Why Factor Risk Models Often Fail Active Quantitative Managers: The completeness conflict

Dan diBartolomeo

It is routine among equity portfolio managers to use linear factor risk models to measure the expected variation of portfolio returns. A wide variety of linear factor risk models are available from several vendors, including Northfield. Typically, the active variation from some benchmark index is of greatest concern. While the theoretical and empirical benefits of linear factor models for measuring portfolio risks are well known, it must be admitted that, from time to time, the predictions of these models turn out to be substantially incorrect. Such breakdowns in a model's predictive ability often result in a dispersion of ex-post portfolio returns which exceeds the range of dispersion of expected returns predicted by the model. It is our hypothesis that such failures arise from a simple mathematical conflict which manifests when linear factor risk models are combined with quantitative security selection strategies. With appropriate handling of this conflict, such failures can be avoided.

Linear factor risk models express the expected covariance matrix of security returns in the form of a factor covariance matrix to which each security is exposed and a security-specific portion. The usual mathematical formulation is:

(1) $V_p = \text{SUM}\,[_{i=1 \text{ to } n}\,\text{SUM}\,[_{j=1 \text{ to } n}\, E_{pi}\, E_{pj}\, S_i\, S_j\, R_{ij}\,]] + \text{SUM}\,[_{k=1 \text{ to } m}\;\; W_k^2\, S_k^2]$

(2) $E_{pi} = \text{SUM}\,[_{k=1 \text{ to } m}\, W_k * B_{ki}]$

Where

$V_p =$ variance of portfolio return
n = number of factors in the risk model
m = number of securities in the portfolio
E_{pi} = exposure of the portfolio to factor i
S_i = standard deviation returns attributed to factor i
R_{ij} = correlation between returns to factor i and factor j
W_k = weight of security k in the portfolio
S_k = standard deviation of security specific returns for security k
B_{ki} = beta of security k to factor I

One of the key assumptions of this formulation regards the item called security specific returns. As is implied in the terminology, it is assumed that such returns are indeed specific to a given security and hence are uncorrelated. We are assuming that the factors specified in the model are sufficient to explain all covariance among the securities. In this sense, we presume that the factor specification is *complete*. We should note that completeness is used in a slightly different context than in most academic literature: we are not using "completeness" to refer to the number of factors necessary to describe the equilibrium relationship between risk and return, we make no assumptions about equilibrium but rather about the exclusion of pairwise correlation in residual returns from the model.

We can easily relax this completeness assumption. Let's assume that our factors do not fully specify all of the covariance among securities. In this case, we have:

$$(3)\ V_p = \text{SUM}\left[_{i=1\text{ to }n} \text{SUM}\left[_{j=1\text{ to }n} E_{pi}\, E_{pj}\, S_i\, S_j\, R_{ij}\right]\right] + \text{SUM}\left[_{k=1\text{ to }m}\ \text{SUM}\left[_{l=1\text{ to }m} W_k\, W_l\, Z_k\, Z_l\, P_{kl}\right]\right]$$

Where

Z_k = the standard deviation of residual return for security k

P_{kl} = the correlation between residual returns of security k and security l

It is easy to see that for the special case where $P_{kl} = 0$ for all k not equal to l, then $S_k = Z_k$ and equations (1) and (3) are equivalent.

Now let us turn to the issue of security selection strategies when combined with a linear factor risk model. Active managers seek to produce returns greater than their benchmark index in a variety of ways. One way to do this is to assume that the mean expected return for a factor in the risk model will be positive or negative. While our risk model specifies the expected standard deviation of factor returns, we are still able to make any assumptions about the expected means of factor returns without violating any assumptions of equation (1). So managers can construct "factor bets" by emphasizing exposure to a particular factor without any conflict with risk model assumptions.

Another way to manage actively is to select each security we choose to hold on the basis that each chosen security has some particular attribute that makes the expectation of its stock specific return will be positive. For example, we might believe that one security will produce superior returns due to excellent products while another security might produce superior returns due to an unusually strong balance sheet and a third would produce superior returns due to a recent acquisition of a competitor. As long as the reason for the expectation of superior performance in each case is independent, no correlation in the security specific returns is implied and equation (1) still holds.

However, among quantitatively oriented equity managers there is a tendency to select securities by means of a "multiple-factor model". Unlike a risk model where the factors are chosen to best explain the covariance of securities, the factors in these selection models are meant to identify securities which will provide superior mean returns. If the factors in the selection model are the same as in the risk model (or are highly correlated proxies), the assumptions of equation (1) still hold.

If the factors in the selection model are different from the factors in the risk model we have a conflict. The factors in the selection model are characteristics shared by multiple securities which we believe will cause them to behave alike in that they are expected to provide us with superior returns in the future. In short, using a factor selection model (with different factors than the risk model) can only produce superior returns if returns residual to the risk model factors exhibit covariance. In essence, we must now assume that that our risk model is not complete, therefore equation (1) does not hold and that equation (3) is the proper representation of the problem. Mathematically, it would be possible to have different selection factors and have equation (1) still hold if we can demonstrate that the returns associated with the selection factors are perfectly constant over time. To our knowledge, no financial researcher has ever claimed to have found such a constant return factor.

For practitioners, the problem is hopefully clear: you can't have it both ways. Either equation (1) is appropriate or equation (3) is appropriate. Yet, it is common practice to measure risk using the assumptions of uncorrelated residuals [equation (1)], while using stock selection strategies that can only work if equation (1) does not hold. Since active mangers will be concentrating their portfolios in those securities expected to act alike in providing superior returns, the average residual covariance will be positive. The second term of equation (3) will have a larger positive magnitude than the second term of equation (1). Hence, if we use a risk model which assumes uncorrelated residuals we will have a downward biased estimate of the risks not identified by the factors of our risk model.

There are several things that portfolio managers can do to mitigate this problem. First, a good dose of humility is in order. The skill of portfolio managers is routinely measured using the ratio of excess returns (relative to benchmark) divided by the standard deviation of excess returns (square root of the variance as calculated in our equations, also known as tracking error). This ratio is often called the information ratio. By examination of manager return databases, it is easy to see that successful managers (top quartile) average ex-post information ratios on the order of one half. If the expected information ratio for a portfolio is far greater than one-half, it is quite probable that either the expected excess returns or the expected risk estimates are just too good to be true.

Another useful approach is to simply assume that the risk model's estimate of non-factor risk will be downward biased. By reducing our willingness to take non-factor risks, we can control non-factor risk to have a small magnitude, hence any downward bias in the estimate of non-factor risk will also have small magnitude. Unfortunately, some practitioners take the opposite view, trying to be tolerant of non-factor risk so as to "allow room" for their security selection strategies to operate.

It should be noted that being particularly averse to non-factor risks will also tend to reduce our factor bets as well. In order to reduce non-factor risk we must tend to hold the same securities as the benchmark in similar proportions. In such a portfolio, the factor bets will necessarily be small. It is not possible to have the same stocks in the same weights but have different factor exposures than the benchmark. However, the converse is not true, for it is very easy to construct portfolios with the same factor exposures as the benchmark with entirely different securities.