



# The Inefficient Market Hypothesis

Jason MacQueen, Director of Research

SMART  
PORTFOLIO  
STRATEGIES



# Where Are We Going?

- The main tenets of Modern Portfolio Theory are fairly well-known to most investors, even if they don't always remember all the heroic assumptions!
- Some aspects of MPT have passed into common knowledge (and the CFA syllabus), and affect the behaviour of many fund managers and investors.
- The focus of this talk is on the subversive notion that capitalisation-weighted benchmarks are in some sense Markowitz-efficient portfolios.
- This assumption is deeply embedded in many standard practices:
  - Optimisation relative to a cap-weighted benchmark, targeting Tracking Error
  - Using benchmark weights as a starting point for building an active portfolio
  - *Both can significantly hurt the portfolio performance of skilled active managers.*



# The Tenets of Modern Portfolio Theory

- Modern Portfolio Theory (MPT) rests on three main ideas:-
- Markowitz Efficient Portfolios, in which expected returns are traded off evenly against risk, using a risk aversion parameter  $\Psi$  so that  $(0 < \Psi < \infty)$ .
- Sharpe's Capital Asset Pricing Model (CAPM),  $ER_i = R_f + \beta_{iM} * (ER_M - R_f) + \alpha_i$ .
- Fama's Efficient Market Hypothesis (EMH), which would have us believe that all stocks are (more or less) fairly priced all the time.
- One logical consequence of all this is the entertaining assertion that the market itself is therefore the optimal efficient portfolio (see CAPM).
- Another, even more entertaining consequence, is that the risk-adjusted expected returns of all stocks will be equal (think Security Market Line).



# The Assumptions of Modern Portfolio Theory

- It is worth reminding ourselves of the main assumptions behind MPT
- Markowitz assumes expected returns and risks are known with certainty.
  - In reality, expected returns are usually ranges, and even the best risk numbers are only estimates (and as Dan will tell you, the world isn't stationary anyway).
- CAPM assumes rational, risk-averse investors, frictionless trading, unlimited borrowing and shorting, and homogenous expectations.
  - In reality, most of the above are not even close to being true.
- EMH assumes all information is available to investors simultaneously, and that they all process that information rationally to determine 'fair value'.
  - Even with the internet, it can't be true that all investors get all available information at the same time, and does anyone really believe that all investors are rational?!



# The Case for the Prosecution

- Finance academics like to use the tenets of MPT to argue that the market (represented by some cap-weighted index) can't be beaten.
- All active managers are trying to outperform, but in aggregate their excess returns must sum to zero before transaction costs and management fees, so clearly many will underperform.
- Academics produce numerous surveys of the underperformance of mutual funds and other actively-managed portfolios as supporting evidence.
- Note, however, that this is circumstantial evidence at best; it is certainly not proof “beyond a reasonable doubt”, or, in civil law terminology, proven “on the balance of probabilities”.



# The *Reductio Ad Absurdum* Argument

- Active managers spend vast resources on stock selection, building models, doing research, hiring stock analysts, visiting companies, etc. Is this all wasted effort?
- If active managers gave up trying to outperform and turned passive, there would be far less price discovery and stock prices would quickly become inefficient.
- So for stock prices to be even fairly efficient, there must be a significant amount of stock selection research being done.
- It is highly unlikely that all investors are equally good at this, so it follows that some managers will be better at it than others.
- So it would appear that, for even a mild form of the Efficient Market Hypothesis to hold, there **MUST** be stock selection skill out there . .
- . . . and that some managers must be better at it than others. Q.E.D.



## The Case for the Defense

- But if some active managers really do have good stock selection skill, why doesn't it always show up in their portfolio performance?
- Portfolio performance comes from skill in both stock selection and portfolio construction; both can have a significant impact on performance. (*"On the Value of Portfolio Construction" Northfield Webinar July 2019*).
- So skilled active managers can still underperform their benchmarks if their portfolios are not designed to maximise the impact of their stock selection skill - and minimise other 'noise'.
- From 2003 to 2014, we built custom risk models for Quant managers so they could see clearly, and quantify, the bets they already knew they were making. To our surprise, we found that more often than not, more than 50% of the risks they were taking were unintended bets, a.k.a. 'noise'.



# What is Portfolio Efficiency ?

- Technically, an efficient portfolio is one in which each unit of portfolio risk is balanced by a unit of expected return.
- Perhaps a more useful and practical definition is that in an efficient portfolio, the investor's views on the likely returns to each asset are reflected in the portfolio as accurately as possible. After all, there's no point having stock selection skill if you don't make the best use of it!
- All portfolios have a mix of deliberate and unintended bets, and both will affect the overall performance of the portfolio. An efficiently-constructed portfolio is one which maximises its exposure to the deliberate bets, while minimising its exposure to the unintended bets as far as possible.
- In a long-only portfolio, it is impossible to eliminate all other influences ('noise', or unintended bets) on performance; but they can be minimised.



## What about Efficient Portfolios ?

- Efficient portfolios are like unicorns, mermaids and the Indominus Rex. They don't exist.
- In real life, all portfolios are inefficient all the time. The interesting questions for a fund manager are therefore :-
  - How inefficient is each holding ?
  - How inefficient is the portfolio ?
- Portfolio rebalancing should therefore always be trying to nudge the portfolio towards greater efficiency (i.e. towards lower inefficiency).
- To do this, we need a way to determine the inefficiency of each holding, and a way to measure overall portfolio inefficiency.
- Ideally, we would like to be able to use it to compare the inefficiency of different portfolios.



# The Inefficiency of Individual Holdings

- Derive a set of Implied Returns that make the current portfolio efficient, and compare these Implied Returns  $IR_i$  with the Expected Returns  $ER_i$ . Clearly, the bigger the difference, the more inefficient the holding is.
- Implied Returns are given by the following equation :-

$$IR_i = ER_p + \Psi * S_p * (Beta_{ip} - 1)$$

where

$ER_p$  = Portfolio Expected Return

$S_p$  = Portfolio Risk

$Beta_{ip}$  = Beta of the Stock to the Portfolio

$\Psi$  = Return/Risk trade-off  $(0 < \Psi < \infty)$

- There are many different solutions as  $\Psi$  varies from 0 to infinity. Clearly, we need the one which makes the portfolio look as efficient as possible.



## What is $Beta_{iP}$ ?

- Portfolio risk (as variance) is given by  $V_P = \text{Sum}_i\{\text{Sum}_j(x_i * x_j * C_{ij})\}$  where  $x_i, x_j$  are holdings of  $i$  and  $j$ , and  $C_{ij}$  is the full covariance matrix
- Hence the % contribution of risk (variance) from holding  $i$  is given by :-

$$\begin{aligned} PCV_{iP} &= [100 * \text{Sum}_j\{x_i * x_j * C_{ij}\}] / V_P \\ &= [100 * x_i * \text{Sum}_j\{x_j * \text{Cov}(R_i, R_j)\}] / V_P \\ &= [100 * x_i * \text{Cov}(R_i, \text{Sum}_j\{x_j * R_j\})] / V_P \\ &= [100 * x_i * \text{Cov}(R_i, R_P)] / V_P \end{aligned}$$

- Dividing the % contribution to risk by the % holding size, we get :-

$$PCV_{iP} = \frac{[100 * x_i * \text{Cov}(R_i, R_P)]}{100 * x_i * V_P} = \frac{\text{Cov}(R_i, R_P)}{V_P} = Beta_{iP}$$



## Calculating the Implied Returns

- So  $\text{Beta}_{iP}$  shows whether this holding  $i$  is more or less risky than average in this particular portfolio.  $\text{Beta}_{pP}$  of the portfolio to itself is, of course, 1.
- To make the portfolio as efficient as possible, we calculate  $\text{Min } \Psi$  as:-

$$\text{Min } \Psi = \frac{\text{Sum}_i\{(ER_i - ER_p) * (\text{Beta}_{iP} - 1)\}}{S_p * \text{Sum}_i\{(\text{Beta}_{iP} - 1)^2\}}$$

- Then the corresponding Implied Returns are given by:-

$$(IR_i - ER_p) = \text{Min } \Psi * S_p * (\text{Beta}_{iP} - 1)$$

- This makes it clear that, in an efficient portfolio, the more attractive stocks ( $IR_i > ER_p$ ) will also be the more risky holdings ( $\text{Beta}_{iP} > 1$ ), and there is a constant return/risk trade-off  $\text{Min } \Psi$  throughout the portfolio



# Individual Holding Inefficiency

- The inefficiency of each holding is determined by the difference between its Implied Return and its Expected Return :  $(IR_i - ER_i)$
- If  $IR_i > ER_i$  the holding is overweight, so selling some of it would improve the efficiency of both the holding and the overall portfolio.
- If  $IR_i < ER_i$  the holding is underweight, so buying more of it would improve the efficiency of both the holding and the overall portfolio.
- If  $Abs(IR_i - ER_i) > Abs(IR_j - ER_j)$ , then holding  $i$  is more inefficient than  $j$ .
- Simple, really! So we can now tell how inefficient each holding in a portfolio is, and whether it's too large or too small (incrementally).



