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Advanced Risk Decomposition

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Presentation overview

- Implied Alpha as Risk Metric
  - The implied alphas in a portfolio are a valuable concept in optimization to understand results and communicate risk intuitively across organizations, but it can be tricky to grasp at first for new users
  - We'll walk through several simplified examples of how implied alpha can be used in different contexts
- Risk Decomposition by Asset
  - Users frequently have use cases that warrant customized risk reporting based on non-standard grouping of assets
  - We'll walk through the math of how to decompose risk contribution by asset so that it can then be added together and grouped in any configuration

Stage-setting hypothetical: Weights contain beliefs

- You are surreptitiously and benevolently given the holdings and weights of the world's greatest portfolio manager
- She has a perfect track record. One hundred percent of her predictions have come true, and all future predictions will also be true.
- There are no ethical or regulatory issues involved.
- What are this manager's return expectations for the stocks she has selected?

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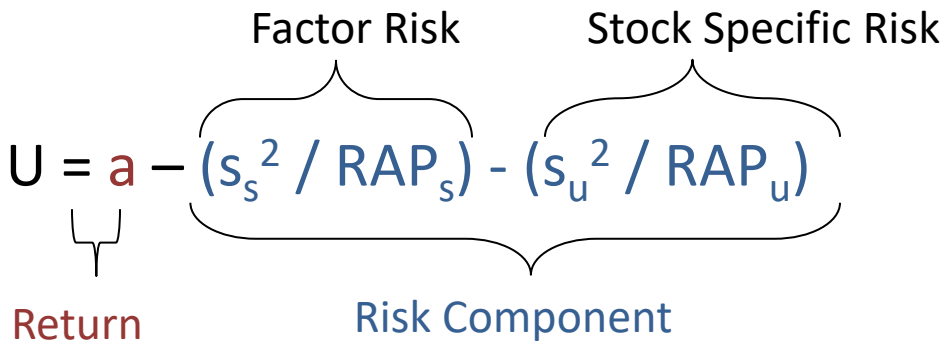
- [Optimizer example]

Calculating Implied Alpha

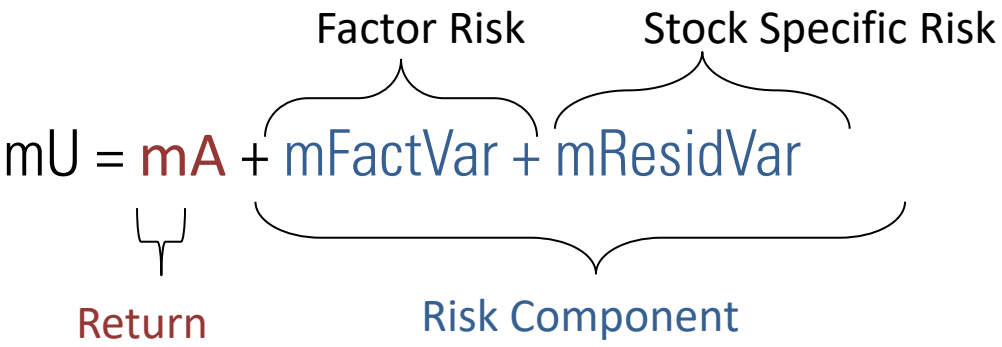
Start with objective function (simplified to assume no transaction costs):



- U =Utility in certainty-equivalent expected return units
- a = the "certainty equivalent" expected portfolio return
- s_s^2 = portfolio variance risk due to common factors (correlation across securities)
- s_u^2 = portfolio variance risk due to stock specific risks
- RAP = risk tolerance

Calculating Implied Alpha

Think about it in marginal terms, i.e. “what would change if I traded this asset”



*sign changes in risk terms to reflect that on a marginal basis, **increasing** the position size **decreases** risk

Calculating Implied Alpha

Add in the assumption that the portfolio is optimal: “If the manager thought the portfolio could be improved, she would trade, and the weights would be different. Therefore, the portfolio cannot be improved, and the marginal utility of any trade is 0.

$$mU = 0 = mA + mFactVar + mResidVar$$

Rearrange terms:

$$mA = -(mFactVar + mResidVar)$$

This is the implied *return* for the security. Since implied *alpha* is benchmark-relative, we subtract the weighted average implied return of the benchmark:

$$\text{Implied Alpha} = -(mFactVar + mResidVar) - \text{BenchIA}$$

Implied Alpha is Reversing an Optimization

- To underscore the point, calculating an implied alpha is just running an optimization backwards. The optimizer normally translates returns and risks into optimal weights, but having any two of those three values lets you solve for the third.

