Abstract
Within asset management, the risk of benchmark relative performance is typically expressed by measures such as “tracking error”, which describes the expectation of times-series standard deviation of benchmark relative returns. This is clearly useful measure for index fund management, where the expectation of the mean for benchmark relative return is fixed at zero. The active management case is problematic, as tracking error excludes the potential for the realized future mean of active returns to be other than the expected value. All active managers must believe their future returns will be above benchmark (or peer group average) in order to rationally pursue active management, yet it is axiomatically true that roughly half of active managers must produce below average results. Following the convention of Qian and Hua (2004), we refer to this additional portfolio risk as “strategy risk”. In this paper, we will first describe our two approaches to estimating the magnitude of strategy risk across asset classes and manager styles, and incorporating this information into portfolio risk assessments.
Introduction
Within asset management, the risk of benchmark relative performance is typically expressed by measures such as “tracking error”, which describes the expectation of times-series standard deviation of benchmark relative returns. This construct is useful for index fund management, where the expectation of the mean for benchmark relative return is fixed at zero. The active management case represents a central paradox in asset management, as tracking error excludes the potential for the realized future mean of active returns to be other than the expected value. All active managers must believe their future returns will be above benchmark (or peer group average) in order to rationally pursue active management, yet it is axiomatically true that roughly half of active managers must produce below average results.

Prior Approach to Strategy Risk
One approach to this issue comes in Qian and Hua (2004), wherein they define “strategy risk”. In essence, it is the risk created because the skill level of the active manager is not constant over time, as evidenced by the mean return being other than expected. They use the terms “active risk” to describe the combination of the tracking error and strategy risk. They formulate active risk as:

$$\sigma_{active} = \sigma_{IC} \times n^{0.5} \times \sigma_{TE}$$

Where

- $\sigma_{IC}$ = time series variation in IC
- $n$ = the number of independent bets in the strategy per unit time
- $\sigma_{TE}$ = tracking error

This formulation arises directly from the Grinold (1989), and the Fundamental Law of Active Management (FLAM). If the manager’s skill level is constant over time, any variation in IC must arise purely from sampling error, making the standard deviation of IC equal to the reciprocal of the square root of breadth. The product of the two factors is unity in this special case, and active risk is equal to tracking error.

If the manager’s skill level is time-varying the dispersion of IC will be greater than the square root of breadth and active risk be scaled upward as a multiple of tracking error. In a simple conceptual sense, tracking error represents the risks that external events will impact our investment result, while strategy risk represents what we can do to ourselves to contribute to an adverse outcome relative to expectations. One stylized example might be a portfolio manager whose work suffers to a failed marriage, and who makes a descent into alcoholism, reducing their IC relative to past values.

There are some practical problems with this approach to active risk. The first is that we actually have to be able to observe the time series variation in the information coefficient. This limits the set of possible users to active managers who make formal alpha forecasts for each investment period. It cannot be used by more fundamental managers, or by “outsiders” such as pension funds who employ the managers. The second is that the property of breadth is notoriously difficult to actually measure, as it
represents a complex function of the number of assets in the prediction universe, the correlation among those assets, portfolio turnover and the extent to which the active strategy in question seeks to exploit information about the correlated or independent portions of asset returns. Finally, there are typically unrealistic assumptions inherited from the FLAM which include that transaction costs are zero, and there are no constraints on portfolio composition.

One way to improve this formulation is to replace the use of the information coefficient (IC) with the Effective Information Coefficient (EIC) as defined in diBartolomeo (2008). Rather than measure the variation in the correlation of forecasts and outcomes, we measure the variation in the correlation between “implied alphas” and outcomes.

\[
\sigma_{active} = \sigma_{EIC} \times n^{0.5} \times \sigma_{TE}
\]

Where

EIC = the effective information coefficient, the correlation between implied alphas and subsequent return outcomes

The benefit of this substitution is that we can eliminate two the problems associated with Qian and Hua method. EIC can be estimated by an “outsider” or by a fundamental manager whose investment process does not involve the expressing security level return expectations in a numerical form. The implied alphas are obtained by inference from the asset positions that are observable for all portfolios. In addition, the estimation of EIC incorporates the effect of constraints on portfolio position size and turnover. Unfortunately, we make no particular headway in terms of the practical difficulties of estimating the breadth of a strategy.