# General Portfolio Inefficiency

- So Holding Inefficiency is  $(IR_i - ER_i)$ , which may be positive or negative.
- We can define **Specific Portfolio Inefficiency (SPI)** for a portfolio as :

$$SPI = \text{Sqrt}\{ \text{Sum}_i[ (IR_i - ER_i)^2 / N ] \}$$

- Note that **SPI** is in units of return. Note also that it is scaled by the returns, so if we double all the returns, **SPI** also doubles. We can rescale it, and generalise it, by dividing by the Expected Return of the Portfolio, to give ourselves a measure of General Portfolio Inefficiency :

$$GPI = \text{Sqrt}\{ \text{Sum}_i[ (IR_i - ER_i)^2 ] / N \} / ER_p$$

- Note that a higher GPI score means a portfolio is more inefficient.

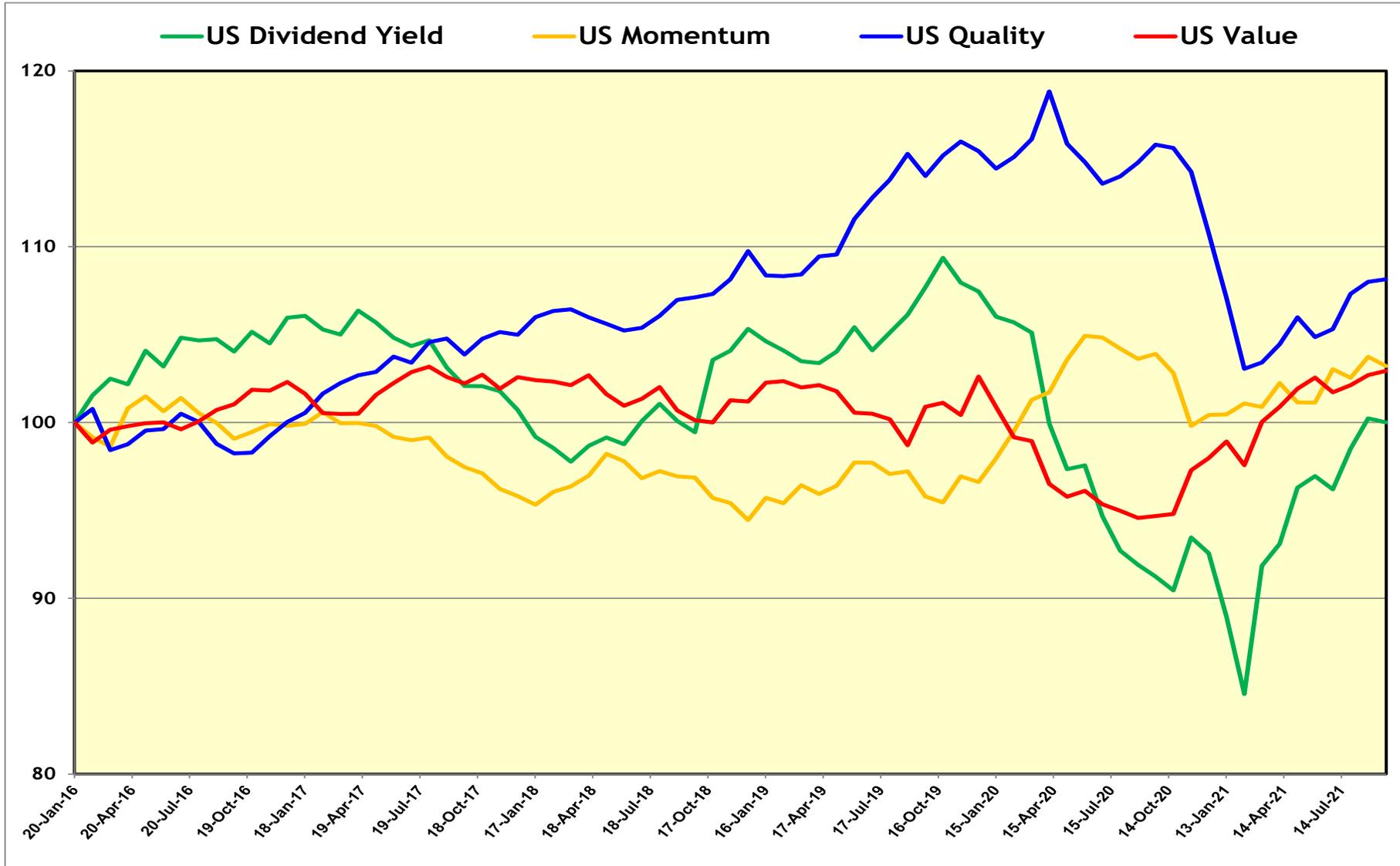


# Style Factors as Expected Return Proxies

- The purpose of this research exercise is to take some Expected Returns, construct a set of portfolios in various different ways, and then look at the measure of General Portfolio Inefficiency in each case, and compare it with a Performance Attribution analysis.
- We will use two different Style factors as proxies for Expected Returns, so we are building two sets of Style factor portfolios.
  - Value                                      B/P, E/P and CF/P
  - Quality                                      ROA, ROE, CF/Sales
- The Stock Selection rule is very simple: at each rebalancing date we rank the S&P 500 stocks high to low by the corresponding Style beta, and then select the top 100 stocks.



# Style Factor Returns Jan 2016 - Sept 2021





# The Inefficiency of Style Factor Portfolios

- In the July 2019 Value of Portfolio Construction webinar, we built Style factor portfolios using various different portfolio construction methods:-
  - **Equal-weighting**
  - **Attribute-weighting** (*where the weight is proportional to the ERI*)
  - **Inverse Volatility weighting**
  - **Markowitz Optimisation**
  - \* **Capitalisation-weighting**
  - \* **Risk Parity weighting**
  - \* **Smart Portfolio Optimisation**
- The point was to show that even though they all used the same stocks, their return and risk (i.e. performance) was very different.
- We are now more interested in the relative Portfolio Inefficiency of some of these strategies.



# Portfolio Construction Methods

- We include the five heuristic portfolio construction methods, which each produce one rebalanced portfolio, as well as Markowitz Optimisation.
- Markowitz Optimisation produces an efficient frontier, of course, with many possible portfolios on it. At the higher end of the frontier, the optimiser is emphasising higher return; at the lower end it is focusing more on minimising risk.
- In the analysis below, we therefore show three unconstrained Markowitz Optimisation results, from different points on the efficient frontier.
- These points are 30%, 50% and 70% in terms of increasing Expected Return.
- To illustrate the effect of constraints on General Portfolio Inefficiency, we also show the same three points from a frontier with a maximum holding constraint of 5%.



## Strategy Parameters

- Each strategy was started with \$100 million in January 2016, and all the portfolios were rebalanced every 12 weeks (think quarterly). The results shown below assume round-trip transaction costs of 30 bps, and an annual management fee, payable quarterly, of 20 bps.
- Over this period, Quality did quite well for just over 4 years, then turned negative for 10 months, and finally picked up again in the last 8 months.
- Value, of course, was not a good strategy to be in for most of this period. It was mildly positive for the first 19 months, negative for the next 38 months, and then recovered quite strongly over the last 12 months.
- These are quite good proxies for a manager's stock selection skills, but without any factor timing skill, which would also have been useful!

