Some Nuances of Return Modeling: The Long and the Short of It

by

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Institutionally managed portfolios have different acceptable turnover requirements. The turnover rate of a portfolio will dictate the optimal forecast horizon for returns. It is difficult, and generally undesirable, to generate separate return forecasts for each portfolio. We discuss a solution to this problem—through the decomposition of returns into short and long horizons. We also discuss some related nuances of the return modeling process.

Let: r_1 = one month forward return
r_2 = 2-12 month forward return

Consider the following simplified system (we focus on a buy-decision followed by a sell-decision for concreteness):

1. We can consider a single stock in isolation
2. Transactions occur only at the beginning of a month
3. Transactions costs C_B (buy) and C_S (sell) are not affected by the size of the trade
4. If a stock is bought at the beginning of month 1, it will be sold at the beginning of the month j with probability:

$$p_j = (1-p) \cdot p^{j-2}, \qquad j = 2, 3, \ldots$$

These probabilities reflect the rate at which market conditions change. The mean number of months a stock is held is:

$$E(J-1) = 1 / (1-p)$$

from which p can be estimated empirically, as

$$p = 1 - 1 / E(J-1).$$

5. Expected returns:

Expected return in month 1 = $\mu_1 = E(r_1)$
Expected return in months 2-s = $\mu_2 = E(r_2)$
Expected return in later months = 0

6. The risk for month 1 is σ_1 and for months 2-s is σ_2 per month.

Net Return = Total Return - C_B - C_S

$$\text{Total Return} = \sum_{j=2}^{s+1} P(\text{Sell in month } j) \cdot (\text{Cumulative return by month } j)$$

$$= \sum_{j=s+2}^{\infty} (1-p) \cdot p^{j-2} [r_1 + (j-2) \cdot r_2] + (1-p) \cdot p^{j-2} [r_1 + (s-1) \cdot r_2]$$

...

$$= \mathbf{r_1 + r_2 \cdot p \cdot (1 - p^{s-1}) / (1-p)} \qquad \text{(Eq. 1)}$$

Therefore, $$E\,(\text{Total Return}) = \mu_1 + \mu_2 \cdot p \cdot (1 - p^{s-1}) / (1-p) \qquad \text{(Eq. 2)}$$

$$V\,(\text{Total Return}) = \sigma_1^{\,2} + \sigma_2^{\,2} \cdot p^2 \cdot (1 - p^{s-1})^2 / (1-p)^2 \quad \text{(Eq. 3)}$$

<u>Rules for Combining r_1 and r_2:</u>

1. The simplest scheme is to use the weighting given in Eq. 1. Under the assumptions, transactions costs will affect the original buy decision, but are built in otherwise. More complex and realistic models could, and perhaps should, be developed here.

2. A second approach will be to consider risk-adjusted return:

$$E\,(\text{Total Return}) - \lambda \cdot V\,(\text{Total return}).$$

To come up with the decision rule for weighting will be difficult.

<u>Example:</u>

Assume s = 12 months

Mean Number of Months Held	p	Relative weight of r_2
1	0.0	0
2	0.5	1
3	0.667	1.98
6	0.833	4.33
9	0.888	5.81
12	0.917	6.78

APPENDIX

$$\sum_{j=s+2}^{\infty} (1 - p) \cdot p^{j-2} = p^{s}$$

$$\sum_{j=2}^{s+1} (1 - p) \cdot p^{j-2} = 1 - p^{s}$$

$$\sum_{j=2}^{s+1} (1 - p) \cdot (j-2) \cdot p^{j-2} = p \cdot (1 - p^{s-1}) / (1 - p) - (s - 1)\, p^{s} = Q_s$$

$$\text{Total Return} = [r_1 + (s - 1) \cdot r_2] \cdot p^{s} + r_1 \cdot (1 - p^{s}) + r_2 \cdot [Q_s]$$

$$\text{Total Return} = r_1 + r_2\, [(s-1) \cdot p^{s} + p \cdot (1 - p^{s-1}) / (1-p) - (s-1) \cdot p^{s}]$$

$$\text{Total Return} = r_1 + r_2 \cdot p \cdot (1 - p^{s-1}) / (1-p)$$

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Presented by

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Aeltus®

Investment Management, Inc

Objective

1. Develop Rules for Combining Short-Term and Long-Term Return Forecasts

2. Consider a few other Issues in Return Modeling

ST+LT: Notations and Assumptions

Let: r_1 = One Month Forward Return

r_2 = 2-s Month Forward Return

Consider the following simplified system:

A “Buy” Decision followed by a “Sell” Decision

ST+LT: Notations and Assumptions

- We can consider a single security in isolation
- Transactions occur at the beginning of the month
- Transactions costs C_B (buy) and C_S (sell) are not functions of size

Probability Distribution

If a stock is bought at the beginning of month 1, then it will be sold at the beginning of month j with probability:

$$p_j = (1 - p)\, p^{j-2}$$

These probabilities reflect the rate at which valuations change

Estimating p

Mean Holding Period (in months) =

$$E\,(J-1) = 1/(1-p)$$

From which *p* can be estimated empirically.

$E(J-1) = 1/(1-p)$

Forecast Horizon, Holding Period and p

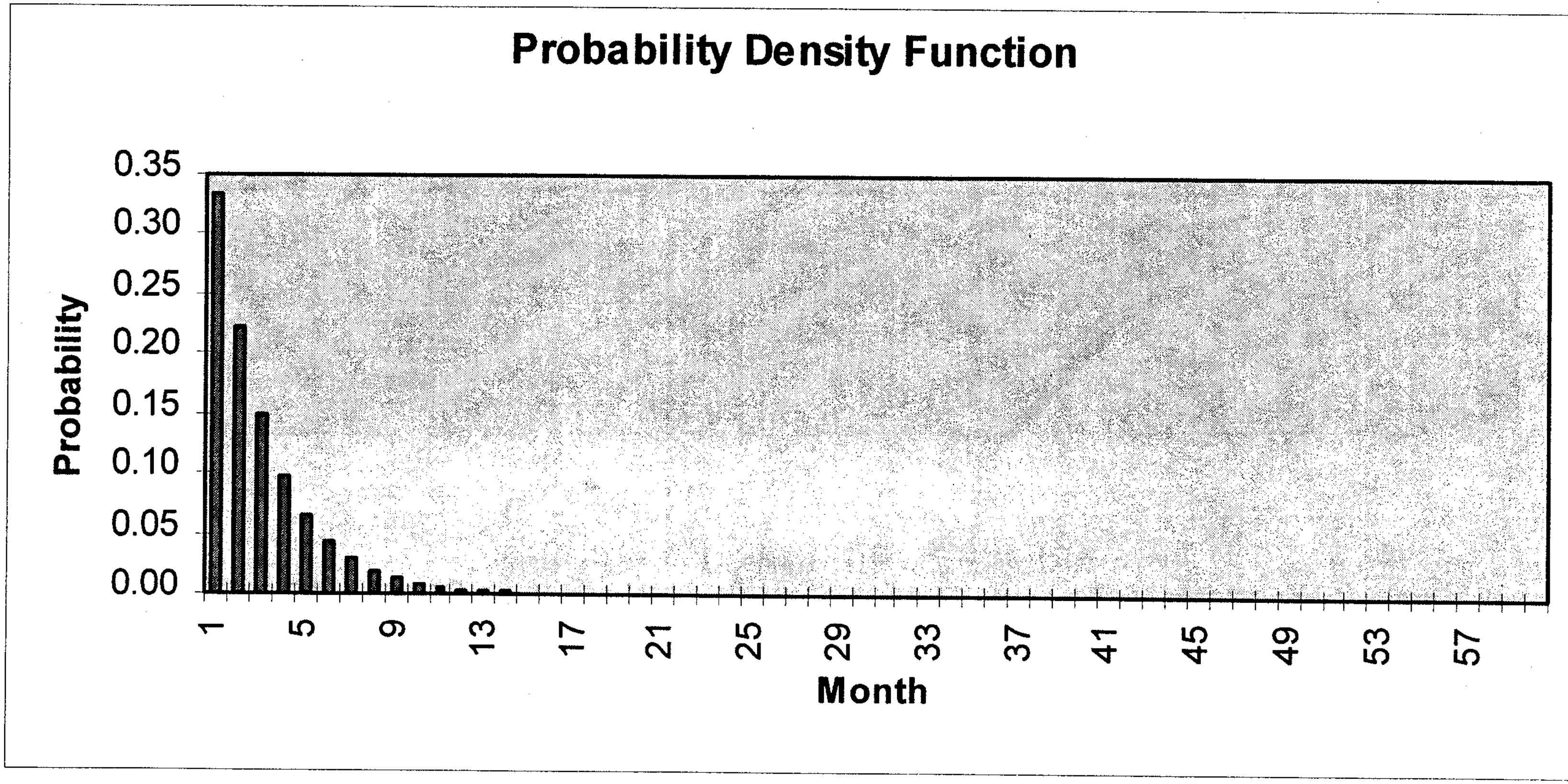
s	j-1	p
12	3	0.67
12	6	0.83
12	9	0.89
12	12	0.92

