

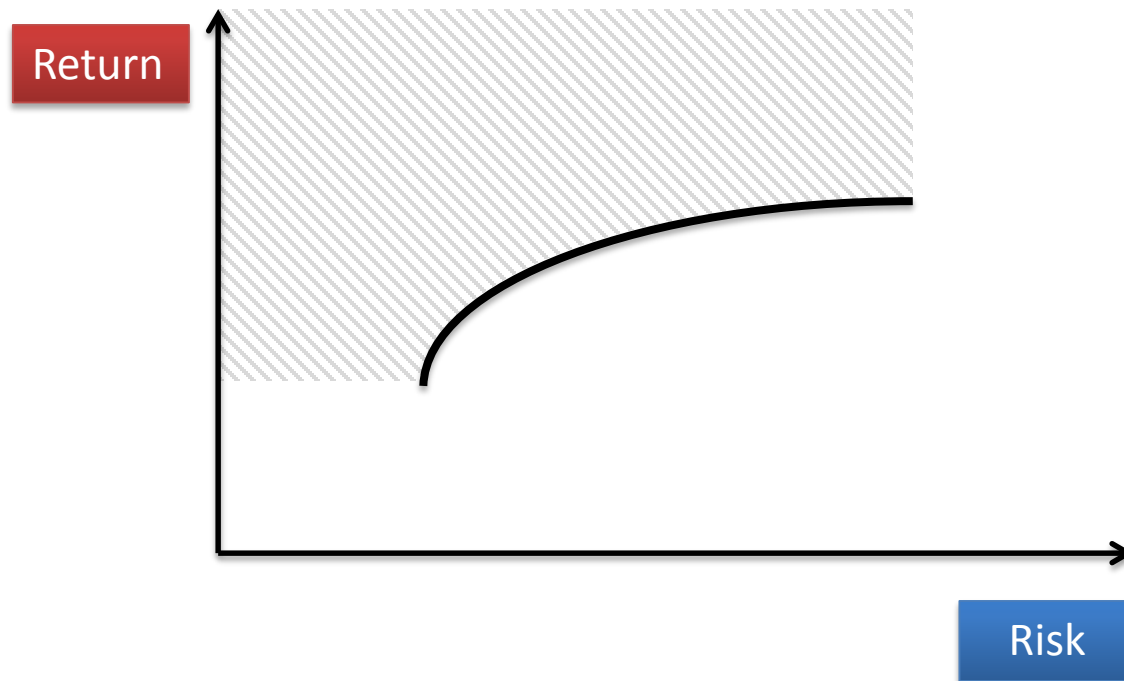
Optimization 101

Steven Dyer
February Online Workshop
15 February 2018

Goals

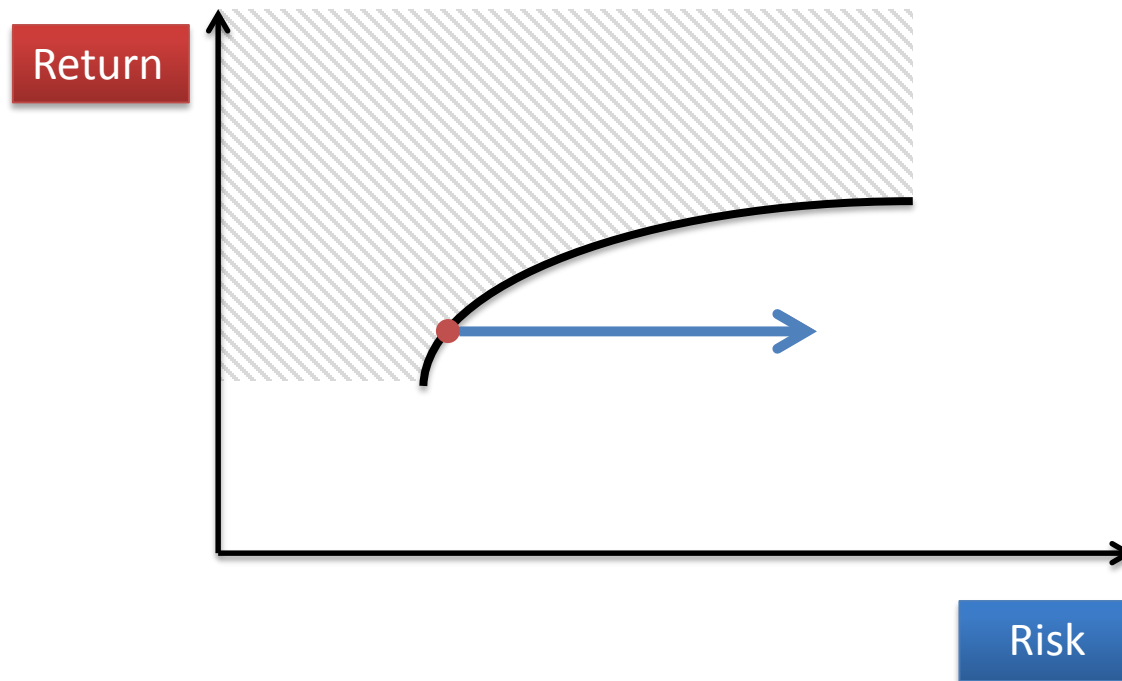
- Provide a short introduction to the concept of portfolio optimization
- Describe the Northfield methodology of optimization
- Use almost no math and any math will be easy and any hard math you can ignore
- Have some fun.