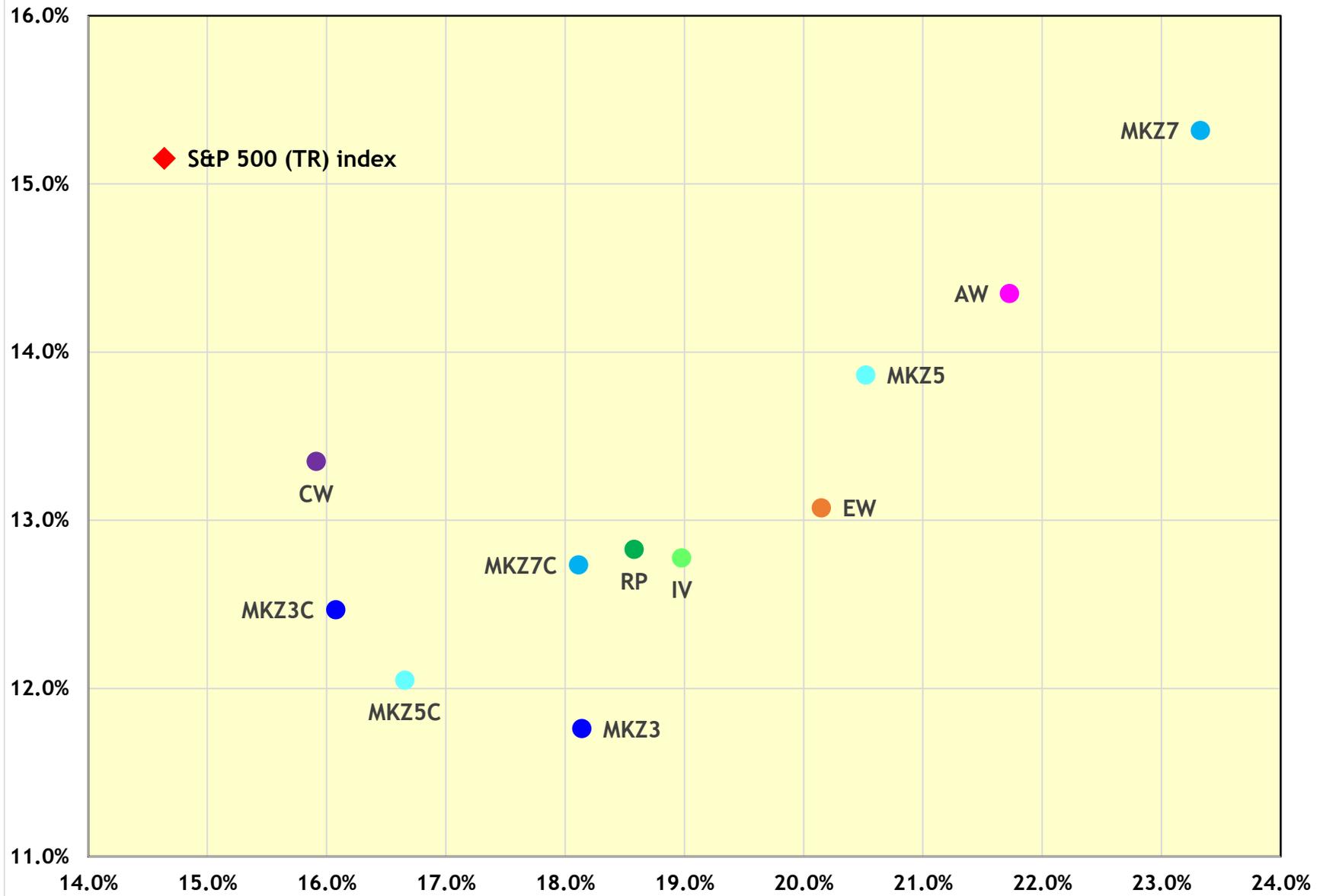


## What Are We Looking For?

- We use Value and Quality factor betas as proxies for Expected Returns, and we are assuming the corresponding factor premia will be positive
- Since this is a back test, we already know, of course, that neither Style did very well over this period, so we are not expecting great performance.
- However, the point of the test is to see how well each strategy did at getting the manager's 'stock selection skill' into the portfolio, so we will be focusing more on what percentage of the average portfolio return and risk over the period came from exposure to Value or Quality respectively.
- The General Portfolio Inefficiency scores from each strategy will also tell us how each strategy did at reflecting the manager's Expected Returns.



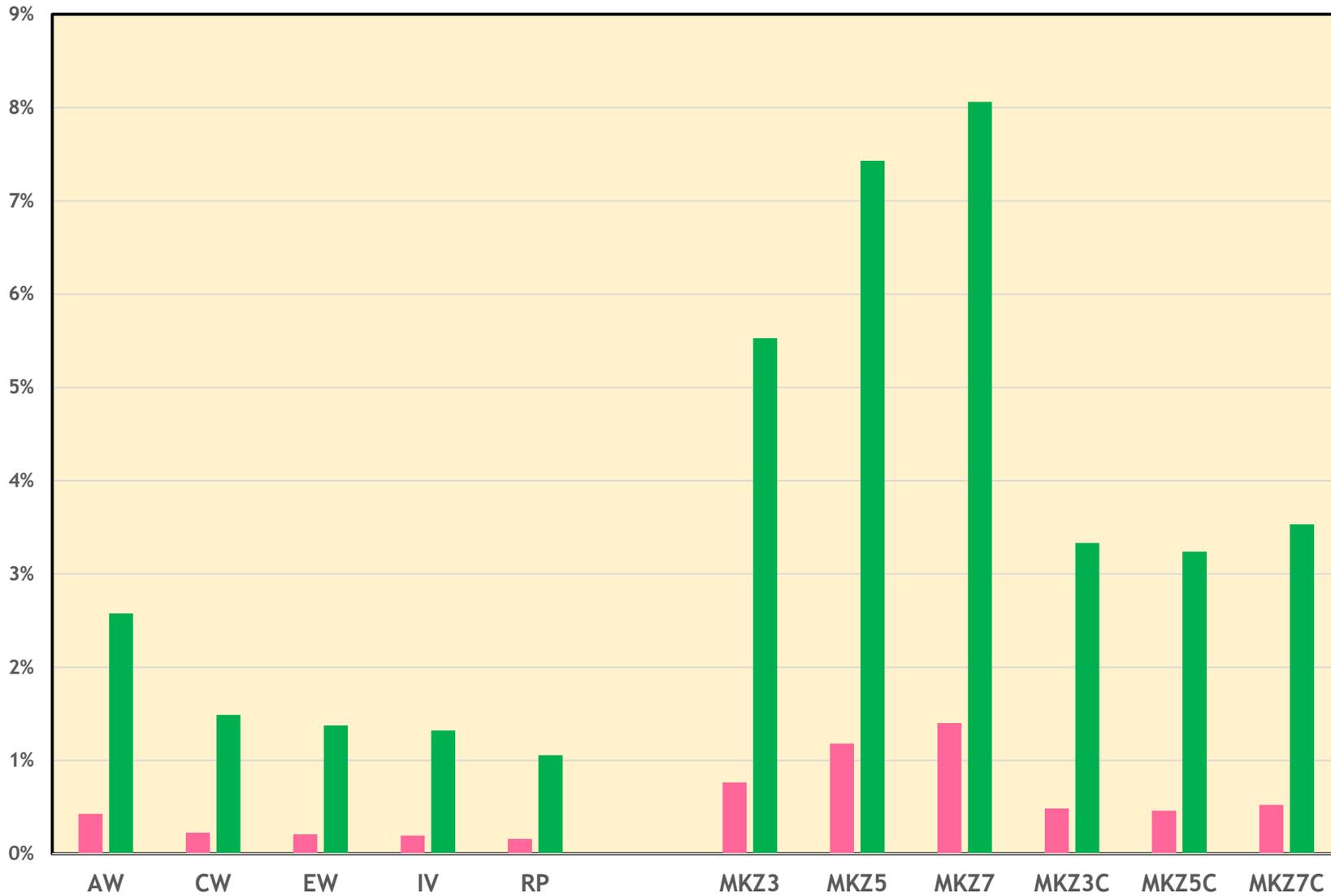
# VALUE Summary - Return vs Risk - Jan 2016 - Sep 2021