[Optimizer example]

Complications

- This doesn't work out so cleanly in real-world examples
- Anything other than risk contribution and return expectations in the optimization that affect optimal weights will prevent the implied alpha from exactly matching the input expected alpha
  - Leverage limits, including long-only
  - Transaction costs, including taxes
  - Constraints of any kind – position limits, industry/sector, cash, etc.

Using the complications to your advantage

- The implied alpha for positions can be used to figure out if any securities are held at a non-optimal weight
- If a position has a high implied alpha, the position is contributing a disproportionately high amount of risk. It is probably being prevented from being sold.
- If the position has a low/very negative implied alpha, it has a disproportionately low contribution to risk. It is probably being prevented from being purchased.
- If all implied alphas are very high or very low, it will point to your risk acceptance value (RAP) not being set rationally
- You can compare the "Implied Alpha" and your actual input alpha. If the portfolio is optimized, then your implied alpha and your input alpha should agree unless the portfolio composition is being impacted by constraints. The more the implied and input alphas don't agree, the more the constraints are costing you.
  Cost of Constraints in Optimization | northinfo.com.

Using the complications to your advantage (cont'd)

- Implied alphas can also be useful for managers using less formal portfolio construction methodologies
- If you don't have input alphas, but you rank stocks, you can compare your stock ranking to the ranking by Implied Alpha. If the rankings don't match, you want to increase your position size when the input rank is better than the implied alpha rank, or decrease position size when input rank is worse than implied alpha rank.
- Some further reading on this topic for managers of funds-of-funds:
  A Radical Proposal for the Operation of Multi-Manager Investment Funds | northinfo.com

Implied Alphas of Factors

- This same logic can be applied to factors – since active factor bets/tilts contribute to risk, they also have an implied return

We have a portfolio with annual tracking error of 5 or tracking variance 25. We will assume that 20 units are factor variance and 5 units are specific variance.

Using our rule of thumb, our estimated RAP = 6 * 5 = 30

Now let's say that of the 20 units of tracking variance, 10 units come from the Oil Price factor, and that our active factor exposure to Oil Prices is .15. If we are willing to take 10 units of risk from exposure to oil with a RAP of 30, we must believe that we will get 10/30 units of alpha (excess return) from Oil Prices, or .33 over the next year.

Active exposure * factor alpha = factor return impact = .33
.15 * factor alpha = .33
Factor alpha = .33/.15 = 2.2%

So the inference is that we acting as if we are certain of a 2.2% return to Oil Prices over the next 12 months. (Alternatively, if we believed there was a 10% correlation between our predictions and actual returns, this would be equivalent to saying we expect that oil prices will rise 22%, but we're not very confident we're right.)

Risk Decomposition by Asset

- What percent of my total risk/active risk is coming from any individual asset?
- What percent of my risk is coming from a sector?
- What percent of my beta risk is coming from my fixed income sleeve?

Risk Decomposition by Asset - Easy way

- What is the contribution to active risk of a given stock in my portfolio?
- Simple calculation of active weight and marginal variance
- If you make the simplest possible assumptions, it's just marginal variance times half the active weight.
- Using nomenclature in the Security Marginal Contribution report, it's

MV * (ActWt(%)/2)

(see "Complications" slide)

Risk Decomposition by Asset: Complicated way

- Allows you to decompose risk by stock and by factor, which can then be summed up by any grouping: risk by sector, by asset class, or by asset group *and* factor (e.g. "Large Cap contribution to Market risk")
- Requires a bit of spreadsheet math