Efficient frontier



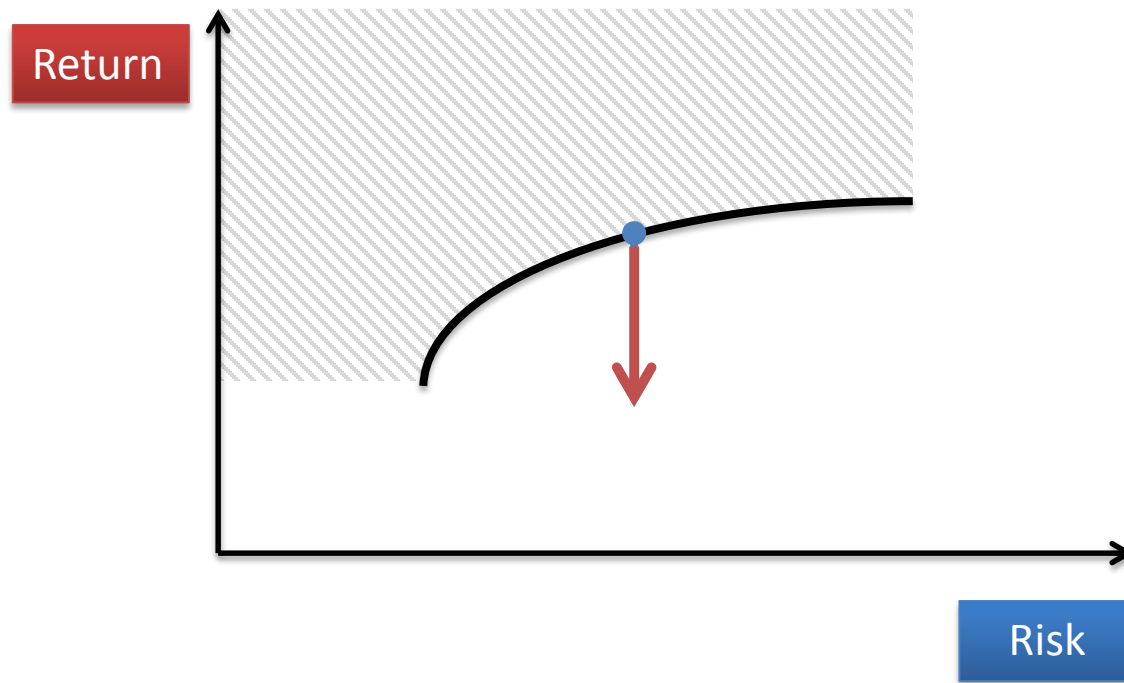
Efficient frontier

For any level of **return**, the least amount of **risk**

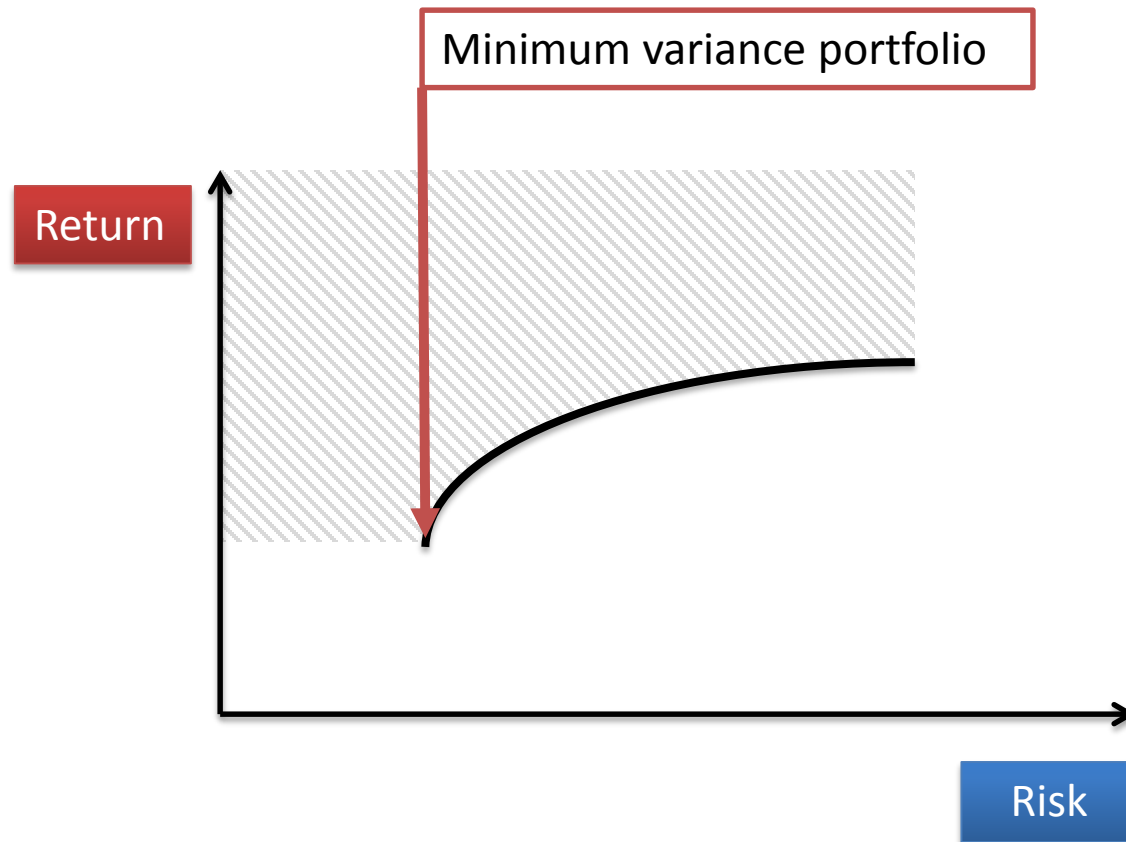


Efficient frontier

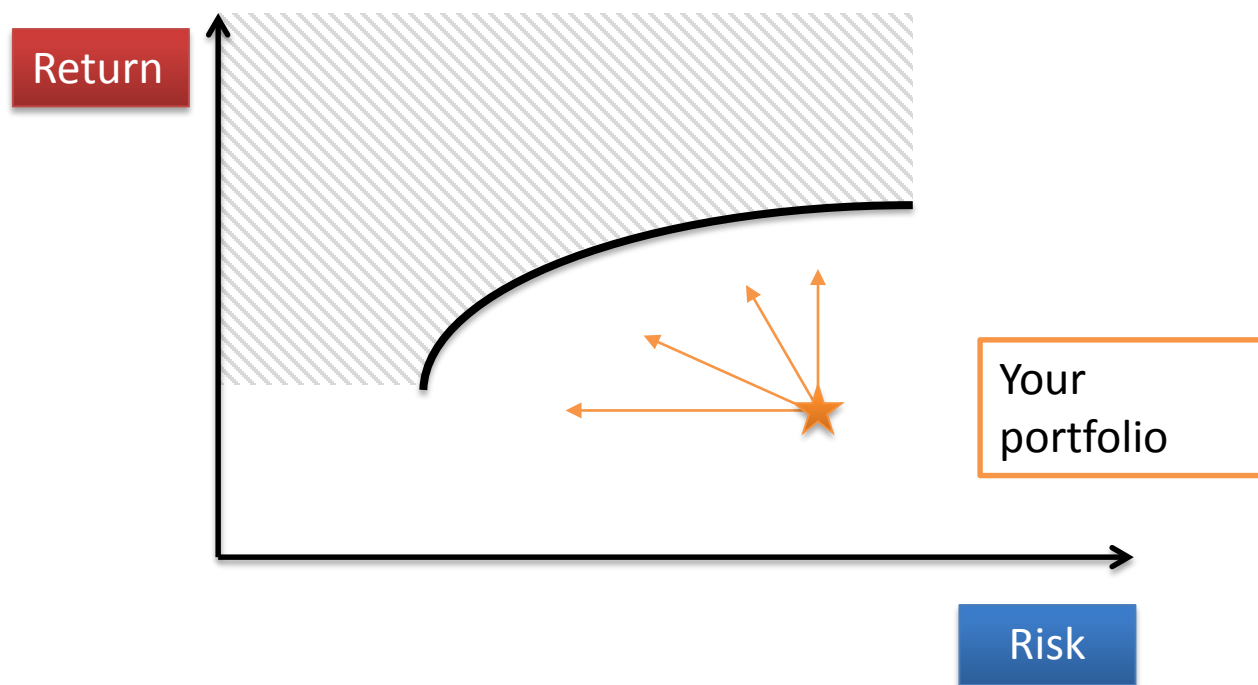
For any level of **risk**, the most amount of **return**



