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Why Maximizing IR is Wrong

Tracking Error and the Paradox of Active Management

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Statement of the Problem

- Within asset management, the risk of benchmark relative performance is typically expressed by measures such as "tracking error", which describes the expectation of times-series standard deviation of benchmark relative returns. This is useful for index fund management, where the expectation of the mean for benchmark relative return is fixed at zero.
- The active management case is problematic, as tracking error excludes the potential for the realized future mean of active returns to be other than the expected value. *All active managers must believe their future returns will be above benchmark (or peer group average) in order to rationally pursue active management, yet it is axiomatically true that roughly half of active managers must produce below average results.*

Three Sources of Active Risk

- The usual representation of active risk is tracking error. This metric is defined as the expected dispersion of benchmark relative returns. This concept is frequently applied to both active and passive strategies. You can think of it as the volatility of a long short portfolio where you are long your portfolio and short the benchmark.
- The second source of risk is undertaking active management requires a belief that the manager be skillful so that there is expectation of positive alpha. If the level of skill can vary over time, we refer to this as "strategy risk".
- The third source is the aforementioned paradox. All active managers must believe they will achieve positive alpha, but it is mathematically impossible for everyone to be above average. This fact is must be recognized and incorporated in the risk metric.

Qian and Hua

- Tracking error alone is sufficient under the EMH because true alpha is zero for all managers

- Qian and Hua (2004) defines “strategy risk”. In essence, it is the risk created because the skill level of the active manager is not constant over time, as evidenced by the mean return being other than expected. They use the terms “active risk” to describe the combination of the tracking error and strategy risk.   They formulate active risk as:

$$\sigma_{active} = \sigma_{IC} * n^{.5} * \sigma_{TE}$$

Qian and Hua 2

- This formulation arises directly from the Grinold (1989) Fundamental Law of Active Management. If the manager's skill level is constant over time, any variation in IC must arise purely from sampling error, making the standard deviation of IC equal to the reciprocal of the square root of breadth. The product of the two factors is therefore unity, and active risk is equal to tracking error.

- If the manager's skill level is time-varying the dispersion of IC will be greater than the square root of breadth, and active risk be scaled upward as a multiple of tracking error. **In a simple conceptual sense, tracking error represents the risks that external events will impact our investment result, while strategy risk represents what we can do to ourselves to contribute to an adverse outcome relative to expectations.**

Qian and Hua 3

- There are some serious practical problems with this approach to active risk. The first is that we actually have to be able to observe the time series variation in the information coefficient. This limits the set of possible users to active managers who make formal alpha forecasts for each investment period. It cannot be used by more fundamental managers, nor by "outsiders" such as pension funds who employ the managers.

- The second is that the property of breadth is notoriously difficult to actually measure, as it represents a complex function of the number of assets in the prediction universe, the correlation among those assets, portfolio turnover and the extent to which the active strategy in question seeks to exploit information about the correlated or independent portions of asset returns.

- Finally, there are typically unrealistic assumptions inherited from the FLAM which include that transaction costs are zero, and there are no constraints on portfolio composition

An Improvement

- One way to improve this formulation is to replace the use of the information coefficient (IC) with the Effective Information Coefficient (EIC) as defined in diBartolomeo (2008). Rather than measure the variation in the correlation of forecasts and outcomes, we measure the variation in the correlation between "implied alphas" and outcomes.

$$\sigma_{active} = \sigma_{EIC} * n^{.5} * \sigma_{TE}$$

Where

EIC = the effective information coefficient, the correlation between implied alphas and outcomes

An Improvement 2

- The benefit of this substitution is that we can eliminate two the problems associated with Qian and Hua method. EIC can be estimated by an "outsider" or by a fundamental manager whose investment process does not involve the expressing security level return expectations in a numerical form. The implied alphas are obtained by inference from the portfolio positions that are observable for all portfolios.

- In addition, the estimation of EIC incorporates the effect of constraints on portfolio position size and turnover.

Representing Breadth

- Breadth is meant to define the number of independent opportunities to generate alpha. It involves both turnover and the nature of the unrelated opportunities.
- To quantify the opportunity set we consider a hypothetical world where all securities are of the same risk, all positions are of equal magnitude and returns are uncorrelated.
- In this special case, the volatility of the portfolio is the volatility of the individual securities divided by the square root of the number of positions.
- We can apply this formulation in reverse to infer the available number of independent bets in *any* given portfolio. The number of independent bets available is the square of the ratio of average of volatility of the securities divided by volatility of the portfolio.