Risk Breakdown by Sector

	Sector	Active Weight%	Portfolio Weight%	Benchmark Weight%	Active Contribution%	Portfolio Contribution%	Benchmark Contribution%
1	Unclassified	-0.28	0.19	0.47	0.00	0.00	0.00
2	Materials	3.48	5.34	1.85	5.54	6.24	1.86
3	Industrials	0.32	6.78	6.45	4.41	7.66	6.98
4	Telecom	-1.80	1.48	3.28	1.99	1.30	2.19
5	Consumer Discretionary	-0.46	13.16	13.62	17.29	15.11	14.23
6	Consumer Staples	-5.14	4.24	9.38	4.84	3.55	7.18
7	Energy	-0.16	4.13	4.29	2.02	5.04	4.48
8	Financials	25.57	40.85	15.27	36.65	39.97	15.23
9	Healthcare	-7.78	5.67	13.45	7.67	6.05	13.08
10	Information Technology	-18.16	10.04	28.20	16.54	10.36	32.53

Risk By Asset

Marginal Systematic Tracking Variance	Marginal Unsystematic Tracking Variance	Marginal Tracking Variance	Marginal Tracking Error	Contribution to Systematic Tracking Variance	Contribution to Unsystematic Tracking Variance	Contribution to Tracking Variance
mTV_{sys}	mTV_{unsys}	mTV	mTE	TV_{sys}	TV_{unsys}	TV

$\mathbf{e}_P$ = vector of portfolio's (absolute) factor exposures

$\mathbf{e}_B$ = vector of benchmark's factor exposures

$\mathbf{e}_s$ = vector of security *s*'s (absolute) factor exposures

$\mathbf{C}$ = matrix of factor covariances

σ_s^2 = security *s*'s unsystematic (asset-specific) variance

w_s^P = weight of security *s* in the portfolio

w_s^B = weight of security *s* in the benchmark

let $\mathbf{v}$ = $2\mathbf{C}(\mathbf{e}_P - \mathbf{e}_B)$
= vector of factor marginal tracking variances
(in variance per unit exposure)

$mTV_{SYS} = 2\mathbf{e}_s^T\mathbf{C}(\mathbf{e}_P - \mathbf{e}_B) = \mathbf{e}_s^T\mathbf{v}$

$mTV_{UNSYS} = 2\sigma_s^2(w_s^P - w_s^B)$

$mTV = mTV_{SYS} + mTV_{UNSYS}$

$mTE = mTV / (2 * TE)$

$TV_{SYS} = (w^{sP} - w_s^B)\,\mathbf{e}_s^T\mathbf{C}(\mathbf{e}_P - \mathbf{e}_B)$
$= \frac{1}{2}(w_s^P - w_s^B)\,\mathbf{e}_s\mathbf{T}\mathbf{v}$
$= \frac{1}{2}(w_s^P - w_s^B)\,mTV_{SYS}$

$TV_{UNSYS} = (w_s^P - w_s^B)\,{}^2\sigma_s^2$
$= \frac{1}{2}(w_s^P - w_s^B)\,mTV_{UNSYS}$

$TV = TV_{SYS} + TV_{UNSYS}$

Risk by Stock: Complications

- Contra-asset problem
  - The biggest error users make when decomposing risk by asset, and the biggest hurdle to its usefulness in providing actionable information to a manager, is what is referred to as the "contra-asset problem."
  - If a security is 5% of my weight and 8% of my risk, if I take it out of my portfolio, what do I replace it with? Portfolios have to add to 100%.
    - If I replace it with cash, this causes problems if I am managing against a benchmark. Often this will result in output where removing a risky asset and replacing it with cash causes tracking error to go **up,** relative to a non-cash benchmark. (This is Northfield's standard)
    - If I reweight the other names in the portfolio, the risk contribution from every other asset will also change, so making any 1-to-1 comparison between assets can't happen.
    - If I use the benchmark as the contra-asset, the output will be generally be more intuitive, but if the asset is part of the benchmark, you buy back some of the asset you just sold

Risk by Stock: Complications

- Marginal Tracking Error/Contribution to Tracking Error
  - Since these are squared terms, they are not additive and difficult to deal with.

Conclusions

- Don’t ignore Implied Alpha as a risk metric!
- Risk decomposition by asset, despite its limitations, can be a helpful way to produce customized reporting for a different view of your portfolio
- The risk by stock spreadsheet will be distributed as a thank you for attending.

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