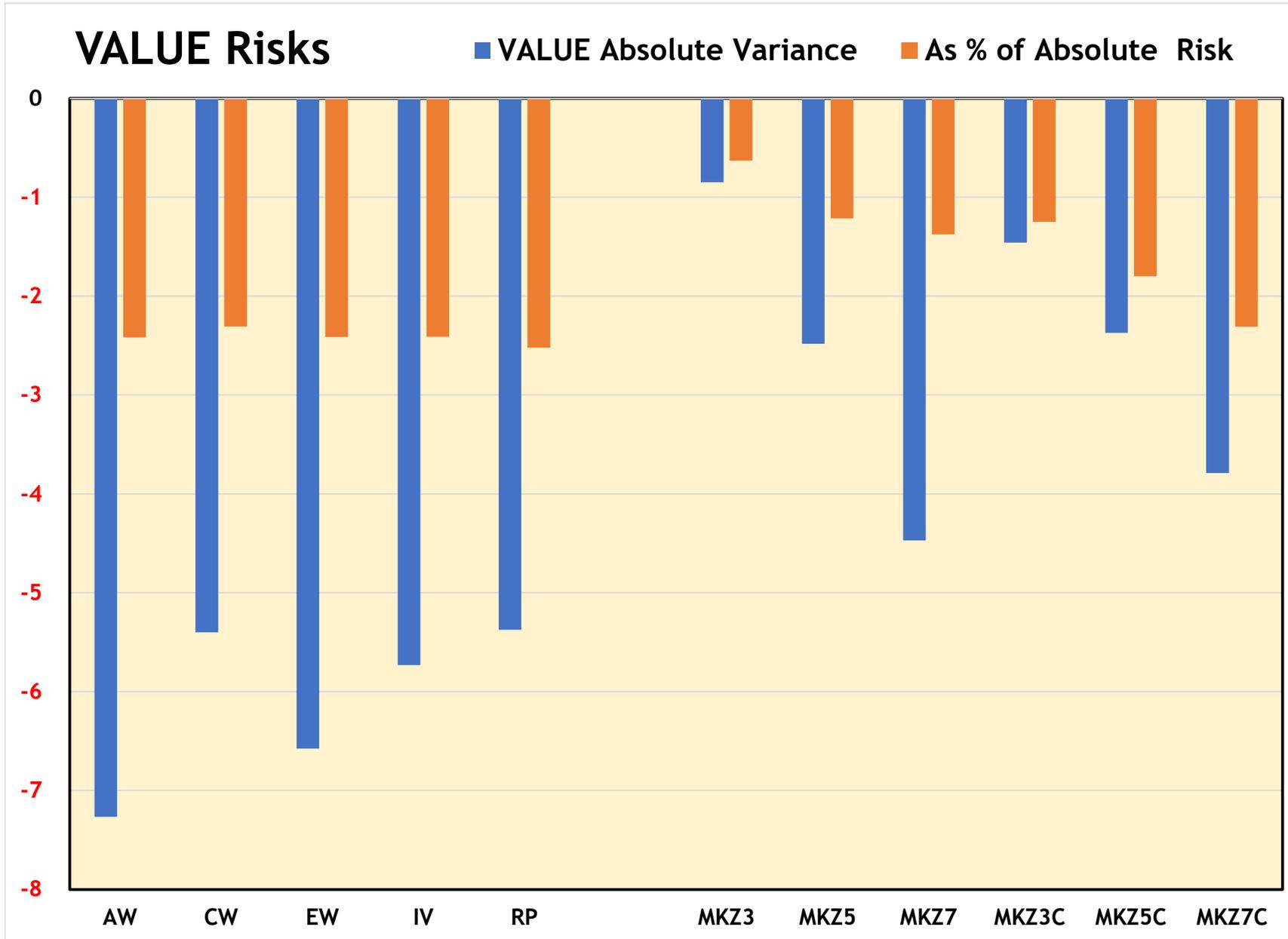


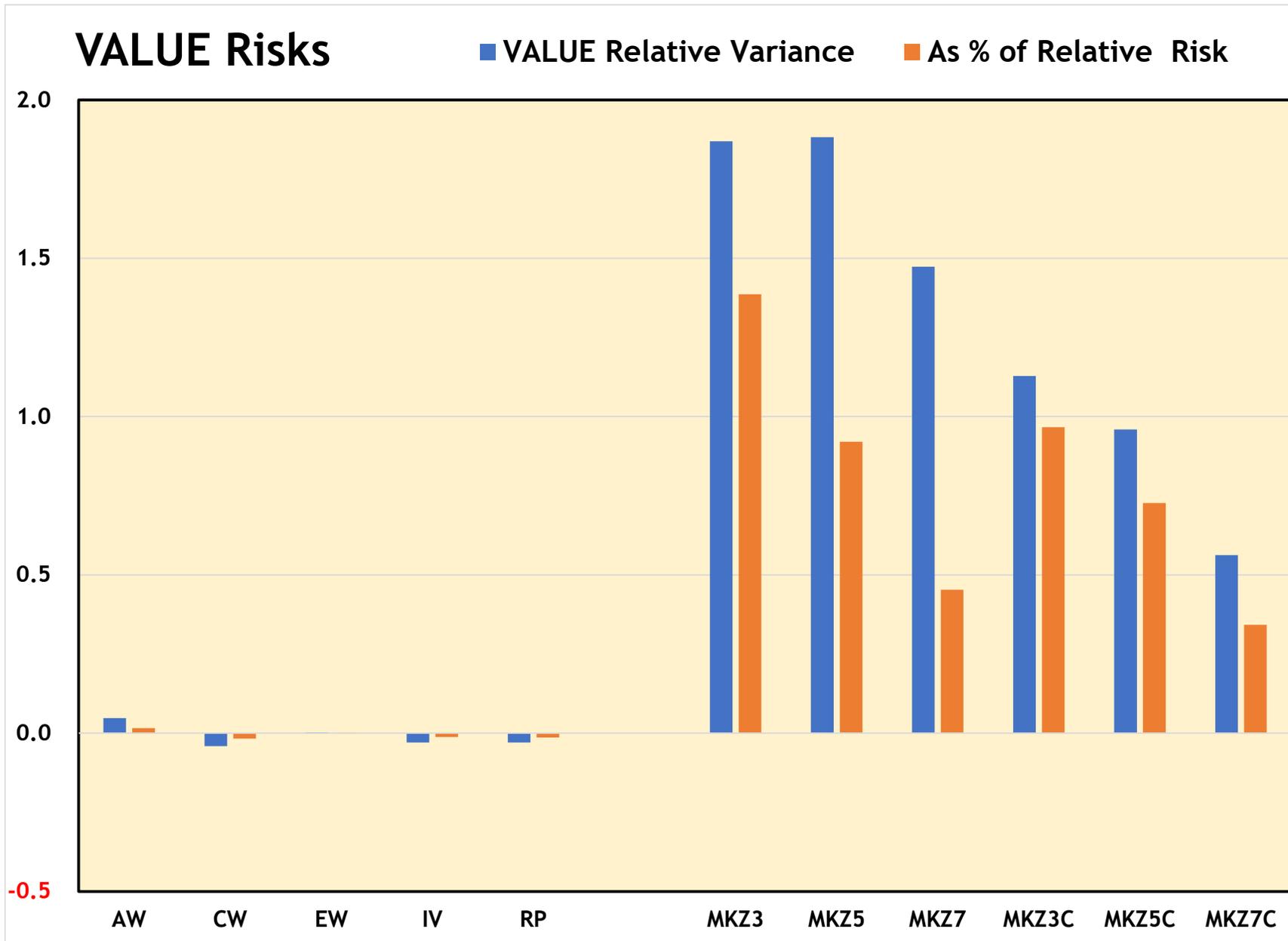


# VALUE Returns

■ VALUE Actual Return ■ As % of Total Return









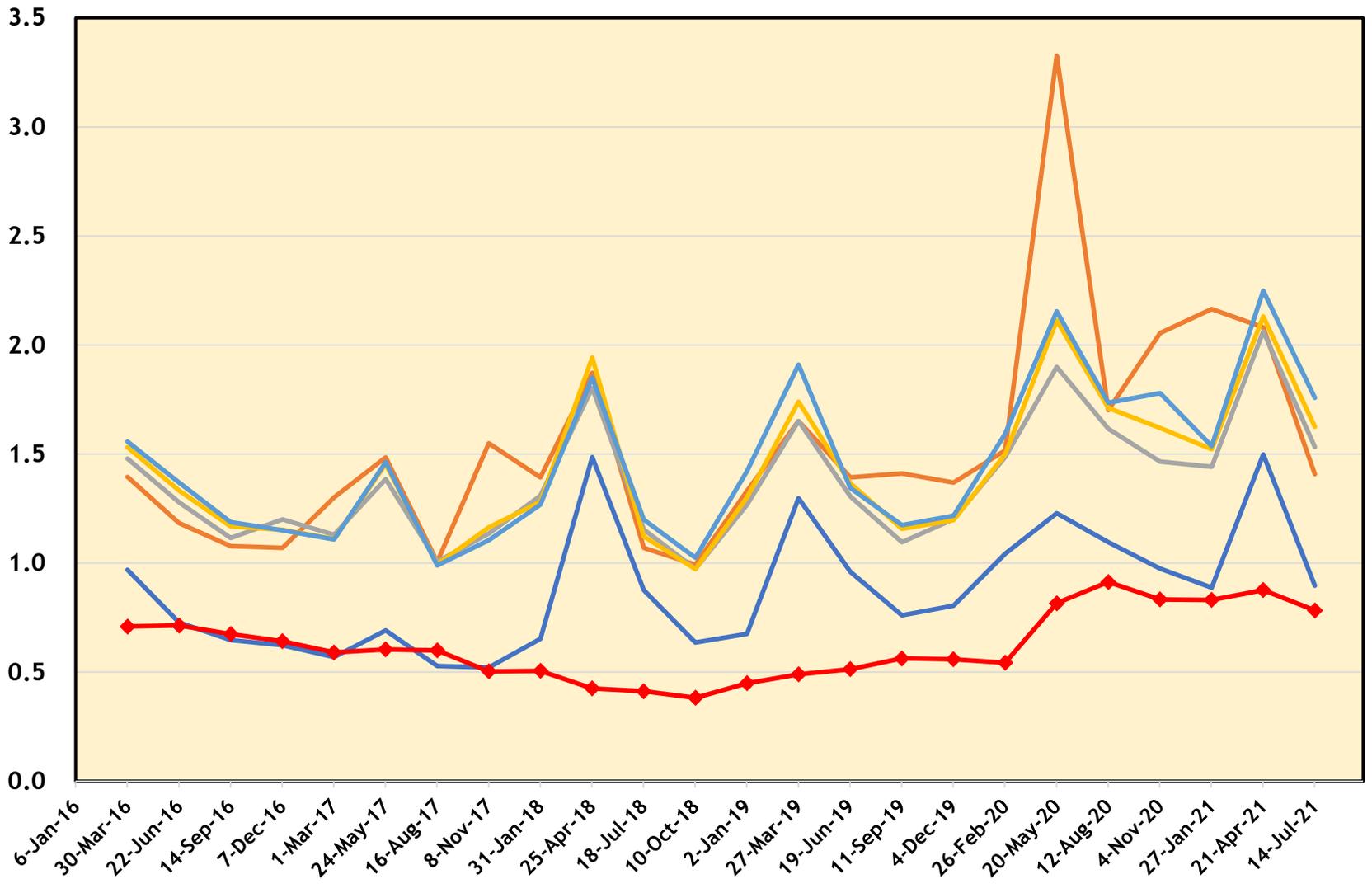
## Comments on VALUE Return & Risks

- The Returns chart shows very small contributions to portfolio return from the five heuristic strategies, very large contributions from unconstrained Markowitz strategies and still fairly substantial contributions from the constrained Markowitz strategies.
- The Absolute Risks chart shows that in the five heuristic strategies, the exposure to VALUE was actually diversifying quite a lot of the portfolio risk; in the Markowitz strategies, it had a smaller diversifying effect.
