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EXPLO:
A Unified Model of Investor Utility, Valuation, and Liquidity

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“..Do you want to be a Quant, or do you want to look like a Quant…”

~ *Paraphrase of Police Captain Charlie Queenan, “The Departed”* ~

Agenda:

- Provide the results of an Experiment that suggests a new and improved utility form
- Trace the roots of a Theory that may explain the results of the Experiment
- Incorporate additional elements of in investor behavior that make the assumptions of the Theory more realistic
- Build-up a new Utility function in a multi-period setting
- Examine the sources of investor risk aversion and linkage to liquidity requirements, leverage, and time horizon
- Outline applications

A Valuation Experiment:

- Observe three popular traded asset composites:
  - Shares Core S&P 500 ETF – from 2006.11 to 2020.06, monthly
  - iShares Russell 3000 ETF – from 2006.11 to 2020.06, monthly
  - iShares FTSE 250 UCITS ETF – from 2010.10 to 2020.06, monthly

- For each observe or estimate per point in time: *current annual dividend, average historical growth, historical standard deviation*

- Simulate the evolution of dividends over 20 years under 10^{50} paths and estimated the cumulative future value of dividends reinvested the composite's contemporaneous periodic returns

- Using the actual traded price P_{T-1} at time T-1 and corresponding statistics ${\mathrm{E}_{profit}}_{T-1}$ and ${\mathrm{E}_{loss}}_{T-1}$calculated using the simulation, infer a coefficient λ that satisfies :

$$P_{T-1} * [(1 + r_{T-1})^{20} - 1] = {\mathrm{E}_{profit}}_{T-1}(W - P_{T-1}) - \lambda_{T-1} * {\mathrm{E}_{loss}}_{T-1}(P_{T-1} - W)$$

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An Experiment (cont'd):

- Using λ from prior step and the statistics statistics E_{profit_T} and E_{loss_T} calculated for time T, compute an estimated price $P_T{}^*$ that satisfies:

$$P_T{}^* * [(1 + r_T)^{20} - 1] = \mathrm{E}_{profit_T}(W - P_T{}^*) - \lambda_{T-1} * \mathrm{E}_{loss_T}(P_T{}^* - W)$$

- At time T, compare the estimated price at time $P_T{}^*$ with the actual price P_T to evaluate the valuation model ability to precisely calculate the market price. Given the dependence of the model to Expected Profit and Loss we will call it *EXPLO*

- Using the actual traded price P_{T-1} at time T-1 and the corresponding statistics $E_{T-1}[W]$ and $Variance_{T-1}[W]$ calculated using simulation, infer a coefficient λ that satisfies the following equation. The equation can be shown to be equivalent to the mean variance certainty equivalent over the chosen time horizon T.

$$P_{T-1} * (1 + r_{T-1})^{20} = E_{T-1}[W] - \lambda_{T-1} * \frac{Variance_{T-1}[W]}{P_{T-1}}$$

An Experiment (cont'd):

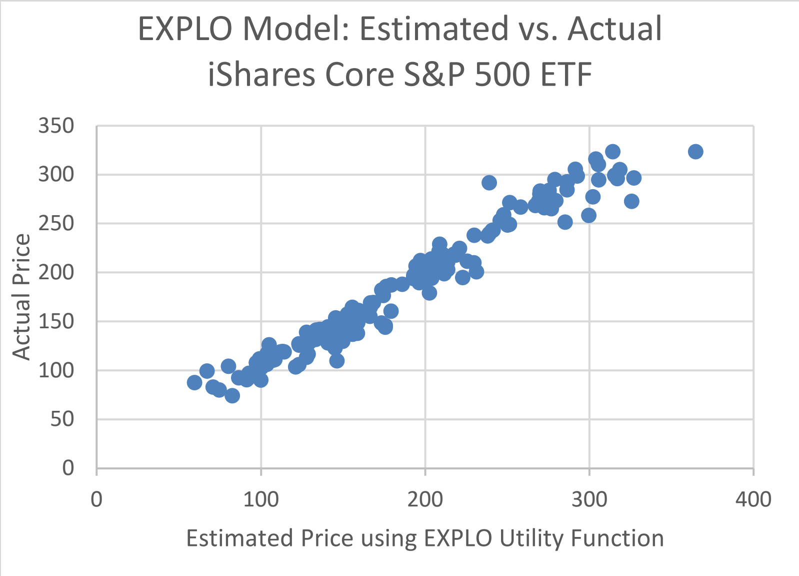
- Using λ from prior step and $E_T[W]$ and $Variance_T[W]$ calculated for time T, compute an estimated price ${P_T}^*$ that satisfies:

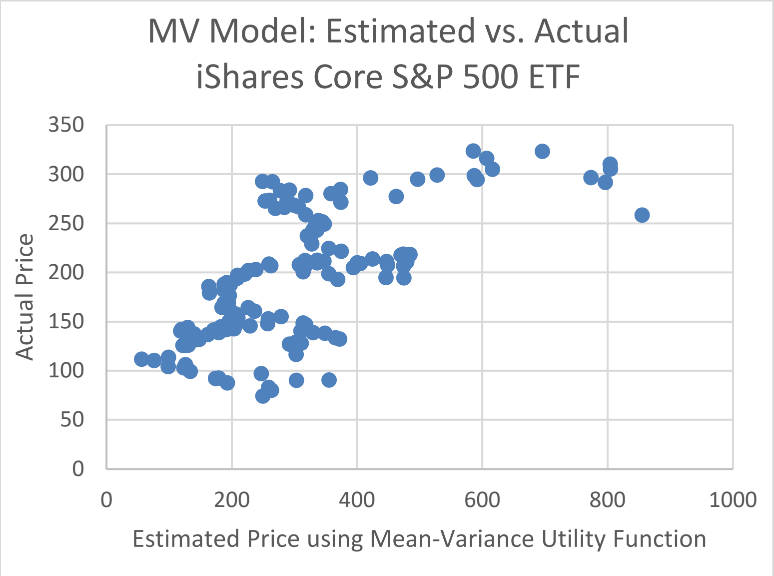
$$ {P_T}^* * (1 + r_T)^{20} = E_T[W] - \lambda_{T-1} * \frac{Variance_T[W]}{{P_T}^*} $$

- At time T, compare the estimate price at time ${P_T}^*$with the actual price P_T to evaluate the ***MV*** valuation model ability to precisely calculate the market price.

- The following examples demonstrate clearly that the EXPLO model dominates the MV model in the precision with which it estimates the market price of the investment instruments.

Results (A):

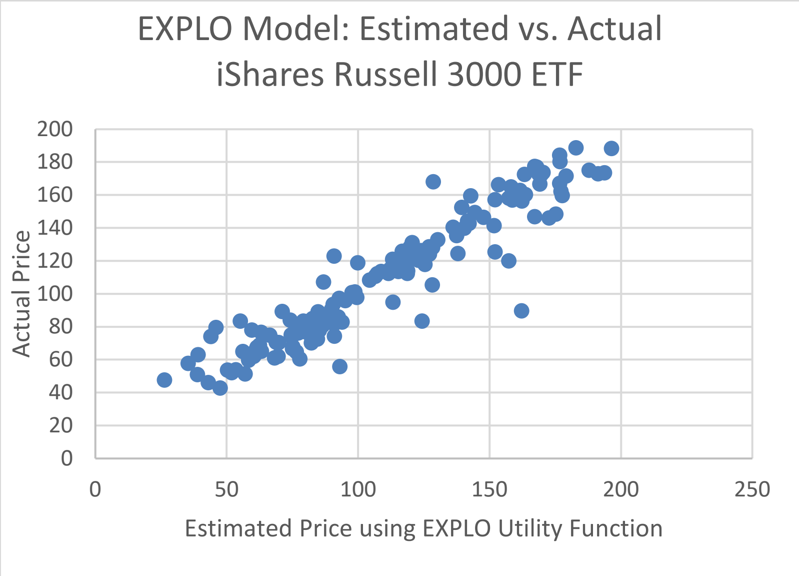


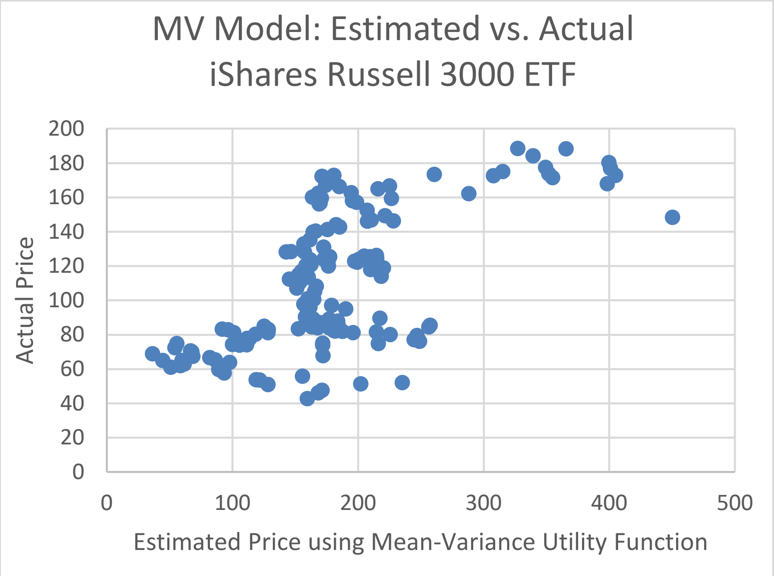


	Coefficients	Standard Error	t Stat	P-value
Intercept	7.84	3.15	2.49	0.01
Slope	0.95	0.02	59.56	0.00
R-Squared	0.96			