$E(J-1) = 1/(1-p)$

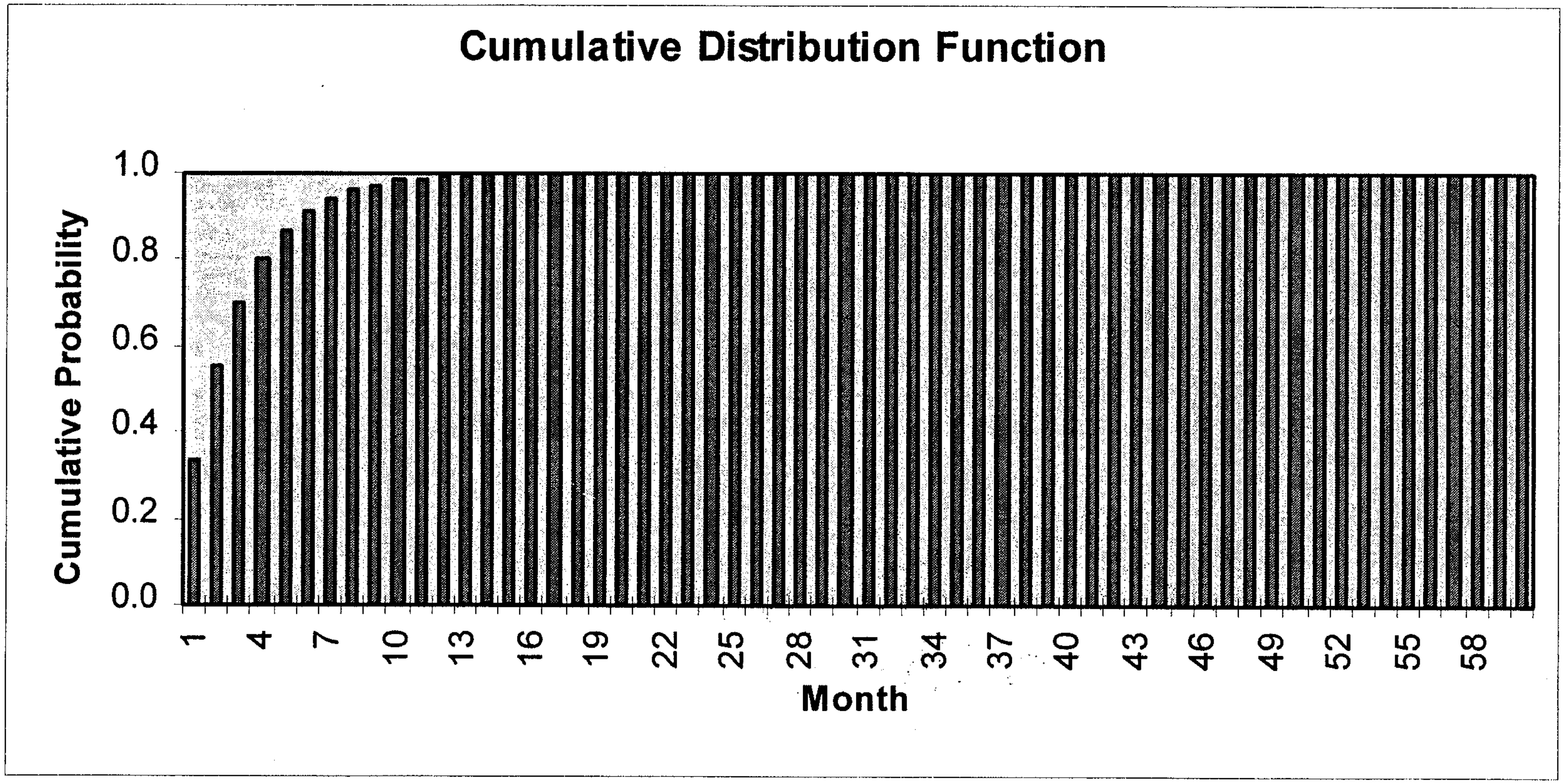
Forecast Horizon, Holding Period and p

s	j-1	p
12	3	0.67
12	6	0.83
12	9	0.89
12	12	0.92
24	3	0.67
24	6	0.83
24	9	0.89
24	12	0.92
24	24	0.96
36	3	0.67
36	6	0.83
36	9	0.89
36	12	0.92
36	24	0.96
36	36	0.97
60	3	0.67
60	6	0.83
60	9	0.89
60	12	0.92
60	24	0.96
60	36	0.97
60	48	0.98
60	60	0.98