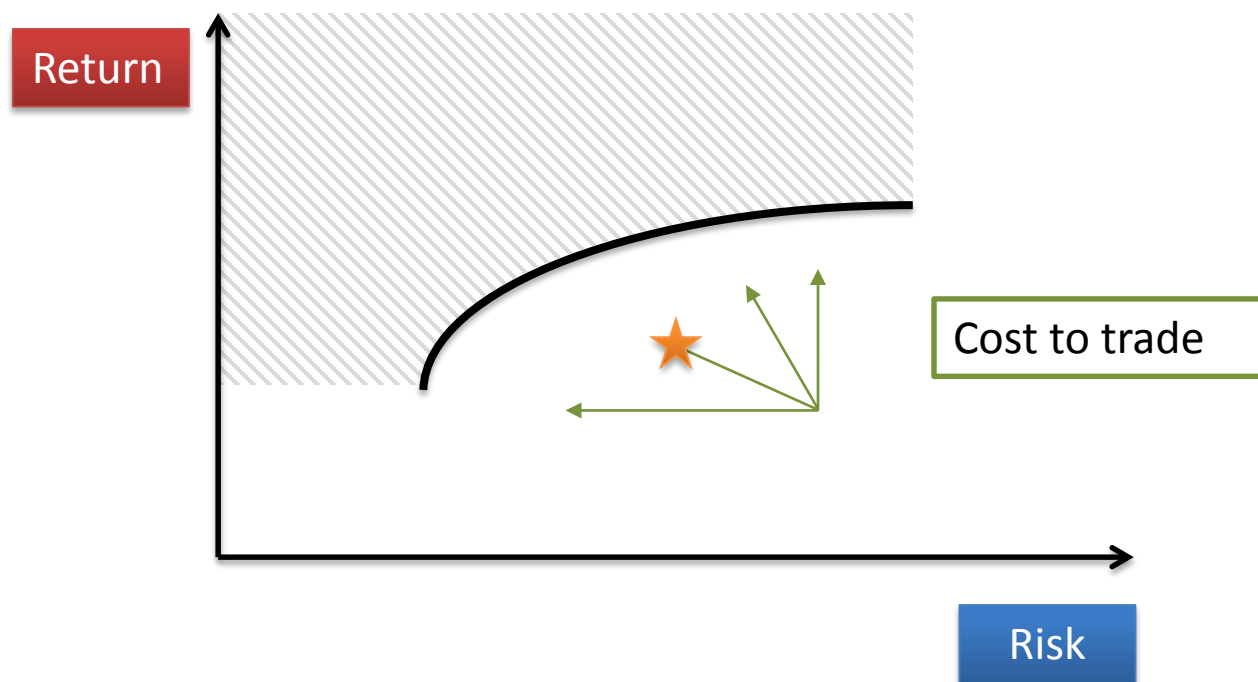
Efficient frontier



Efficient frontier

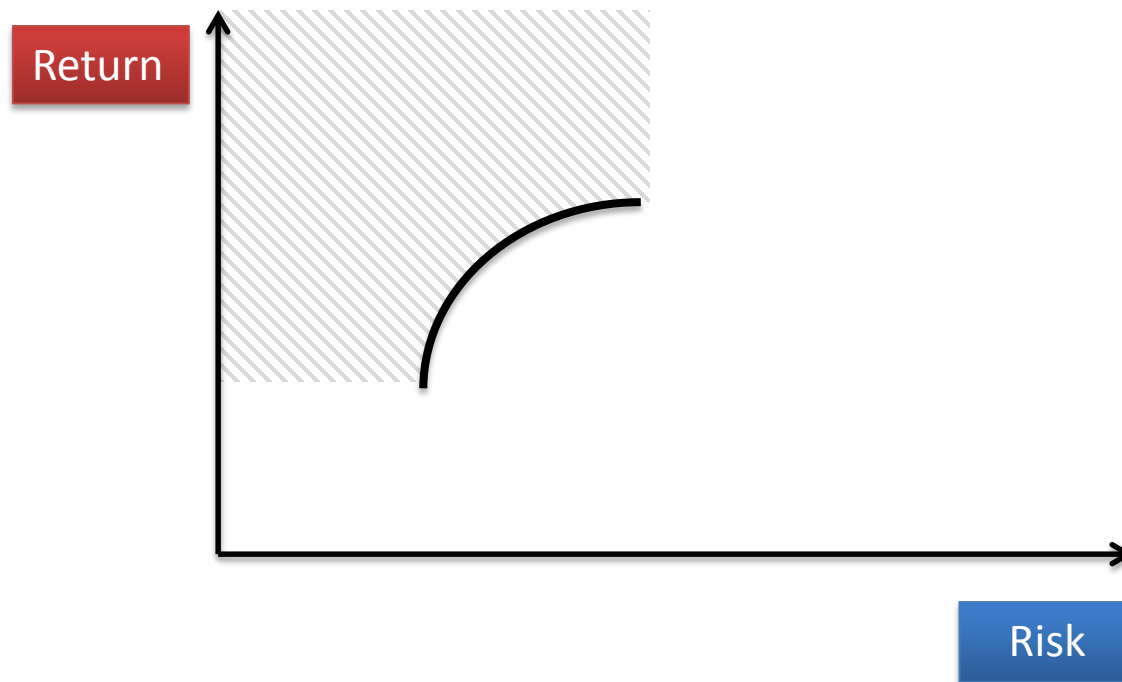


Efficient frontier



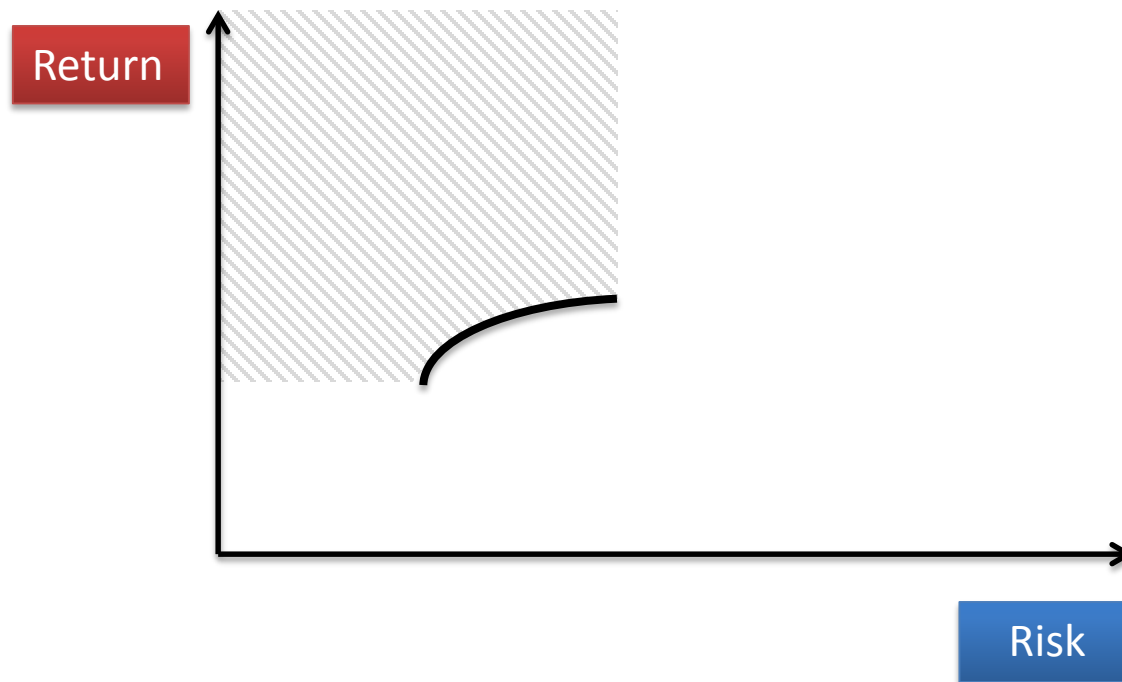
Efficient frontier

(The efficient frontier will have a different size and shape depending on your universe!)



Efficient frontier

(The efficient frontier will have a different size and shape depending on your universe!)



Optimization Objective Function

- Investor objective is to maximize risk- adjusted returns net of costs:

$$\text{Utility} = \text{Return} - \text{Risk} - \text{Costs}$$

- Most commercial portfolio optimizers uses the objective function described in Levy and Markowitz (1979)
- Portfolio return variance is the proper measure of risk because the difference between the arithmetic average rate of return and the geometric average rate of return is proportional to the variance (see Messmore, 1995)

Optimization Objective Function

$$\text{Utility} = \text{Return} - \text{Factor Risk} - \text{Stock Specific Risk} - \text{Costs} - \text{Taxes}$$

Market: +6%

FB: Buy

GOOGL: Hold

TWTR: Sell

TSLA: -2% ± 11%

Market Bet ↑

Sector Bet ↓

Region Bet ↑

Mkt Cap Bet ↓

Currency Bet ↔

Bid/Ask spread

Commissions

Liquidity costs

Long-term?

Short-term?

Time until long-term?

Gain? Loss?

Wash sale?