A More General Approach
A more general conception of the problem would be to think of active risk as the square root of total active variance

\[
\sigma_{active} = (\sigma_{mean}^2 + \sigma_{TE}^2 + 2 \times \sigma_{mean} \times \sigma_{TE} \times \rho)^{0.5}
\]

Where

\( \sigma_{mean} \) = uncertainly of the true mean relative to expectation of the mean
\( \rho \) = correlation between uncertainty and tracking error

Many papers, such as Grinold (1994), have been written in the finance literature based on the prior belief that if a manager has no skill or markets are wholly efficient, the expectation of benchmark (or peer) relative excess returns will be zero. An expectation of expected return is then formulated as Bayesian combination of the manager’s actual belief about expected returns and the prior belief of zero.

Let us consider the concept of active management as an exercise in flipping a coin. To pursue active management, the manager (and their investors) must have two separate
beliefs. First, they must believe that the coin is not a fair coin, but rather is weighted to favor one of the two possible outcomes. Secondly, they must believe that they possess valid information as to which way the coin is weighted, as compared to other investors who also believe the coin is weighted but to the opposite side.

If active managers are incorrect about market being not wholly efficient, then we can assume our coin is indeed fair, and the prior belief of zero excess returns is very sensible. On the other hand, if a given manager is correct that markets are not wholly efficient, but wrong in the belief that their side of the coin is favorably weighted, it must be true that the other side of the coin is weighted. If we believe that a coin will come up “heads” on the next flip (i.e. our investment time horizon) and we are wrong, then the coin must come up “tails”. Our prior belief cannot sensibly be that the coin will fall and yet stand on edge.

One way model this problem is to consider a binary distribution for the active return of a manager. We assume that each manager has a benchmark relative return expectation of portfolio alpha \( \alpha_p \) with a probability \( w \) of being correct. If the manager’s forecast is wrong, they have a probability of \( (1-w) \) of realizing \(-\alpha_p\). With this framework, the value of \( \sigma_{\text{mean}} \) is:

\[
\sigma_{\text{mean}} = ((1-w) \times 4 \times \alpha_p^2)^{.5}
\]

Where
- \( w \) = is the probability of realizing the expected alpha
- \( \alpha_p \) = manager’s expectation of portfolio alpha

For \( w = .5 \) we obtain the simple expression, \( \sigma_{\text{mean}} = 2^{.5} \times \alpha_p \)

**Active Risk and IR**

It is the frequent custom of the asset management industry that the information ratio is used as a proxy for manager skill.

\[
\text{IR} = \left( \frac{\alpha_p}{\sigma_{\text{TE}}} \right)
\]

\( \alpha_p = \text{IR} \times \sigma_{\text{TE}} \)

\[
\sigma_{\text{mean}} = ((1-w) \times 4 \times (\text{IR} \times \sigma_{\text{TE}})^2)^{.5}
\]

For \( w = .5 \), we obtain, \( \sigma_{\text{mean}} = 2^{.5} \times \text{IR} \times \sigma_{\text{TE}} \)

Let’s make the simplifying assumption that \( \rho = 0 \). We will consider two managers, K and L. Both managers have tracking error equal to 5. Manager K is a traditional asset manager that purports to clients that their information ratio is .5. Manager L is fund that purports to their investors that their information ratio is 3. If we take the claimed information ratio values as proxies for manager skill, Manager L is **six times** as good as Manager K.
However, an extravagant claim for a manager’s information ratio leads to a very high estimate of active risk. Let’s consider what happens at $w = .5$:

For Manager K we get:

$$\sigma_{\text{active}} = (2 \times .25 \times 25 + 25)^5 = 6.125$$

For Manager L we get:

$$\sigma_{\text{active}} = (2 \times 9 \times 25 + 25)^5 = 20.61$$

Our new active risk value is Manager K is about 23% greater than original TE, and the revised IR is now .41. For Manager L, the active risk value is more than four times the original tracking error, and the adjusted IR is now just .73. It should be noted that if more aggressive managers with higher tracking errors tend to also have more uncertainty in their means (i.e. $\rho > 0$), then it is possible that the adjusted IR for Manager L will actually approach the lower value of the much more conservative manager K.