- The Relative Risks chart shows that the heuristic strategies had basically the same VALUE risk as the benchmark, whereas the Markowitz optimised strategies actually had modest positive contributions to Tracking Error.
- When looking at the percentage contributions, remember that these are percentages of different Tracking Errors, so they are not directly comparable with each other.



# HEURISTIC METHODS - PORTFOLIO INEFFICIENCY - VALUE

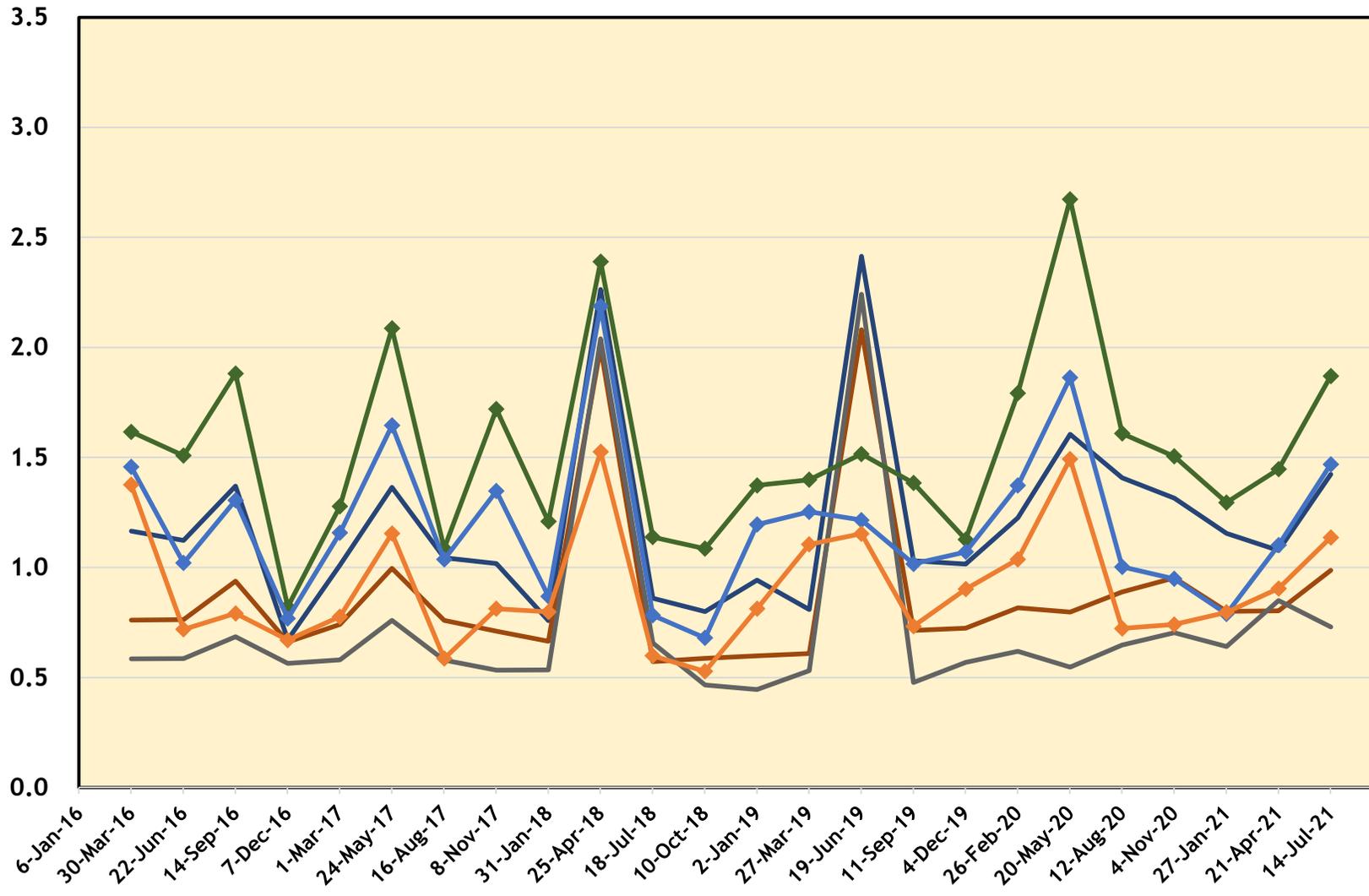
— AW    — CW    — EW    — IV    — RP    ◆ S&P