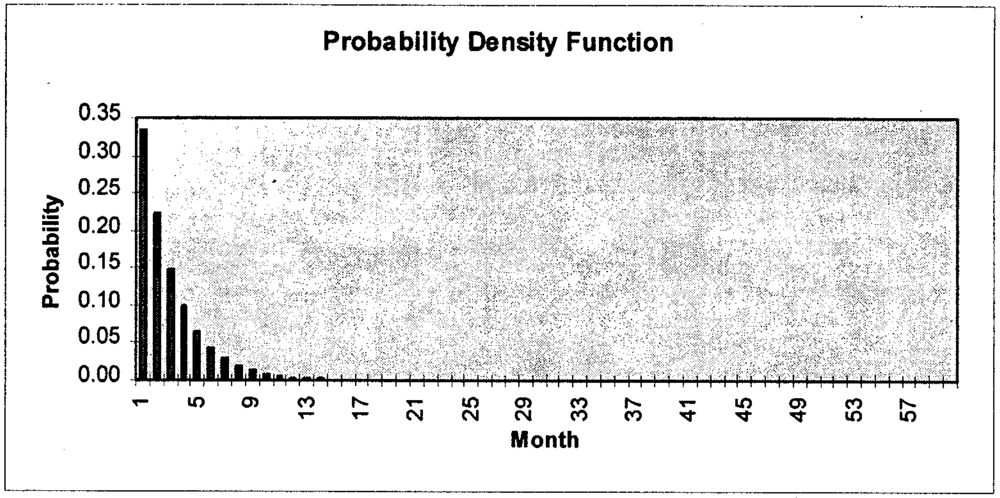
pdf: Mean Holding Period = 3 months

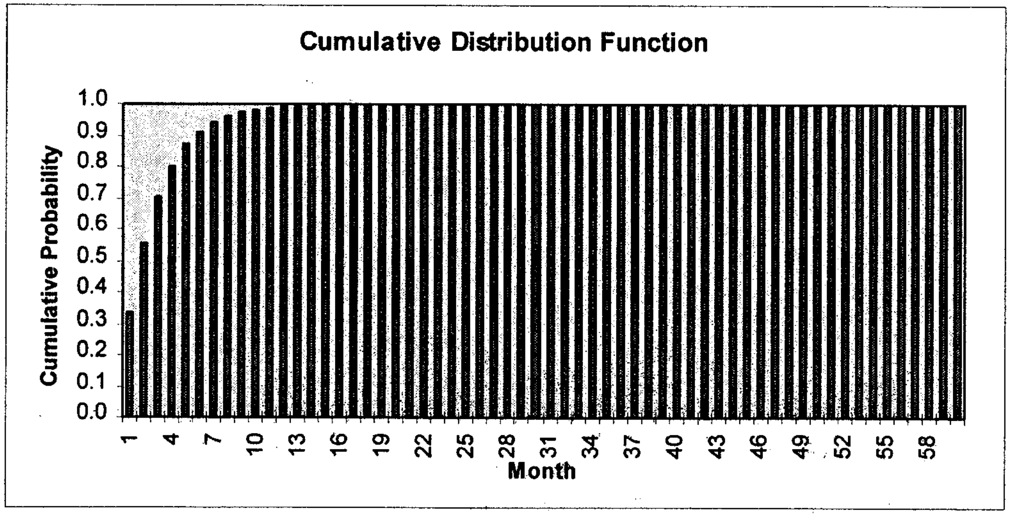


CDF: Mean Holding Period = 3 months

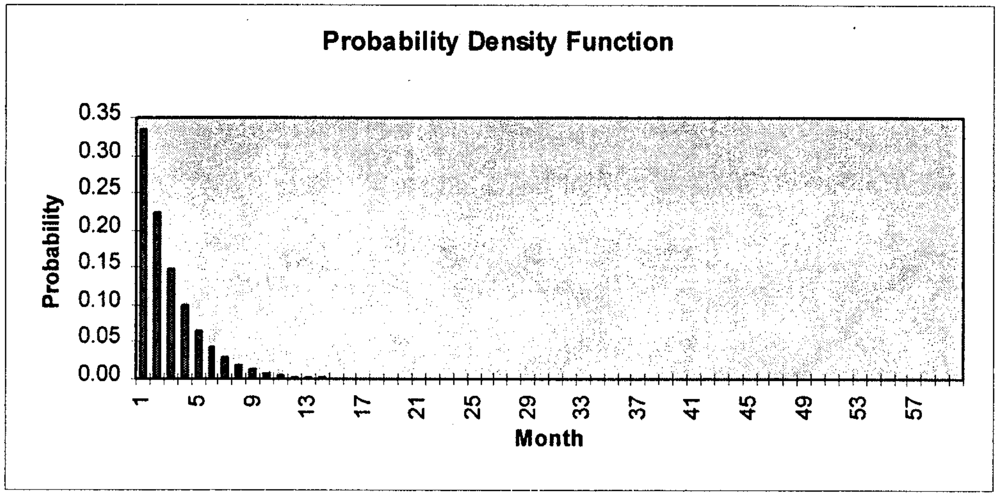


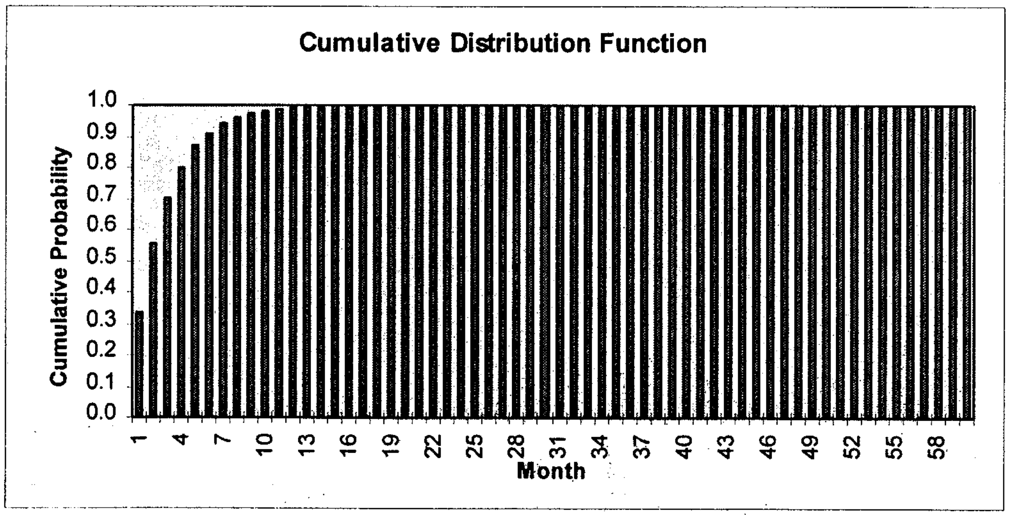
pdf, CDF: Mean Holding Period = 6 Mos