	Coefficients	Standard Error	t Stat	P-value
Intercept	107.57	8.88	12.12	0.00
Slope	0.28	0.03	10.79	0.00
R-Squared	0.45			

Results (B):

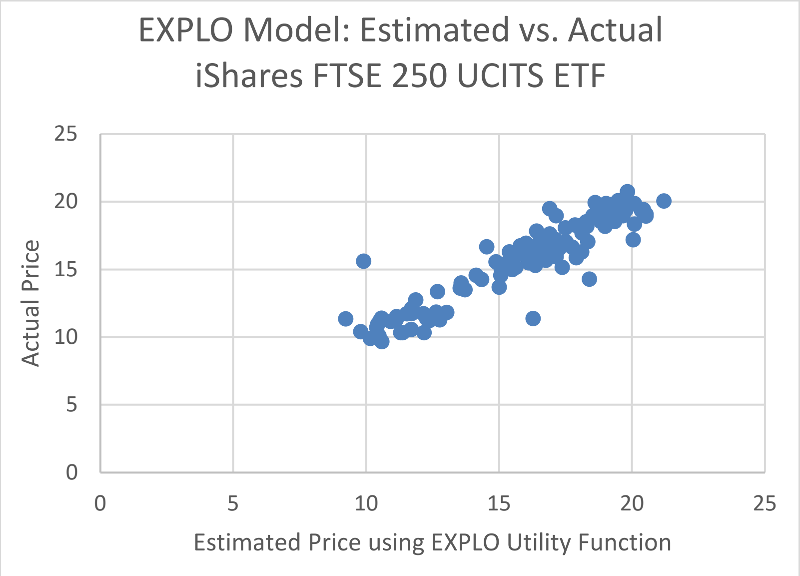


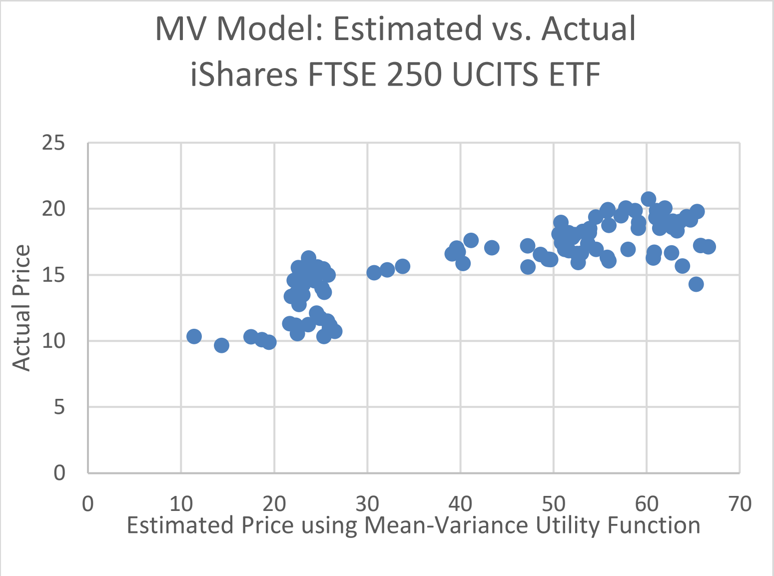


	Coefficients	Standard Error	t Stat	P-value
Intercept	11.47	2.76	4.16	0.00
Slope	0.89	0.02	37.47	0.00
R-Squared	0.90			

	Coefficients	Standard Error	t Stat	P-value
Intercept	50.42	6.20	8.14	0.00
Slope	0.33	0.03	10.42	0.00
R-Squared	0.41			

Results (C):





	Coefficients	Standard Error	t Stat	P-value
Intercept	1.35	0.57	2.37	0.02
Slope	0.90	0.04	25.67	0.00
R-Squared	0.85			

	Coefficients	Standard Error	t Stat	P-value
Intercept	9.92	0.43	22.95	0.00
Slope	0.14	0.01	15.27	0.00
R-Squared	0.69			