All of these terms are in the same units of %/year, so we can make *direct comparisons* between risk, return, and costs. All of this sums up to a **simple number**.

Objective Function Explained

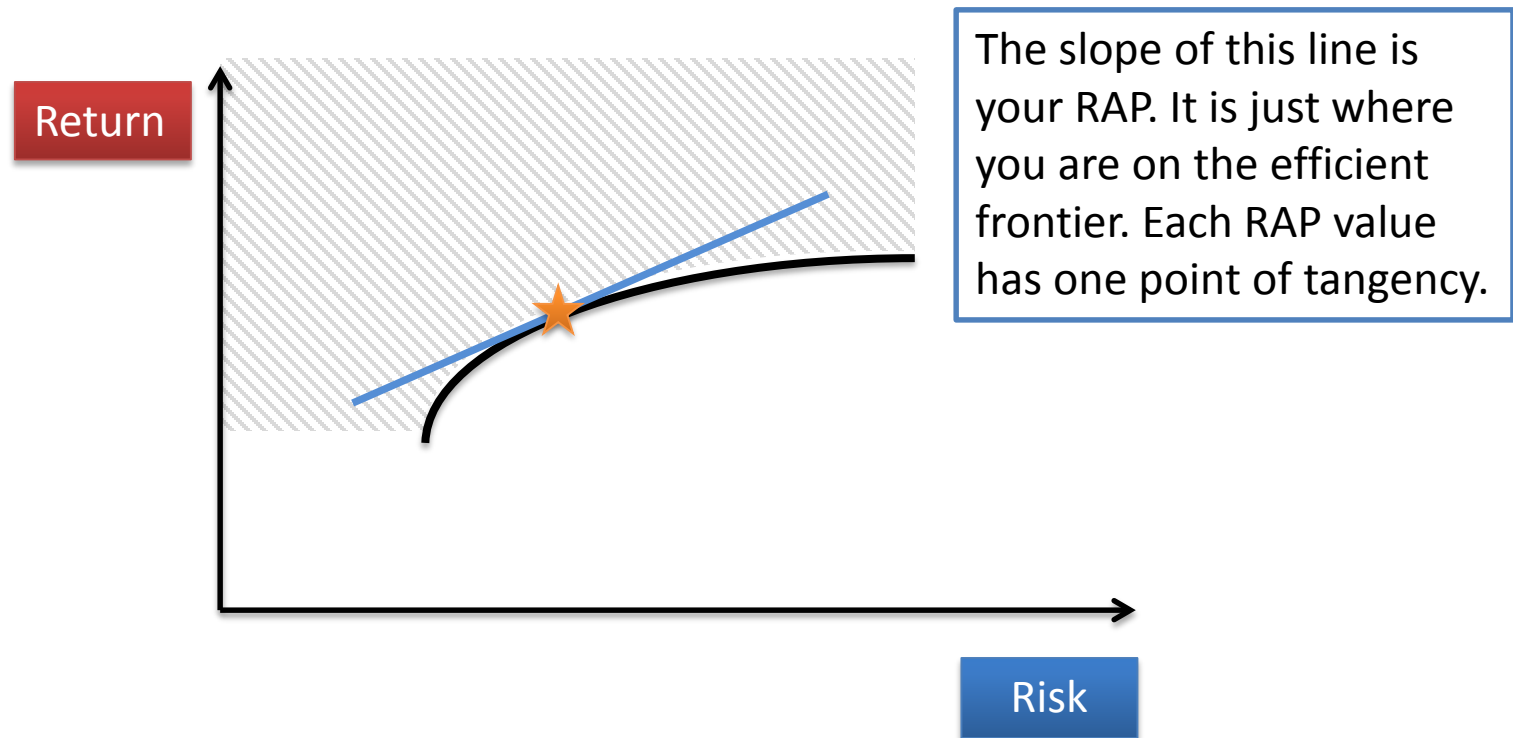
$$U = \underbrace{\alpha}_{\text{Return}} - \underbrace{\left(\frac{\sigma_s^2}{RAP_s} \right)}_{\text{Factor Risk}} - \underbrace{\left(\frac{\sigma_u^2}{RAP_u} \right)}_{\text{Stock Specific Risk}} - \underbrace{((C + T) * A)}_{\text{Implementation Cost}}$$

Risk Component

- α = the “certainty equivalent” expected portfolio return
- σ_s^2 = portfolio variance risk due to common factors (correlation across securities)
- σ_u^2 = portfolio variance risk due to stock specific risks
- RAP = risk tolerance
- C = transaction costs for the optimization
- T = capital gain taxes for the optimization
- A = amortization constant

Note that this is a convex function and the various scalars transform all terms into certainty equivalent expected return units

Efficient frontier – Risk Acceptance Parameter



Risk Tolerance in Optimizations:

<http://www.northinfo.com/documents/736.pdf>

An Optimization in Steps

- Our goal is to adjust the security weights to maximize the objective function, given your input settings.
- Northfield uses a Pairwise Swap Gradient method.
This means:
 - Buy a little of the best asset, sell a little of the worst asset.
 - Repeat until you can't trade any more or make any more improvements.