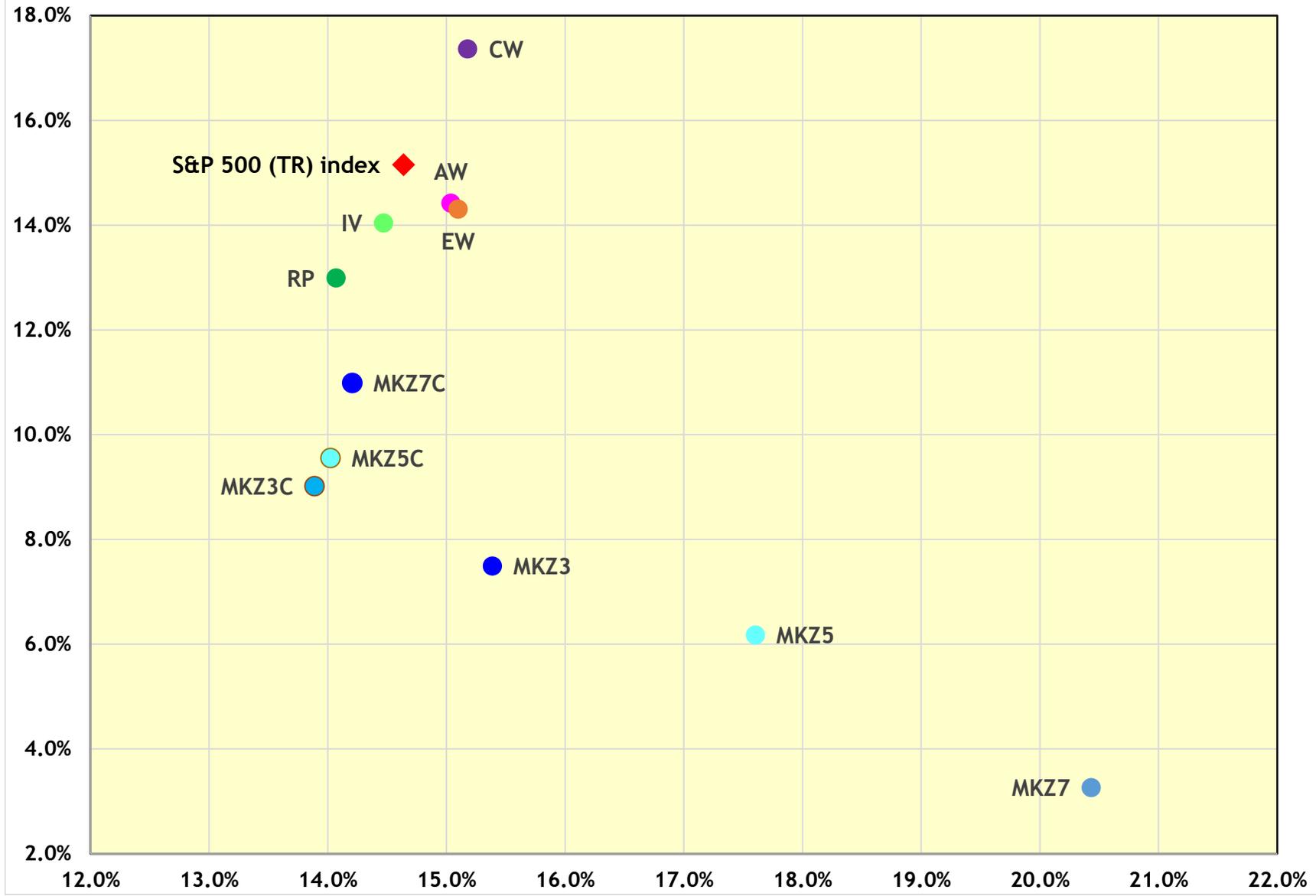
# MARKOWITZ OPTIMISATION - PORTFOLIO INEFFICIENCY - VALUE

—MKZ3 —MKZ5 —MKZ7 —MKZ3C —MKZ5C —MKZ7C





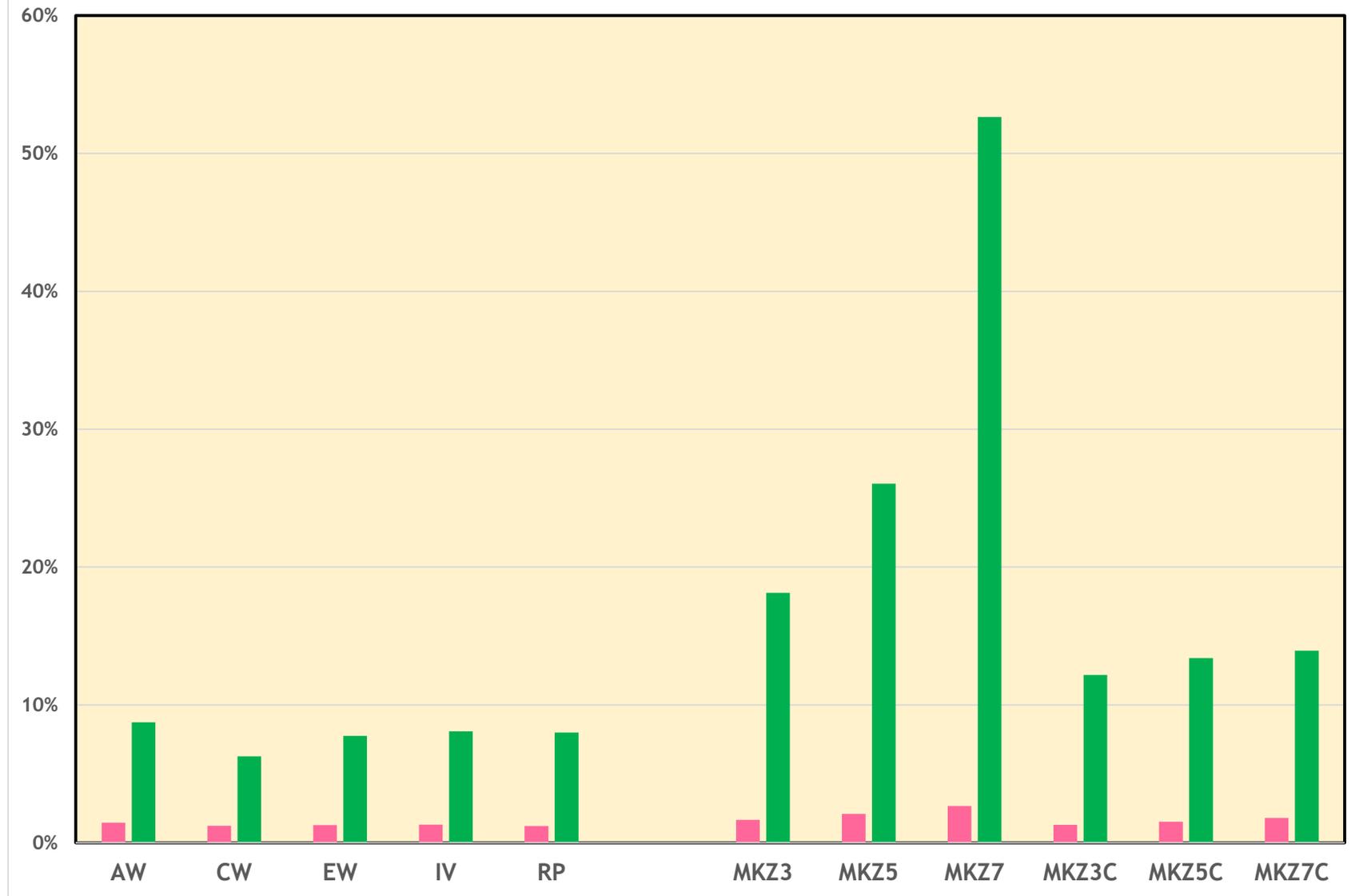
# QUALITY Summary - Return vs Risk - Jan 2016 - Sep 2021