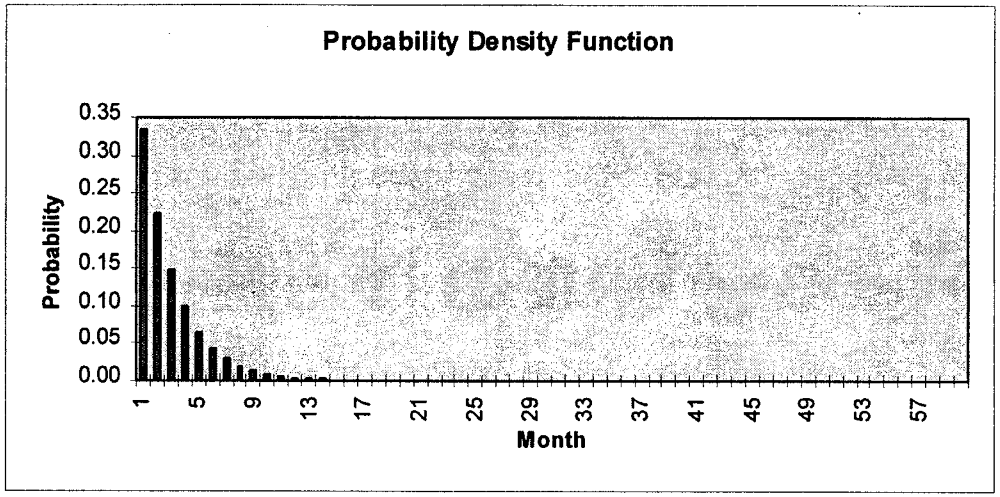


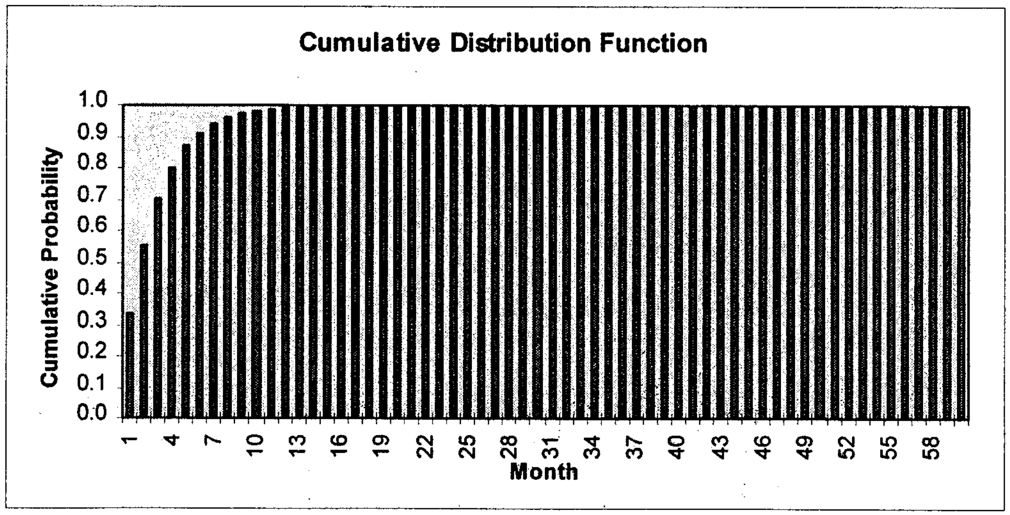
pdf, CDF: Mean Holding Period = 9 Mos





pdf, CDF: Mean Holding Period = 12 Mos





Returns

$p_j = (1 - p)\, p^{j-2}$

Expected Return in month 1 $= \mu_1 = E(r_1)$

Expected Return in month 2-s $= \mu_2 = E(r_2)$

Expected Return in later months = 0

Net and Total Returns

Net Return = r, Total Return = R

$$r = R - C_B - C_S$$

$$R = \sum_{j=2}^{s+1} P\,(\text{Sell in month } j)\,(\text{Cumulative return by month } j)$$

$$= \sum_{j=s+2}^{\infty} (1-p)\,p^{j-2}\,[r_1 + (j-2)\,r_2] + (1-p)\,p^{j-2}\,[r_1 + (s-1)\,r_2]$$