An Optimization in Steps

- Buy the Bank of America for risk, sell Berkshire for taxes:

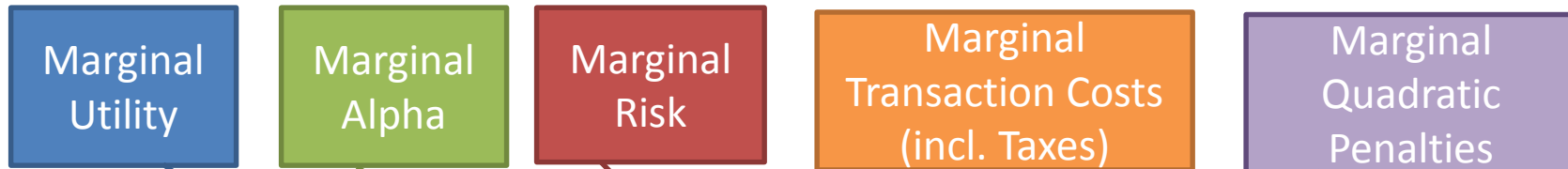
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=====
2>> Assets To BUY: BAC=0.000%, Swap = 1.5317916 (sorted by MuBuy)
N Ticker  MUBUY      MA      MF/RAP    MR/RAP    MTRANS    MIND
1  *$$$      0         0        -0         -0         -0         -0
2  *BAC     -0.094    0         0.011     0.14      -0.25     -0
3  REGN     -0.11     0         0.044     0.095     -0.25     0
4  GOOGL   -0.12     0         0.023     0.11      -0.25     0
5  GILD     -0.12     0         0.025     0.1        -0.25     0