# QUALITY Returns

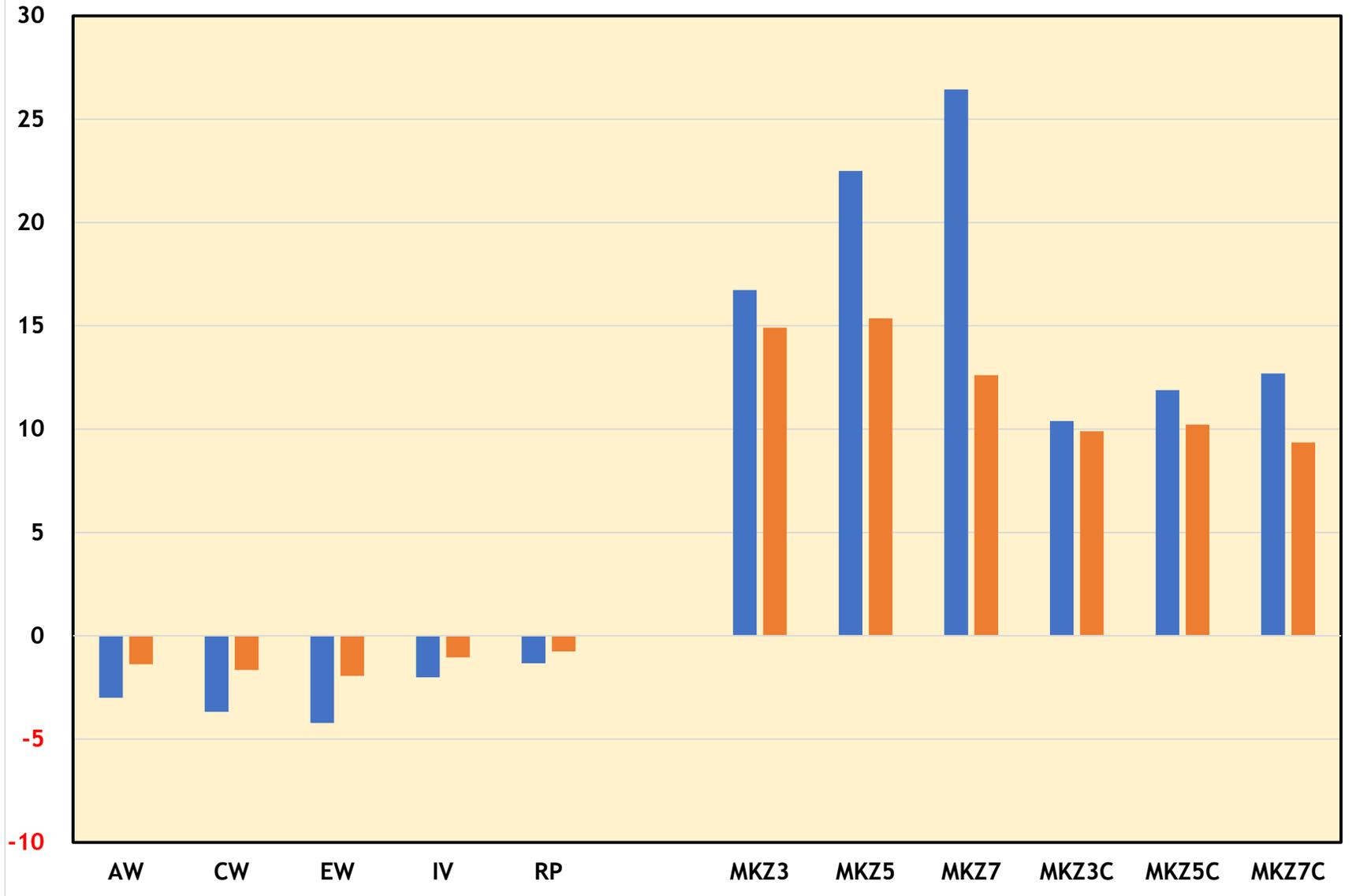
■ QUALITY Actual Return ■ As % of Total Return