The Story of the Logarithm of Wealth

- In the 18th century Bernoulli proposed that utility from a quantity of a resource is based on a logarithmic function of that quantity
- In the 1950s, J.L. Kelly introduced the idea that the price of a bet is found by maximizing the expected value of the logarithm of wealth from the bet
- The underlying principle is that in the long run, maximizing the expectation of the logarithm of wealth maximizes the long term geometric return median return of a portfolio
- In this presentation we introduce another even more fundamental connection between the expectation of the log of wealth and the long term geometric return of a portfolio

Logarithm of Wealth (cont'd)

- One of the major criticism to an utility function simply based on maximizing the logarithm of wealth lacks an explicit reference to the risk aversion differentiating investors
- Some of the most pointed critiques of the "Kelly criterion" applied to investing was by Samuelson observing that Arrow-Pratt utility risk aversion goes to zero in the vast majority of cases
- The possibility of large losses that can lead to investor ruin, preventing the possibility of a long run "recoup" - survival in the short run is a necessary condition for the success of any long term strategy
- The presence of possible ruin outcomes is missing in the simple logarithmic utility

Logarithm of Wealth (cont'd)

- Various academics and practitioners have proposed ways to address some of the criticisms of an utility function based on expected logarithm of wealth
- Rubinstein (1976) develops a logarithmic utility model where the objective is to maximize residual wealth above a shortfall level
- Ziemba et al. (2006) suggests a "fractional" logarithmic utility model where the aberrations of the Kelly optimality are buffered with a proportion of a risk-free asset
- Wilcox (2003) builds on some of the concepts in Rubenstein's work, but recognizes that utility should maximize discretionary wealth, rather than residual or absolute wealth and derives coefficients of risk aversion that have direct linkage to investor leverage

Finite Time Determinacy Principle

- A fundamental observation is hiding in plain sight:
  - *The population of all possible outcomes is bound to occur over some long term horizon reflecting all outcomes' relative frequencies*
  - *We do not know how long this horizon is, and we don't know the order of occurrence of outcomes, but if we assume the existence of defined and unique distributional moments of all orders of population outcomes for a particular random variable, then this uniquely defines a vector of frequency of occurrence for all outcomes*
  - *The fact that the frequency of occurrence is defined also necessitates the existence of a minimum finite time horizon T_{min} over which the relative frequency of occurrence is bound to occur*

- The importance of this observations to log of wealth utility could not be exaggerated: ***The sum of the log of wealth population outcomes over the time horizon T_{min} results in the deterministic wealth accumulation over that time horizon, making the certain wealth accumulation over that horizon proportional to the first order expectation of the outcomes.***

Motivation

- The *Finite Time Determinacy Principle* can be thought in analogy to a very strong form of the Central Limit Theorem, and the log of wealth utility provides a clear and deterministic objective according to this principle

- Even when the time horizon implied by the finite time determinacy principle (unknown to investor) is too long, then the *Central Limit Theorem* will assure that maximizing the log of wealth will maximize the long term median wealth.

- The works of Wilcox and Rubinstein provide useful ideas that address some the limitations of the original log of wealth utility form and we can build on them to derive a new log of wealth utility form

- We will find that our Experiment provides strong support for a log of wealth utility model which we will describe next

Model Approach

- Similar to Wilcox we incorporate ***Leverage***. In our model leverage can be explicit or implicit
  - Explicit leverage is due to ***borrowing*** from an external lender
  - Implicit leverage is debt due to self – i.e. ***non-discretionary future expenses***
- In distinction to prior models, we introduce ***discretionary consumption*** as a proportion of periodic profits
- Introduce explicit reference to a ***defined time horizon*** when debt – explicit and implicit – is due to be redeemed fully
- Introduce the ***impact of possible default*** in each period as well as cumulatively over all time horizons

Model Approach (cont'd)

- Our assumption is that the outcomes of the log of wealth as measured by an unlevered asset portfolio W are independent and identically distributed in each period
- There is no limitation to a certain type of statistical distribution of **ln(**W**)** in each period
- A proportion of profits φ is used for discretionary consumption in each period, and the remainder of profits $(1-\varphi)$ is kept in the portfolio to be reinvested for subsequent periods
- After a default in a single period, all investor equity is assumed lost and no further reinvestment of any amount is assumed to occur in future periods
- The maximum period at which liabilities are due is denoted by τ

Model Approach (cont'd)