A More General Approach

- A more general conception of the problem would be to think of active risk as the square root of total active variance

$$\sigma_{active} = (\sigma_{mean}{}^2 + \sigma_{TE}{}^2 + 2 * \sigma_{mean} * \sigma_{TE} * \rho)^{.5}$$

Where

σ_{mean} = uncertainly of the true mean relative to expectation of the mean

ρ = correlation between uncertainty and tracking error

Actual Uncertainty of Mean

- The resulting distribution is bimodal with modes at α_p and $-\alpha_p$
- Distribution has skew and kurtosis relative to the manager's expectation of α_p
- Adjust uncertainty of mean for higher moments using Cornish-Fisher expansion

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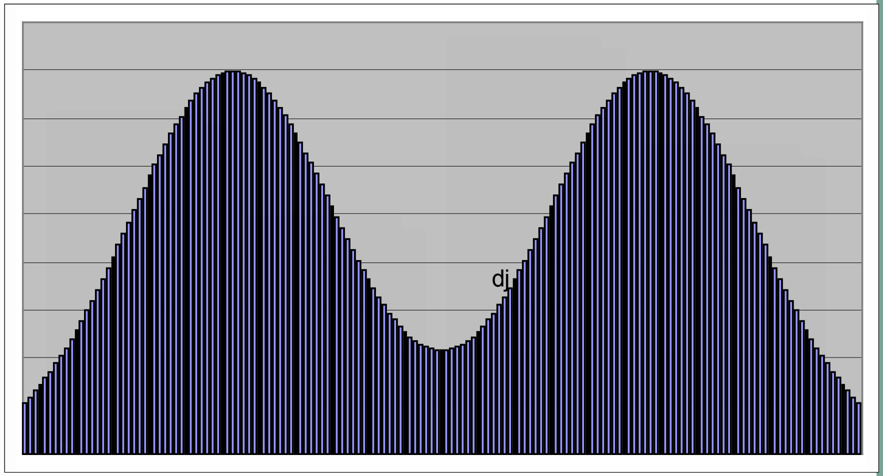

Getting Good Numbers for IR

- We can estimate the moments of the bi-modal distribution as a mixture of two normal distributions.
- For example, we might believe that we have a 75% chance of really being a positive alpha manager and a 25% chance of being a negative alpha manager.
- We can now calculate the four moments of this distribution with values for a given alpha, negative alpha and tracking error.
- Using the Cornish-Fisher (1937) expansion, we can convert the four moment distribution back to a "best fit" adjustment mean and adjusted standard deviation.
- *The mean alpha values will be lower, and standard deviation will be higher in estimating an "adjusted" IR.*

Mixture Distributions

- The moments of a mixture of normal distributions are:

$$\mu = \sum_{i=1}^{n} p_i \mu_i$$

$$\sigma^2 = \sum_{i=1}^{n} p_i(\sigma_i^2 + \mu_i^2) - \mu^2$$

$$skew = \frac{1}{\sigma^3} \sum_{i=1}^{n} p_i(\mu_i - \mu)[3\sigma_i^2 + (\mu_i - \mu)^2]$$

$$kurtosis = \frac{1}{\sigma^4} \sum_{i=1}^{n} p_i[3\sigma_i^4 + 6(\mu_i - \mu)^2\sigma_i^2 + (\mu_i - \mu)^4]$$

Cornish Fisher Expansion

- Using the method of Cornish and Fisher (1937) we can approximate the quantiles of a distribution given the moments of that distribution. The rather lengthy algebraic details are given at:
  - https://en.wikipedia.org/wiki/Cornish–Fisher_expansion
- Widely used in evaluating option risk for known delta/gamma.
- We've added a couple of our own "bells and whistles" this method including some monotonicity requirements.
- If we know the mean, standard deviation, skew and excess kurtosis of an "oddly shaped" distribution (which is what we get from the mixture of normal formula on the previous slide), *we can estimate the "adjusted" expected alpha and volatility to get an adjusted expectation for the information ratio.*

We Want to Maximize Adjusted IR, NOT IR

- Adjusted IR values are lower

- More importantly, the inclusion of skew and kurtosis means that the relationship between traditional IR and adjusted IR is highly non-linear

- If strategies become more aggressive in order to increase IR, it is possible that *the adjusted IR will actually decrease.*

A Short Cut Example

- One way to approach this problem is to consider a binary distribution for the active return of a manager.

- We assume that each manager has a benchmark relative return expectation of portfolio alpha α_p with a probability w of being correct.

- If the manager's forecast is wrong, they have a probability of (1-w) of realizing $-\alpha_p$.