2>> Assets To SELL: BRK.B = 1.939, Swap = 1.5317916 (sorted by MuSell)
N Ticker  MUSELL     MA      MF/RAP    MR/RAP    MTRANS    MIND
1  *BRK.B   -33        0       -0.031    -0.022    -33       -0
2  CVS     -17        0       -0.0093   -0.034    -17       -0
3  APC     -9.4       0        0.033    -0.083    -9.4      -0
4  INTC    -7.4       0       -0.016    -0.039    -7.3      -0
5  XOM     -3.8       0       -0.014    -0.017    -3.8      -0
```

(This lot of Berkshire Hathaway turns long term in 2 weeks)

An Optimization in Steps

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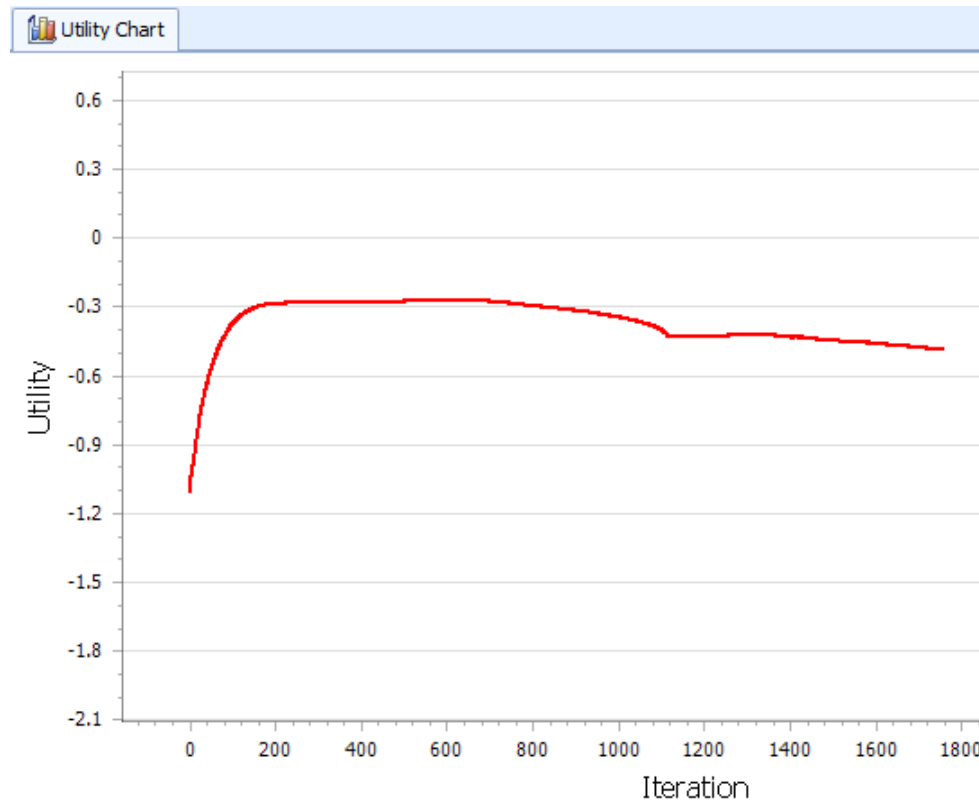
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An Optimization in Steps

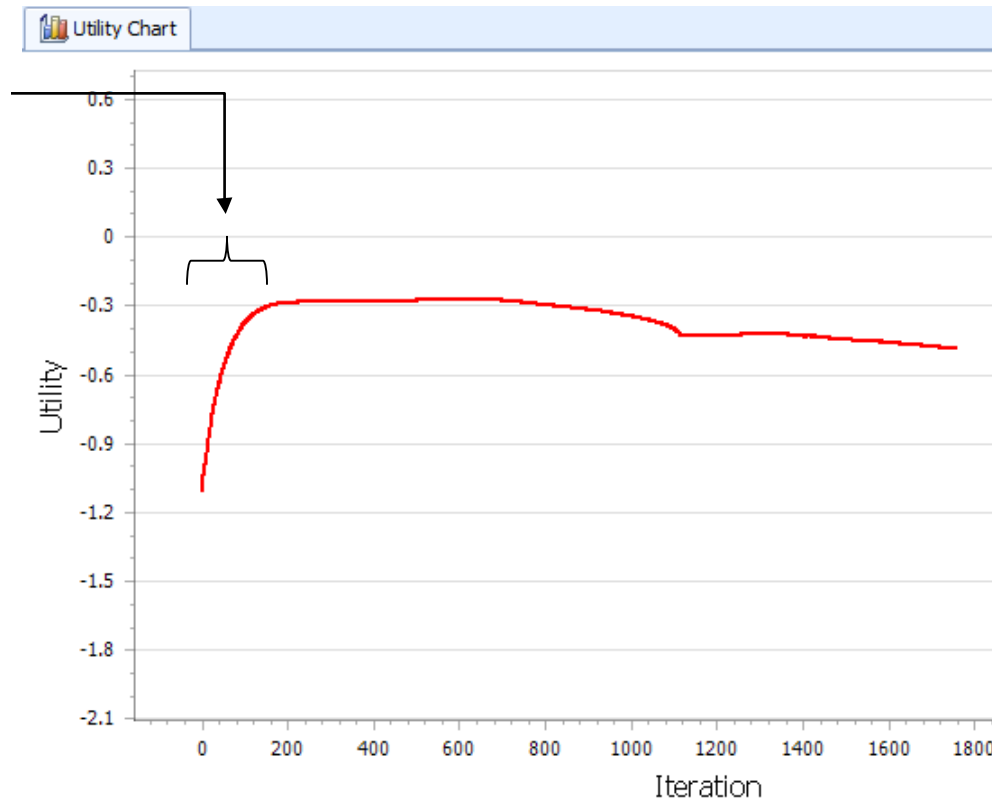
This method produces intuitive, tractable optimization paths that closely mimic a trader's methodology.



An Optimization in Steps

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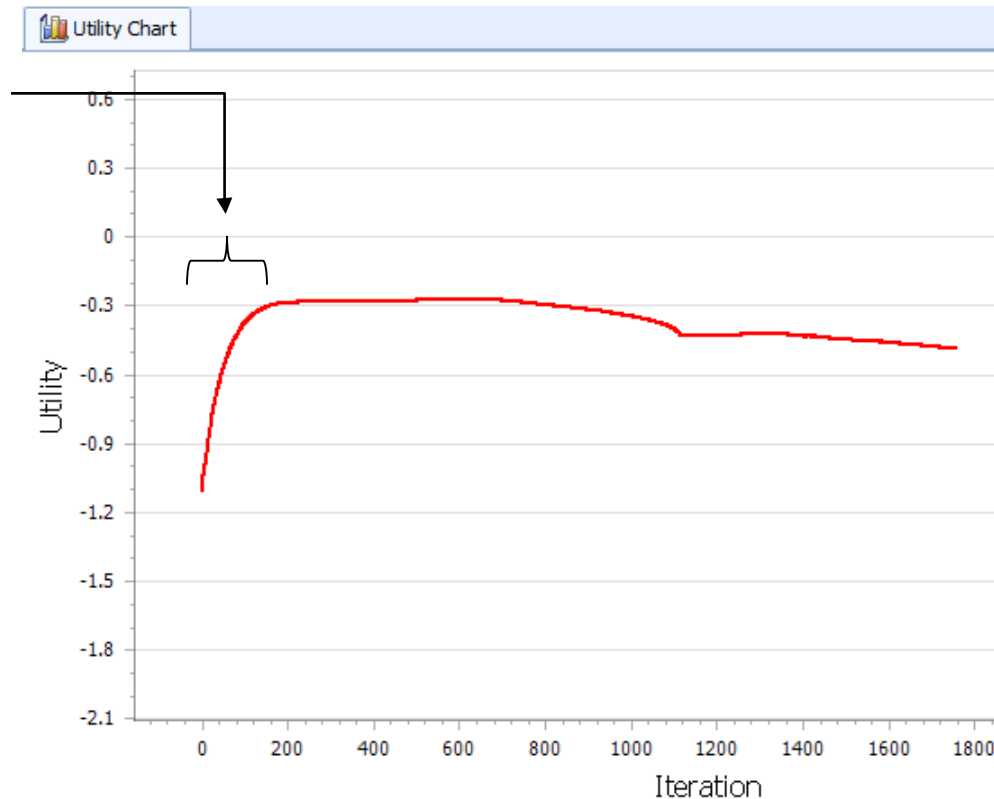
Trades with the most impact on the portfolio happen first.



An Optimization in Steps

This method produces intuitive, tractable optimization paths that closely mimic a trader's methodology.

Trades with the most impact on the portfolio happen first.



Makes understanding the magnitude of costs of certain constraints obvious

