good the other players are, or what their financial resources may be.

A Multi-Period Illustration

- We estimate portfolio factor exposures, specific risk and total volatility of a typical institutional (multi-asset class) portfolio using the Northfield Everything Everywhere Model as of the end of 2014
  - Under the assumption that return process is IID and has a known mean, we would expect the standard deviation of return to increase with the square root of the time horizon
- Using the EE factor realizations of the past 60 months as the seed data, we use "bootstrapping with replacement" to simulate plausible alternative sequences of events.
  - Each path has 60 months from which we calculate five year cumulative return

Simulation Outcomes

- Estimated portfolio volatility is 14%
  - Using the "square root of time" rule, the expected value of the standard deviation of simulated five year cumulative returns is 31%
  - With 100 simulated paths, the standard deviation of the five year return is actually 65%, more than double what we would expect.
- The past we have lived through is only one path of what could have been an infinite number of alternative paths even if we assume the distribution to be unchanged.
  - Over short time horizons, the uncertainty of the mean has small influence on outcomes, but over long term horizons of many periods, it is at least as important as our usual conception of volatility risk

Factor Selection To Reduce Mean Uncertainty

- The econometric literature is extensive with respect to the concept of "cointegration"
  - This is a situation where you can combine two or more non-stationary time series and the resultant series is stationary (the distributional parameters like mean and standard deviation are constant across time)
  - What you would like is a set of factors that when combined with your portfolio would make the resultant return series be cointegrated
- Related papers and presentations on our website
  - http://www.northinfo.com/Documents/573.pdf
  - http://www.northinfo.com/Documents/59.pdf

Conclusions

- Investors have multiple conceptions of portfolio risk
- Two or more metrics of risk may be simultaneously part of the objectives of a single portfolio
  - These can usually be easily accommodated
- Some widely used common factors may be unintuitive in absolute risk applications.
- Multi-period evaluation suggests that some factors we would usually overlook may add material value
  - This may arise from either being negatively correlated with broad market returns or by have autocorrelation properties that produce cointegrated return series.