# QUALITY Risks

■ QUALITY Absolute Variance ■ As % of Absolute Risk

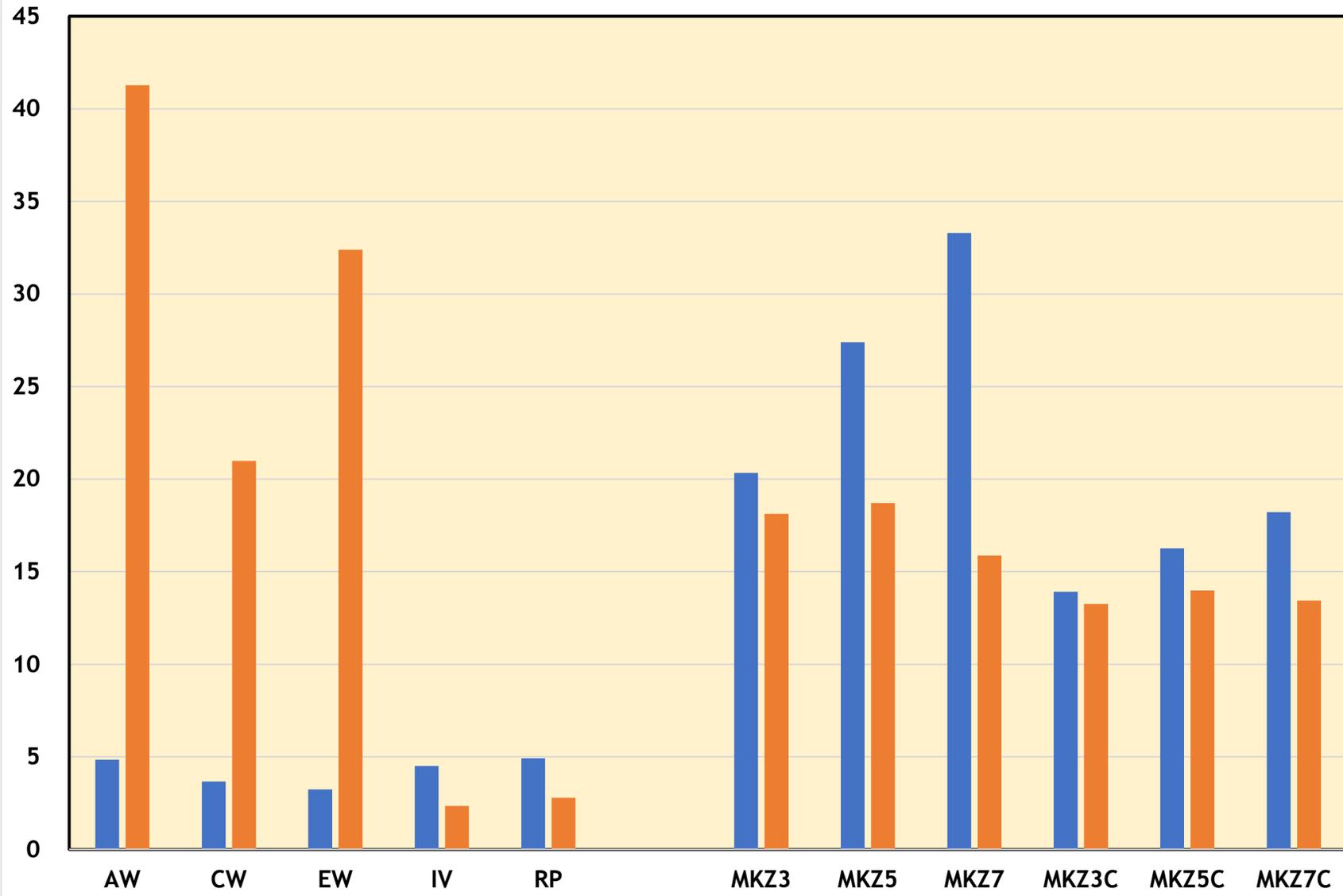




# QUALITY Risks

■ QUALITY Relative Variance

■ As % of Relative Risk





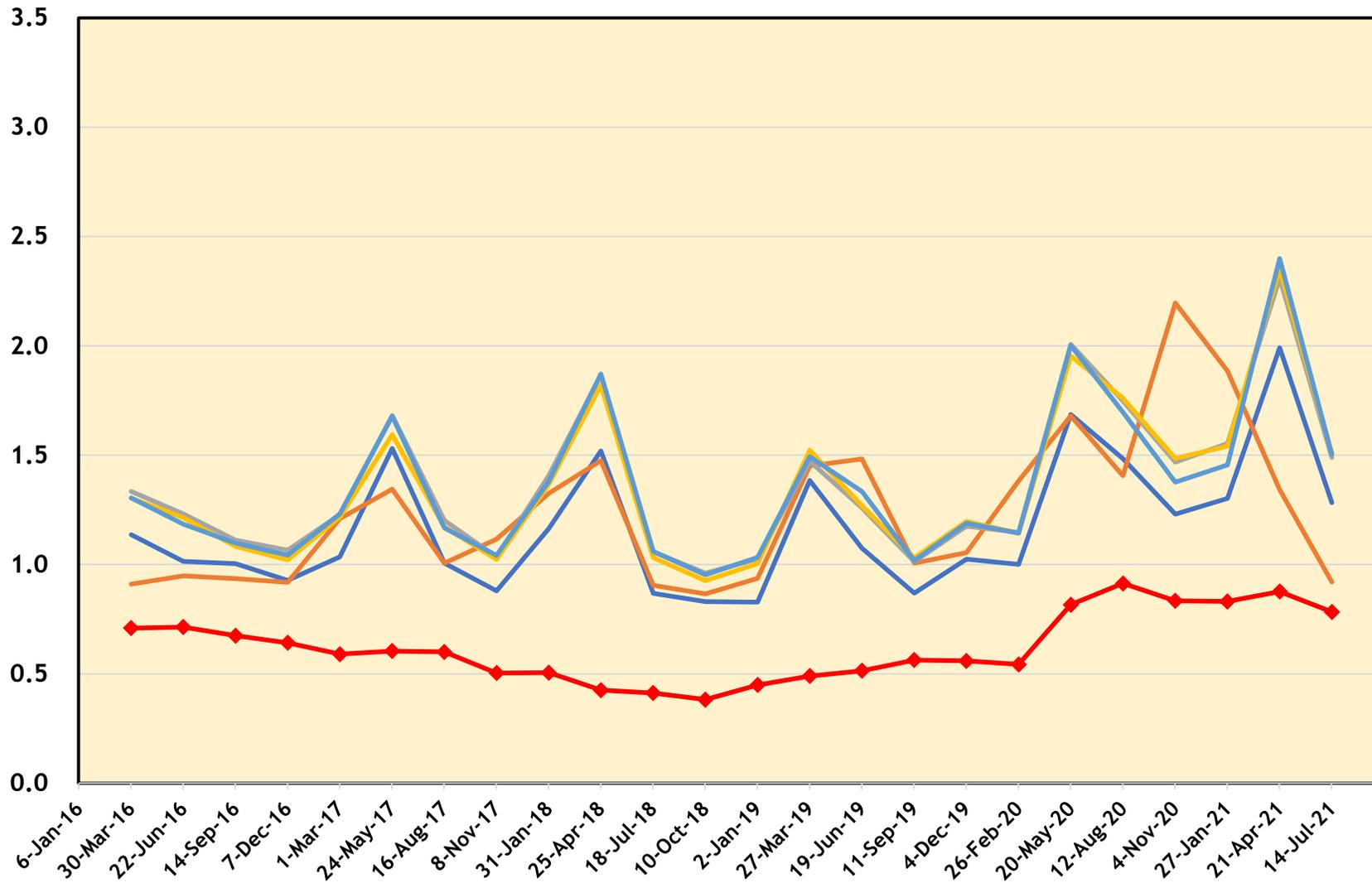
## Comments on QUALITY Return & Risks

- The Returns chart shows very small contributions to portfolio return from the five heuristic strategies, very large contributions from unconstrained Markowitz strategies and still substantial contributions from constrained Markowitz strategies.
- The Absolute Risks chart shows that in the five heuristic strategies, the exposure to QUALITY was actually diversifying some of the portfolio risk, whereas in the Markowitz strategies, it made a significant contribution.
- The Relative Risks chart shows that all strategies had more QUALITY risk than the benchmark, although again, the Markowitz optimised strategies had far more than the five heuristic strategies. When looking at the percentage contributions, remember that these are percentages of different Tracking Errors, so they are not comparable with each other.



# HEURISTIC METHODS - PORTFOLIO INEFFICIENCY - QUALITY

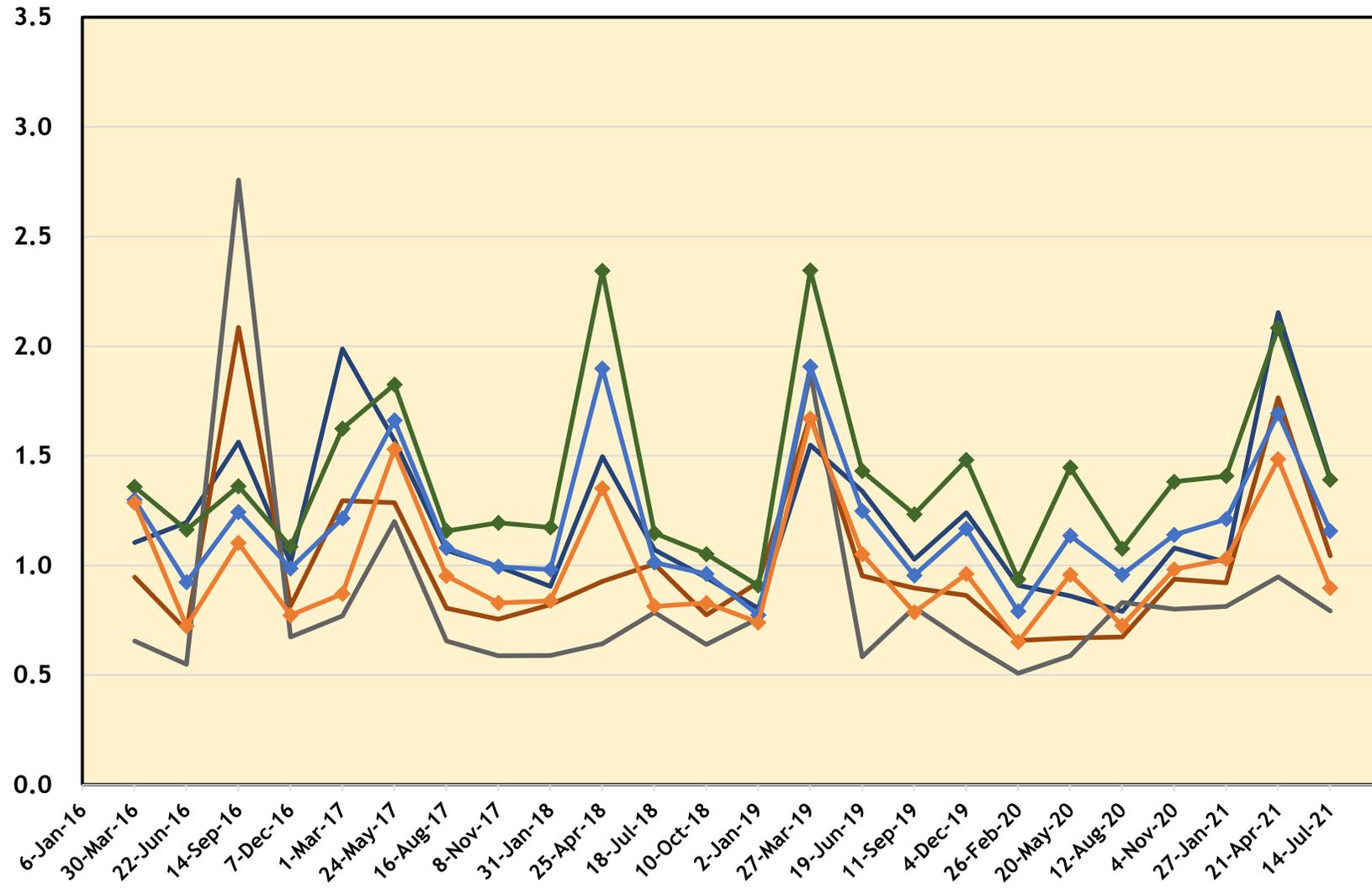
— AW    — CW    — EW    — IV    — RP    ◆ S&P 500





### MARKOWITZ OPTIMISATION - PORTFOLIO INEFFICIENCY - QUALITY

—MKZ3 —MKZ5 —MKZ7 —MKZ3C —MKZ5C —MKZ7C





# The S&P 500 (Capitalisation-Weighted) Index

- We also consider the S&P 500 index portfolio, which is capitalisation-weighted, and like most such indices, is basically unconstrained.
- Bear in mind Rob Arnott's observation that in a simple cap-weighted and unconstrained index, overpriced stocks will be overweight, and underpriced stocks will be underweight, thereby strongly suggesting that such a portfolio would be inefficient.
- This, of course, begs the question of whether there are any such things as overpriced or underpriced stocks anyway.
- Our focus is on the question of whether the S&P 500 really is an efficient portfolio: and if it is not, how inefficient is it?



# Testing the Inefficiency of the S&P 500

- In our review of Modern Portfolio theory above, we noted that the CAPM suggests that stocks' Expected Returns should be linearly related to their beta to the market, giving us the Security Market Line.
- In order to calculate Portfolio Inefficiency, we need to have a set of Expected Returns.
- In this case, the obvious Expected Return proxy would simply be the beta of each stock to the Market factor.
- This what we have used to calculate the Portfolio Inefficiency through this period of the S&P 500 portfolio.



## S&P 500 Portfolio Inefficiency - Comments

- As the charts show, the S&P 500 does appear to be somewhat inefficient, although not as much as quarterly rebalanced Style factor portfolios.
- Note the Market betas probably do not change as much as the Style betas.
- One might argue that the apparent Portfolio Inefficiency is a consequence of using estimated (and imperfect) Market betas and the estimated (and also imperfect) corresponding risk model.
- Or maybe the S&P 500, along with other capitalisation-weighted indices, really are inefficient portfolios.
- This will be our next avenue of research - Watch this space!
- And Thank You for your attention.



[www.smartportfoliostrategies.com](http://www.smartportfoliostrategies.com)

Email : [jasonmacqueen@smartportfoliostrategies.com](mailto:jasonmacqueen@smartportfoliostrategies.com)

Phone : 609 737 3576