Constraint Basics

- Any optimization can be run with a variety of three classes of constraints
- Linear combinations of security weights
 - Position limits
 - Industry and sector limits
 - Factor Exposure or Penalty File Variables
- Cardinality Constraints
 - Maximum number of assets
 - Minimum trade sizes
 - Rounding to round lots
- Stopping Criteria
 - Turnover limits
 - Maximum realized capital gains

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Applied *during* optimization

Applied *after* optimization

An Optimization in Steps

- Buy the best asset, sell the worst asset. Skip any trade that would violate a constraint.

1>> Assets To BUY: XOM=20.000%, Swap = 10.0121557 (sorted by MuBuy)

N Ticker	MUBUY	MA	MF/RAP	MR/RAP	MTRANS	MIND	MSECT	MQPEN	TRNSELL+	TRNBUY+
1 *\$\$\$	0	0	0	-0	-0	-0	-99	-0	0	0
2 D	-0.22	0	-0.089	0.012	-0.15	-0	-99	-0	0	0
3 AEE	-0.33	0	-0.075	0.0044	-0.26	-0	-99	-0	0	0
...										
492 *XOM	-5.5	0	-3.2	-2.2	-0.11	-0	-99	-0	0	0

1>> Assets To SELL: AAPL = 19.999, Swap = 10.0121557 (sorted by MuSell)

N Ticker	MUSELL	MA	MF/RAP	MR/RAP	MTRANS	MIND	MSECT	MQPEN	TRNSELL+	TRNBUY+
1 AAPL	-13	0	-4.5	-8.1	0.02	-0	-99	-0	0	0
2 BAC	-10	0	-5.6	-12	6.7	-0	-99	-0	0	0
3 IBM	-10	-2	-4.3	-3.8	0.057	-0	-99	-0	0	0
26 XOM	-3.9	0	-3.2	-2.2	1.4	-0	-99	-0	0	0