Technical Notes

$$\sum_{j=s+2}^{\infty} (1-p)\, p^{j-2} = p^s$$

$$\sum_{j=2}^{s+1} (1-p)\, p^{j-2} = 1- p^s$$

$$\sum_{j=2}^{s+1} (1-p)\, (j-2)\, p^{j-2} = p\, (1- p^{s-1})/(1-p) - (s-1)p^s \;= Q_s$$

Technical Notes: Continued

Total Return, R

$$= [r_1+(s-1)\ r_2]\ p^s+r_1\ (1-p^s)+r_2\ [Q_s]$$

$$= r_1+r_2[(s-1)\ p^s + p\ (1-p^{s-1})/(1-p) - (s-1)\ p^s]$$

$$= r_1+r_2\ p\ (1-p^{s-1})/(1-p)$$

Ergo:

$$R = r_1 + r_2\ p\ (1-p^{s-1})/(1-p)$$

Therefore,

$$E(R) = \mu_1 + \mu_2\, p\ (1-p^{s-1})/(1-p) \qquad \text{(Eq. 1)}$$

$$Var(R) = \sigma_1^2 + \sigma_2^2\, p^2\ (1-p^{s-1})^2 / (1-p)^2$$

Rules for Combining r_1 and r_2

- Simplest Approach
  - Use the weighting in equation 1
  - Transaction costs will affect the original buy decision, but are built in otherwise
  - More complex and realistic models could, and perhaps should, be developed here
- Second Approach
  - Consider risk-adjusted return:

    $$E(\text{Total Return}) - \lambda \cdot V(\text{Total Return})$$

Example

$$E(r) = (\mu_1 + \mu_2\, p\, (1-p^{s-1})/(1-p))$$

Assume s = 12 months

Mean Number of Months Held	p	Relative weight of r_2
1	0.000	0.00
2	0.500	1.00
3	0.667	1.98
6	0.833	4.33
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Sources of Bias

- Risk Factor, Asset Class, and Industry Bias
- Look-Ahead Bias
- Survivorship Bias
- Data Mining Bias
- Data-Snooping Bias

Alpha by Sector (Per Unit StDev)

	Large Cap		MSCap	
Sector	**Monthly**	**Annual**	**Monthly**	**Annual**
1-Basic Material	0.40	4.95%	1.03	13.04%
2-Energy	0.49	6.08%	0.84	10.57%
3-Cons Non-Disc	0.51	6.29%	1.10	14.01%
4-Consumer Disc	0.70	8.75%	1.19	15.22%
5-Comm Svcs	0.81	10.22%	1.17	14.94%
6-Manufacturing	0.47	5.75%	0.81	10.18%
7-Technology	0.82	10.28%	0.84	10.51%
8-Finance	0.35	4.26%	0.62	7.74%
9-Utilities	0.31	3.83%	0.47	5.84%
Average	**0.54**	**6.71%**	**0.90**	**11.34%**

*Stock & Industry Alpha Model: IC's**

Stock Selection Model Performance 1997 YTD

Large Cap Stock Selection Model IC = 6.0 %

SMid Cap Stock Selection Model IC = 7.6 %

Stock Selection Model Performance for Calendar Year 1996

Large Cap Stock Selection Model IC = 9.1%

SMid Cap Stock Selection Model IC = 9.1%

Industry Rotation Model Performance 1997 YTD

Large Cap Industry Rotation Model IC = 11.1%

SMid Cap Industry Rotation Model IC = 9.1%

* As of 9/30/1997

*Performance Summary**

	1996	YTD 1997	Since*** Inception
****Large Cap**	26.62	38.30	75.12
S&P 500 Index	*23.01*	*29.60*	*59.43*
****Mid Cap**	21.71	37.47	67.32
S&P Mid Cap Index	*19.20*	*30.68*	*55.77*
****Small Cap**	17.62	32.10	55.76
Russell 2000 Index	*16.53*	*29.81*	*51.27*

* As of 9/30/1997
** Gross Returns
*** 12/31/1995 Inception