The lesson for asset owners and particularly “fund of fund” managers is that their hiring of high IR managers must be predicated on the belief that the probability that the manager is skillful must be far above one half despite the obvious constraint on the aggregate value of $w$.

Let us now turn to the estimation of $\rho$, the correlation between the uncertainty of mean return of any particular active manager, and their tracking error. Both of these properties arise from related underlying causes, the volatility of security returns, the correlation of security returns and the size of the manager’s active position weights (i.e. bets). dBartolomeo (2006) provides a broad discussion of the relationship between volatility in financial markets and cross-sectional return dispersion (i.e. “variety”).

Numerous studies have shown that security correlations tend to rise during periods of market volatility, suggesting that the correlation between variety and volatility should be less positive, but less than one. In deSilva, Sapra, Thorley (2001), they derive an expression for the expectation of the cross-sectional variance (variety squared) of security returns, and show that it is linearly related to the realized market return in each period. They also show that the variety in active manager returns should be linearly related to the variety of security returns. These results suggest that the there should be a positive, but not linear relationship between our $\sigma_{\text{mean}}$ and $\sigma_{\text{TE}}$ measures. Akrim and Ding (2002) provides an extensive empirical study confirming that the cross-section of active manager returns is very closely related to the cross-section of security returns, implying that active managers have relatively constant “bet” sizes over time.

An Empirical Approach
We can also use empirical data to statistically estimate $\sigma_{\text{mean}}$ and $\rho$ for any particular set of active managers. Let us work through an example. Our sample is 1957 US Large Cap
Growth Managers. We observe monthly returns for the 60 months ending November 30, 2009. We begin by computing the monthly cross-sectional average and subtract from each observation to put observations in “peer relative excess” units. Next, we calculate the cross-sectional standard deviation of excess returns for each period, and the mean 60 month annualized excess return for each manager. Finally, we calculate 60 month realized annualized tracking error (standard deviation of excess returns).

This process yields some interesting empirical results. The average annualized cross-sectional dispersion is 5.76%, while average time series tracking error is 5.70%. The cross-sectional correlation between the absolute value of annualized excess returns (as a proxy for dispersion of mean) and corresponding tracking errors is .21. Using these central tendencies to represent the typical fund in the sample, we obtain:

$$\sigma_{\text{active}} = (5.76^2 + 5.70^2 + 2 \times .21 \times 5.76 \times 5.70)^{\frac{1}{2}} = 8.91$$

This represents an increase of 56% in risk as compared to tracking error alone, implying that investor expectations for the IR of a typical fund should be reduced by at least one third.

Our new measure of active risk can be used in a number of useful ways. First, it can easily be incorporated in risk budgeting and manager evaluation exercises by asset owners. For asset managers, portfolio optimizations can be organized as a conventional mean-variance process, subject to a constraint on the value of active risk. As active risk rises rapidly with expected IR, this sort of optimization procedure reduces bet sizes in an intuitive fashion, much like a Bayesian process or robust optimization. Our approach has the advantage of explicitly considering the potential for alphas to be of the wrong sign, rather than being just overstated in magnitude.

**Conclusions**

Tracking error is an inadequate measure of risk for active managers. Instead, active managers and the asset owners who employ them should evaluate risk with the broader measure of “active risk” in the spirit of Qian and Hua. Active risk can be formulated as the aggregate of tracking error and the uncertainty of the mean return over time. The estimation of active risk can be reasonably parameterized either from empirical data for defined manager styles or from a simple “rule of thumb”

One broad implication of this work is that investor risk assessments of active managers are understated, and that the allocation of funds between active and passive strategies is substantially biased toward active management.
References