479 SBUX	-0.37	3	-3	-5.5	5.1	-0	-99	-0	0	0

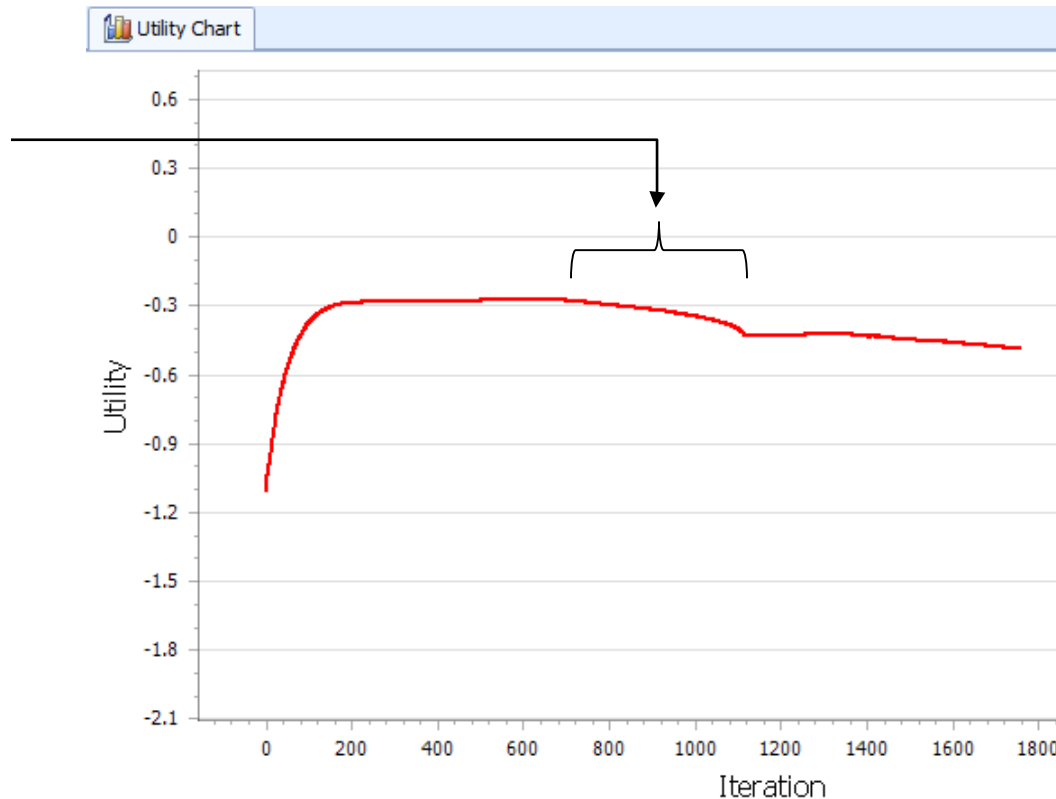
An Optimization in Steps

- Violations of cardinality constraints are checked when maximum objective is achieved
 - Solutions to most cardinality constraints are approximations that hold well only when dealing in small quantities. *Use only when actually needed.*
 - If there are any violations of cardinality constraints that can be fixed, the optimizer will adjust the security weights to fix them

An Optimization in Steps

This method produces intuitive, tractable optimization paths that closely mimic a trader's methodology.

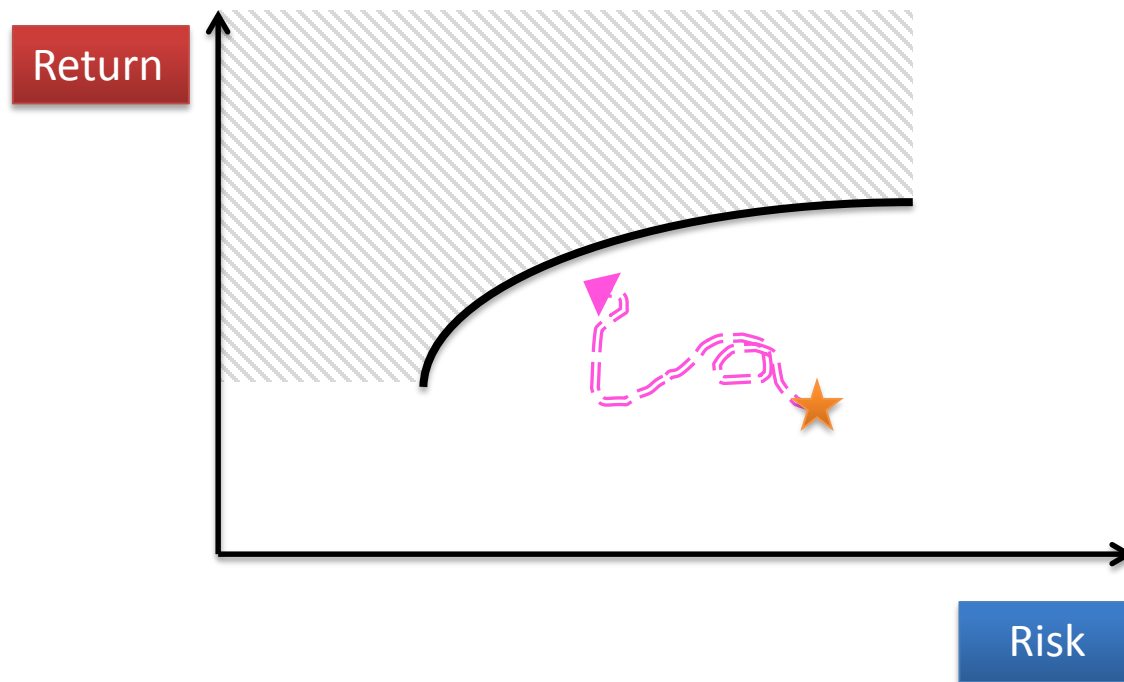
Solving for
Maximum
Assets



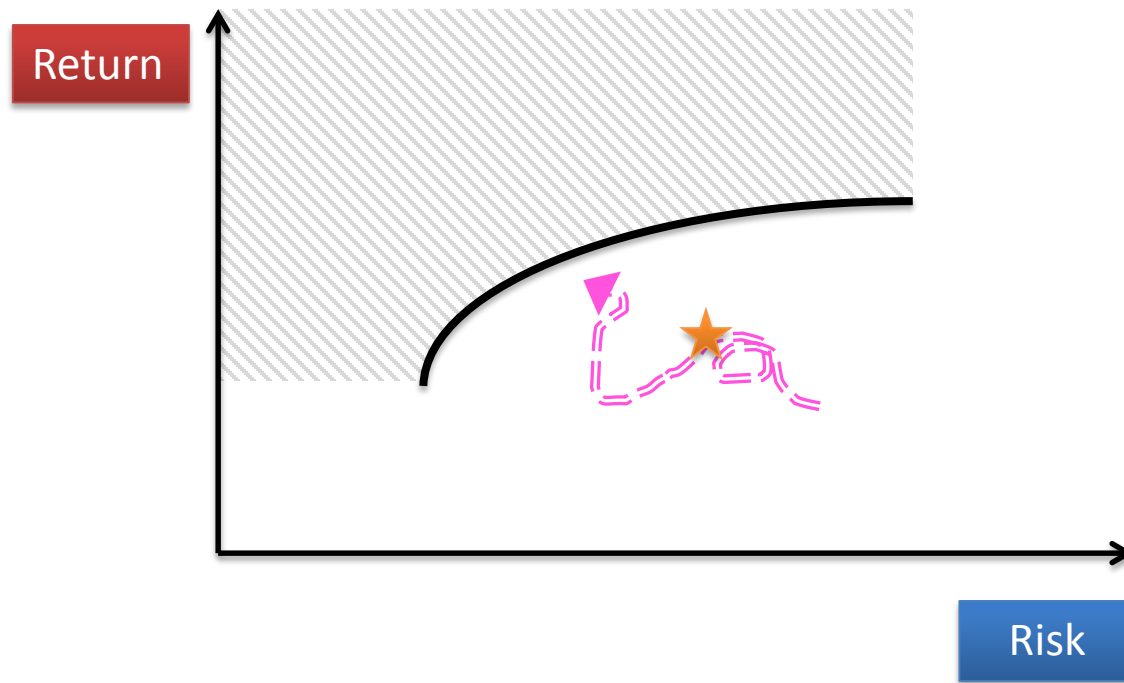
An Optimization in Steps

- You're done when marginal utility of the best and worst positions are equal to within a selected tolerance

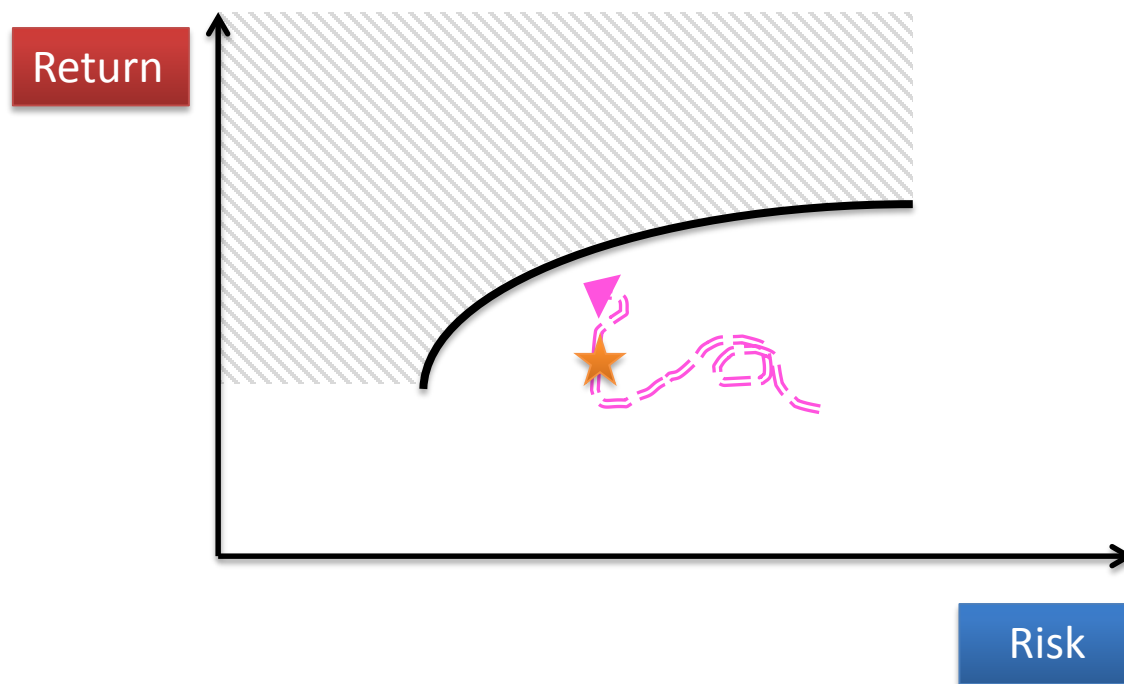
Your portfolio ☹️



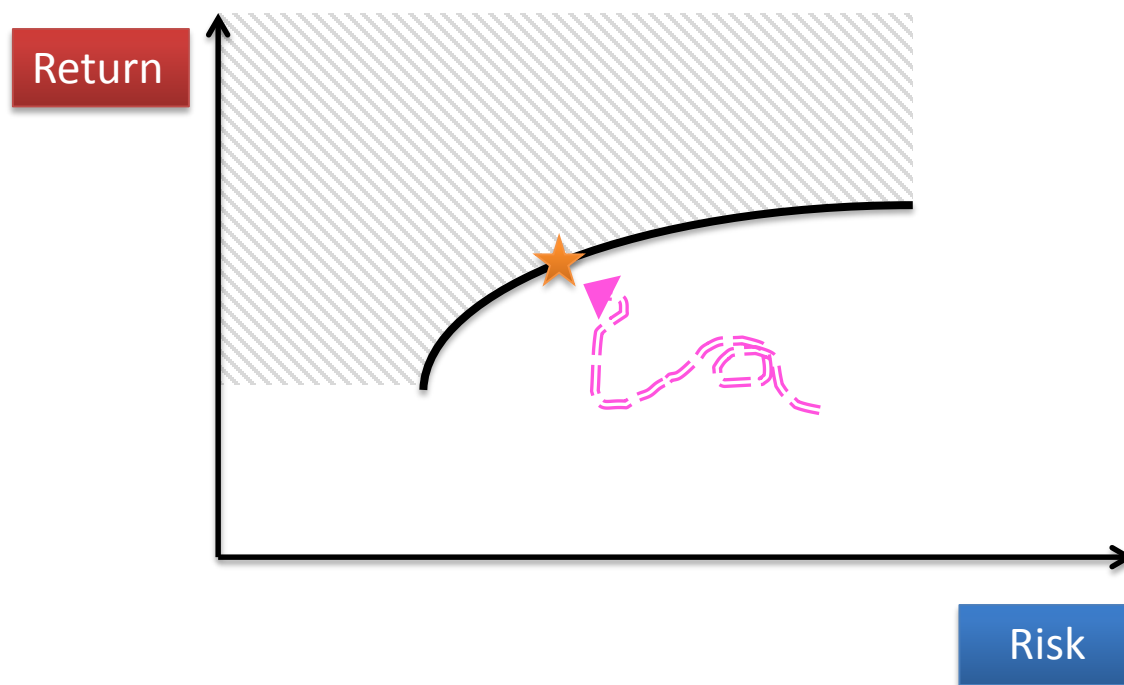
Your portfolio



Your portfolio



Your portfolio 😊



Further explorations

Dan diBartolomeo. Portfolio Construction: Optimizer Advantages, Hazards, and Tips

<http://www.northinfo.com/documents/565.pdf>

Anyone: sdyer@northinfo.com

New discussions: sales@northinfo.com

Current clients: support@northinfo.com