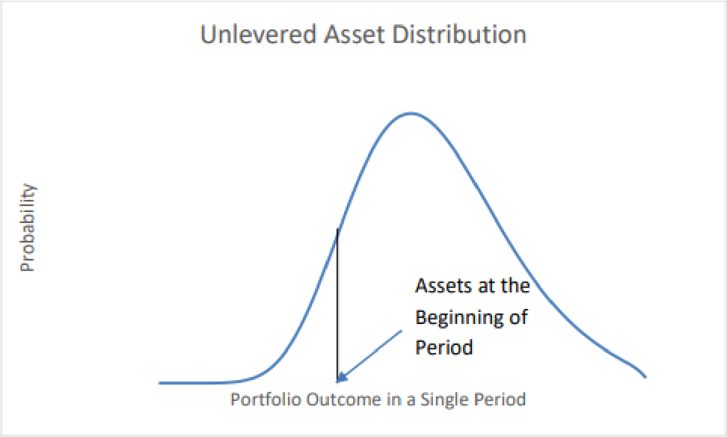
- In our model, leverage **L** is not a simple multiplier of an unlevered asset. It is thought of as a the maximum admissible level of investor leverage to which, after each periodic outcome, investor assets are brought back by selling or acquiring a portion of the same portfolio depending on whether the outcome was a loss or a gain

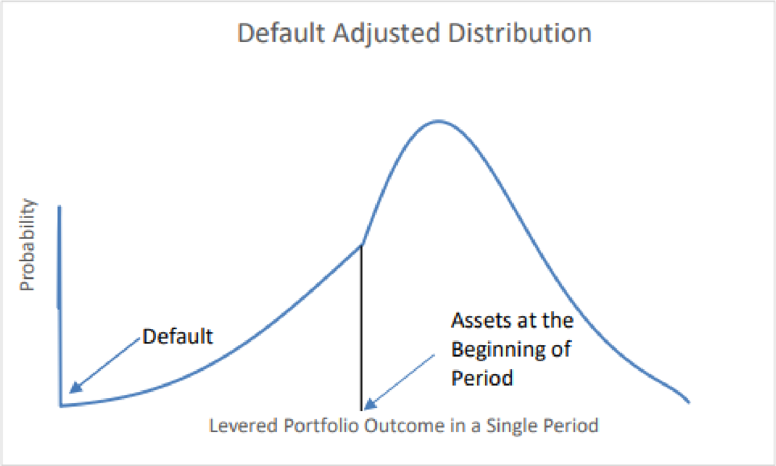
- This has the effect to stretch asset outcomes for the levered asset after each periodic realization with respect the corresponding unlevered assets

- Due to fact that some of the gains are used for discretionary consumption there is an asymmetric behavior of outcomes, even if both gains and losses result in re-levering of investor equity back to **L**

- The separate effects of stretching outcomes and default impact results in a distribution which we will call **Default-Adjusted-Distribution of Assets (DADA)** demonstrated in the next slide

Default-Adjusted-Distribution of Assets (DADA)





First Period Outcome Terms

- For first period we have:

$$E[S_1] = \sum_{i=1}^{N} f_i\left(W_{1_i}\right) * S_{1_i}$$

$$\sum_{i=1}^{N} f_i\left(W_{1_i}\right) = 1$$

- *Type 1* outcomes - ***Gains***:

$$S_{1_i} = \ln\left[L * (1-\phi) * \left(L * W_{1_i} - LP\right) + LP\right] - \ln(LP) \qquad when \qquad LW_{1_i} \in [LP; +\infty)$$

- *Type 2* outcomes – ***Non-default Losses***:

$$S_{1_i} = \ln\{L * [LW_{1_i} - P * (L-1)]\} - \ln(LP) \qquad when \qquad LW_{1_i} \in (P(L-1); LP)$$

- *Type 3* outcomes – ***Default Losses***:

$$S_{1_i} = \ln\left(P(L-1)\right) - \ln(LP) \qquad when \qquad LW_{1_i} \in (0; P(L-1))$$

Interim Period Outcome Terms

- For subsequent periods probabilities and expectations should be adjusted for the Cumulative Probability of Default in prior periods:

$$E[S_k] = (1 - CPD_{k-1}) * \sum_{i=1}^{N} f_i(W_{1_i}) * S_{k_i}$$

- *Due to the IID properties of ln(W) and the constant level of leverage we can write period k terms of S in terms of first period notation:*

- ***Type 1** outcomes - **Gains**:*

$$S_{k_i} = \ln\left[L * (1 - \phi) * \left(L * W_{1_i} - LP\right) + LP\right] - \ln(LP) \qquad when \qquad LW_{1_i} \in [LP; +\infty)$$

- ***Type 2** outcomes – **Non-default Losses**:*

$$S_{k_i} = \ln\{L * [LW_{1_i} - P * (L - 1)]\} - \ln(LP) \qquad when \qquad LW_{1_i} \in (P(L-1); LP)$$

- ***Type 3** outcomes – **Default Losses**:*

$$S_{k_i} = \ln\left(P(L - 1)\right) - \ln(LP) \qquad when \qquad LW_{1_i} \in (0; P(L-1))$$