Short Cut Example 2

- With this framework, the value of σ_{mean} is

$$\sigma_{mean} = ((1-w) * 4 * \alpha_p^{2})^{.5}$$

Where

w = is the probability of realizing the expected alpha
α_p = manager's expectation of portfolio alpha

For w = .5 we obtain the simple expression

$$\sigma_{mean} = 2^{.5} * \alpha_p$$

Short Cut as a Rule of Thumb

It is the frequent custom of the asset management industry that the information ratio is used as a proxy for manager skill.

$IR = (\alpha_p / \sigma_{TE})$

$\alpha_p = IR * \sigma_{TE}$

$\sigma_{mean} = ((1\text{-}w) * 4 * (IR * \sigma_{TE})^{2})^{.5}$

For w = .5

$\sigma_{mean} = 2^{.5} * IR * \sigma_{TE}$

A Tale of Two Managers

- Let's make the simplifying assumption that $\rho = 0$ and consider two managers, K and L (uncorrelated returns).

- Both managers have TE = 5

- Manager K is a traditional asset manager that purports to clients that their IR = .5.

  Manager L is a very aggressive fund that purports to it's investors that their IR = 3

- Manager L's IR is **six times** as good as Manager K.

Hoisted by One's Own Petard

- For w = .5 we obtain:

  For Manager K we get:

  $$\sigma_{active} = (2 * .25 * 25 + 25)^{.5} = 6.125$$

  About 23% greater than original TE, revised IR about .4

  For Manager L we get

  $$\sigma_{active} = (2 * 9 * 25 + 25)^{.5} = 20.61$$

  More than four times the original TE, with adjusted IR = .73

Rule of Thumb-Implications

- However, if more aggressive managers with higher tracking errors tend to also have more uncertainty in their means (i.e. $\rho > 0$), then it is entirely possible that the adjusted IR for Manager L will actually approach the lower value of the much more conservative manager K.

- The lesson for asset owners and particularly “fund of fund” managers is that their hiring of high IR managers must be predicated on the belief that the probability that the managers is skillful must be far above one half despite the obvious constraint on the aggregate value of w.

Relation of Uncertainty of Mean and TE

- Let us now turn to the estimation of ρ, the correlation between the uncertainty of mean return of any particular active manager, and their tracking error. Both of these properties arise from related underlying causes, the volatility of security returns, the correlation of security returns and the size of the manager's active position weights (i.e. bets).

- diBartolomeo (2006) provides a broad discussion of the relationship between volatility in financial markets and cross-sectional dispersion (aka variety).

Related Literature

- Numerous studies have shown that security correlations tend to rise during periods of market volatility, suggesting that the correlation between variety and volatility should be positive, but less than one.

- In deSilva, Sapra, Thorley (2001), they derive an expression for the expectation of the cross-sectional variance (variety squared) of security returns, and show that it is linearly related to the realized market return in each period. They also show that the variety in active manager returns should be linearly related to the variety of security returns. These results suggest that the there should be a positive, but not linear relationship between our σ_{mean} and σ_{te} measures.

- Akrim and Ding (2002) provides an extensive empirical study confirming that the cross-section of active manager returns is very closely related to the cross-section of security returns, implying that active managers have relatively constant "bet" sizes over time.

An Empirical Approach

- We can also use empirical data to statistically estimate σ_{mean} and ρ for any particular category of active manager. Let us work through an example.
- Our sample is 1957 US Large Cap Growth Managers. We observe monthly returns for the 60 months ending November 30, 2009
  - Compute the monthly cross-sectional average and subtract from each observation to put observations in "peer relative excess" unit
  - Calculate the cross-sectional standard deviation for each month
  - Calculate the 60 month annualized excess return
  - Calculate 60 month realized annual tracking error (standard deviation of excess returns)

Empirical Result

- Average annualized cross-sectional dispersion is 5.76%
- Average time series tracking error is 5.70%.
- The cross-sectional correlation between the absolute value of annualized excess returns (as a proxy for dispersion of mean) and corresponding tracking errors is .21.

$$\sigma_{active} = (5.76^2 + 5.70^2 + 2 * .21 * 5.76 * 5.70)^{.5} = 8.91$$

This represents an increase of 56% in risk as compared to tracking error alone, implying that investor expectations for the IR of a typical fund should be reduced by at least one third.

Conclusions

- Tracking error is an inadequate measure of risk for active managers. We should evaluate risk with the broader conception of "active risk" in the spirit of Qian and Hua.
  - Active risk can be formulated as the aggregate of tracking error and the uncertainty of the mean return over time.
  - The paradox that active managers must believe they will have positive alpha and yet roughly half of those beliefs must be wrong has to be included in the estimation or risk and IR.
  - An representation is clearly available by using textbook calculations for a "mixture of normals" and the Cornish Fisher technique.

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