Final Period Outcome Terms

- For subsequent periods probabilities and expectations should be adjusted for the Cumulative Probability of Default in prior periods:

$$E[S_\tau] = (1 - CPD_{\tau-1}) * \sum_{i=1}^{N} f_i(W_{1_i}) * S_{\tau_i}$$

- *Type 4* outcomes – ***Gains and Non Default Losses***:

$$S_{\tau_i} = \ln[LW_{1_i} - P * (L-1)] - \ln(LP) \qquad when \qquad LW_{1_i} \in (P(L-1); +\infty)$$

- *Type 5* outcomes – ***Default Losses***:

$$S_{\tau_i} = \ln(P(L-1)) - \ln(LP) \qquad when \qquad LW_{1_i} \in (0; P(L-1))$$

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All Together

- Due to the independence of the outcomes S_k in each period, after controlling for the probability of default, we can write:

$$E[Portfolio\ log - return\ over\ \tau\ periods]$$
$$= \sum_{k=1}^{\tau} E[S_k] = \sum_{i=1}^{N} f_i\left(W_{1_i}\right) * S_{1_i} + \left\{\sum_{k=2}^{\tau}\left[(1 - CPD_{k-1}) * \sum_{i=1}^{N} f_i\left(W_{1_i}\right) * S_{k_i}\right]\right\}$$

- In this expression the Cumulative Probability of Default $\boldsymbol{CPD_{k-1}}$ can be formulated recursively using the relationships:

$$CPD_{k-1} = CPD_{k-2} + [(1 - CPD_{k-2}) * p_d]$$

$$p_d = \sum_{i=1}^{N} f\left(W_{1_i}\right) * 1\left[LW_{1_i} \in (P(L-1); 0)\right]$$

- In the last expression $\boldsymbol{p_d}$ is the single period probability of default which is constant in all periods:

Maximizing Expectation Using Probabilities

- Maximizing expectations treating probabilities like variables and outcomes as fixed quantities:

$$E[X_i] = f_E(p_1, p_2, p_3 \dots, p_i) = \sum_{i}^{MAX} X_i * p_i$$

$$\Delta f_E(p_1, p_2, p_3 \dots, p_i) = \frac{df_E}{dp_1}\Delta p_1 + \frac{df_E}{dp_2}\Delta p_2 + \frac{df_E}{dp_3}\Delta p_3 \dots \frac{df_E}{dp_i}\Delta p_i$$

$$\Delta f_E(p_1, p_2, p_3 \dots, p_i) = X_1\Delta p_1 + X_2\Delta p_2 + X_3\Delta p_3 \dots X_i\Delta p_i$$

- If Δp_i If is the smallest possible increment or decrement in a p_i variable, a change Δp_i in $p_i$requires an equal in size and opposite in sign change i.e. -Δp_j in *one and only one* such other probability variable p_j

- An increase of the expectation entails a decrease Δp in the probability associated with a lower value of X_i and a simultaneous increase Δp in a higher value X_j

Maximizing DADA Expectation vs. p_d

- In the expectation $E[S_k]$, the outcomes of default:

$$S_{k_i} = \ln\big(P(L-1)\big) - \ln\,(LP) \qquad when \qquad LW_{1_i} \in (P(L-1);0)$$

provides a floor of the portfolio distribution at a complete loss of equity equal to:

$$\ln\big(P(L-1)\big) - \ln\,(LP)$$

- Given that the region of default for S_{k_i} is the lowest possible value of the portfolio distribution in a single period, all of the shifts that maximize the expectation of DADA will ether move probability out of the default outcome or leave that probability unchanged.

Maximizing DADA Expectation vs. CPD

- Given that maximizing the DADA expectation also minimizes p_d, we are interested on its effect on CPD for each period:

- It is intuitive, but it can be shown that CPD in each period is a increasing function with respect to p_d, so therefore any objective that minimizes the latter will also minimize the former.

- Therefore in the following expression the terms (1-CPD) can be dropped because they are not affecting the maximization path of the expression:

$$E[Portfolio\ log-return\ over\ \tau\ periods]$$
$$= \sum_{k=1}^{\tau} E[S_k] = \sum_{i=1}^{N} f_i(W_{1_i}) * S_{1_i} + \left\{ \sum_{k=2}^{\tau} \left[(1 - CPD_{k-1}) * \sum_{i=1}^{N} f_i(W_{1_i}) * S_{k_i} \right] \right\}$$

- This results in the following objective function:

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]} = \sum_{k=1}^{\tau} E[S_k] = \sum_{k=1}^{\tau} \sum_{i=1}^{N} f_i(W_{1_i}) * S_{k_i}$$

Simplifying the Utility Function

- Recognizing the different types of S_k outcomes they can be grouped as follows:

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]} = \sum_{k=1}^{\tau} E[S_k]$$

$$= \sum_{k=1}^{\text{Type 4}} f_i(W_{1_i}) * S_{k_i} + (\tau - 1) * \left(\sum_{k=1}^{\text{Type 1}} f_i(W_{1_i}) * S_{k_i} \right) + \tau * \left(\sum_{k=1}^{\text{Type 2} \cup \text{Type 3}} f_i(W_{1_i}) * S_{k_i} \right)$$

- The next step is to present each of the types of S_k in a more convenient form

Transforming Logarithmic Terms

- It can be shown the Taylor series of each of the Types 1,2,3, and 4, and consequently taking their expectations results in :

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]}$$
$$= \mathrm{E}_{Type\ 4}\left\{L\left[\frac{(W_1-P)}{P}\right]\right\} - \mathrm{E}_{Type\ 4}\left\{L^2\left[\frac{(W_1-P)^2}{2*P^2}\right]\right\} + \mathrm{E}_{Type\ 4}\left\{L^3\left[\frac{(W_1-P)^3}{3*P^3}\right]\right\} \ldots + (\tau-1)$$
$$*\left(\mathrm{E}_{Type\ 1}\left\{[L*(1-\phi)]\left[\frac{(W_1-P)}{P}\right]\right\} - \mathrm{E}_{Type\ 1}\left\{[L*(1-\phi)]^2\left[\frac{(W_1-P)^2}{2*P^2}\right]\right\}\right.$$
$$\left.+\mathrm{E}_{Type\ 1}\left\{[L*(1-\phi)]^3\left[\frac{(W_1-P)^3}{3*P^3}\right]\right\} \ldots\right) + (\tau)$$
$$*\left(\mathrm{E}_{Type\ 2\ \&\ 3}\left\{L\left[\frac{(W_1-P)}{P}\right]\right\} - \mathrm{E}_{Type\ 2\ \&\ 3}\left\{L^2\left[\frac{(W_1-P)^2}{2*P^2}\right]\right\} + \mathrm{E}_{Type\ 2\ \&\ 3}\left\{L^3\left[\frac{(W_1-P)^3}{3*P^3}\right]\right\} \ldots\right)$$

Transforming Logarithmic Terms (cont'd)

- Recognizing that some of the terms represent gains and some losses, they can be further grouped in the following fashion:

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]}$$

$$= \mathrm{E}_{Profit}\left\{\{L + (\tau - 1) * [L * (1 - \phi)]\}\left[\frac{(W_1 - P)}{P}\right]\right\}$$

$$- \mathrm{E}_{Profit}\left\{\{L^2 + (\tau - 1) * [L * (1 - \phi)]^2\}\left[\frac{(W_1 - P)^2}{2 * P^2}\right]\right\}$$

$$+ \mathrm{E}_{Profit}\left\{\{L^3 + (\tau - 1) * [L * (1 - \phi)]^3\}\left[\frac{(W_1 - P)^3}{3 * P^3}\right]\right\} \ldots + \mathrm{E}_{Loss}\left\{L(\tau)\left[\frac{(W_1 - P)}{P}\right]\right\}$$

$$- \mathrm{E}_{Loss}\left\{L^2(\tau)\left[\frac{(W_1 - P)^2}{2 * P^2}\right]\right\} + \mathrm{E}_{Loss}\left\{L^3(\tau)\left[\frac{(W_1 - P)^3}{3 * P^3}\right]\right\} \ldots$$

Simplifying Utility (cont'd)

- Taking only the first order terms :

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]}$$
$$= \{L + (\tau - 1) * [L * (1 - \phi)]\} * \mathrm{E}_{Profit}\left\{\left[\frac{(W_1 - P)}{P}\right]\right\} + L(\tau) * \mathrm{E}_{Loss}\left\{\left[\frac{(W_1 - P)}{P}\right]\right\}$$

- Dividing both sides by a constant:

$$U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]}$$
$$= \mathrm{E}_{Profit}(W_1 - P) - \frac{L(\tau)}{L\{1 + (\tau - 1)(1 - \phi)\}} * \mathrm{E}_{Loss}(P - W_1)$$

- Which results in the *EXPLO* utility formula shown earlier:

$$\boldsymbol{U_{E[Portfolio\ log-rerurn\ over\ \tau\ periods]} = E_{Profit}(W_1 - P) - \lambda * E_{Loss}(P - W_1)}$$

- Where:

$$\lambda = \frac{(\tau)}{\{1 + (\tau - 1)(1 - \phi)\}}$$

Expected Loss and The Effect of Leverage

- In the loss aversion expression we see that the variables that play a role are the time horizon and proportion of profit for discretionary consumption:

$$\lambda = \frac{(\tau)}{\{1 + (\tau - 1)(1 - \phi)\}}$$

- The Leverage constant L has disappeared from the expression of utility. Does leverage not play an effect in the utility form?

- The answer is that it actually does play a role, and a significant one at that.

- When we consider the expected loss in the EXPLO utility formula – i.e.:

$$E_{Loss}(P - W_1)$$

this is not the expected loss of the original unlevered asset, but one that has been modified by the effect of leverage on the levered portfolio through default and the total loss of equity for the investor in the event of default

Default-Adjusted-Distribution of Equity (DADE)

- We may recall that the type of outcomes:

$$S_{k_i} = \ln\big(P(L-1)\big) - \ln(LP) \qquad when \qquad LW_{1_i} \in (P(L-1);0)$$

put a lower bound on the outcomes for assets in the event of default.

- The $P(L-1) = Debt$ is the point, in asset value space, where the investor will have used all of its equity to cover losses before the asset losses start to be covered by the lender

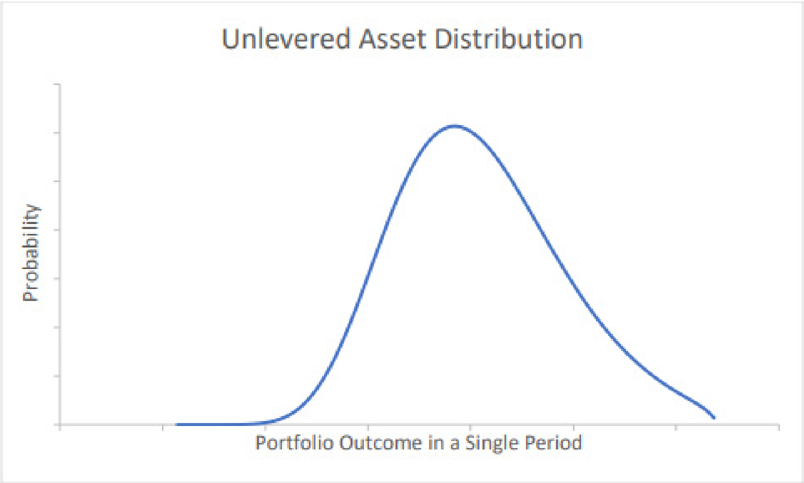
- In investor equity value space this point occurs when:

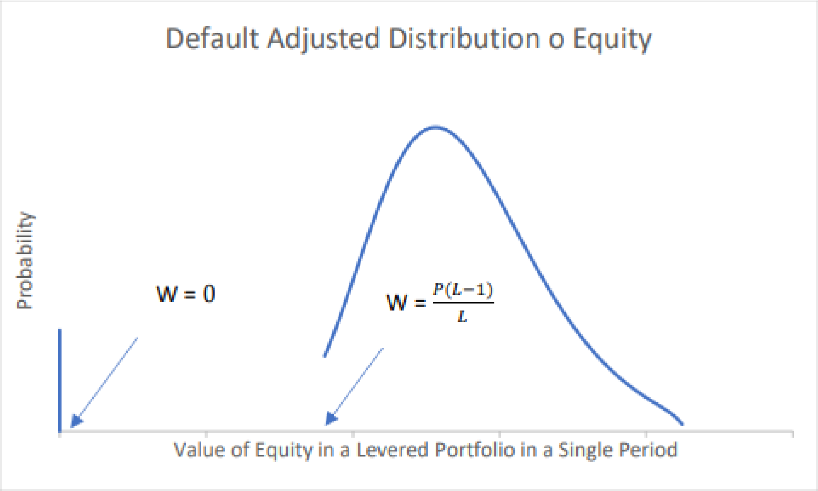
$$W = \frac{P(L-1)}{L}$$

- At this point and all lower points from the original unlevered asset distribution, the levered investor will get a zero outcome in equity space. In other words the investor expected loss gets increased by the value of W under the cutoff default point:

$$E_{Loss}(P-W) = \int_0^P (P-W) * f(W)dW + \int_0^{\frac{P(L-1)}{L}} (W) * f(W)dW$$

Default-Adjusted-Distribution of Equity (DADE)





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Expected Loss Revisited

- It can be shown that given:

$$E_{Loss}(P-W) = \int_0^P (P-W) * f(W)dW + \int_0^{\frac{P(L-1)}{L}} (W) * f(W)dW$$

after a proportional change in **W** and keeping leverage constant, the ratio:

$$\theta = \frac{\int_0^{\frac{P(L-1)}{L}} (W) * f(W)dW}{\int_0^P (P-W) * f(W)dW}$$

also remains constant. Therefore:

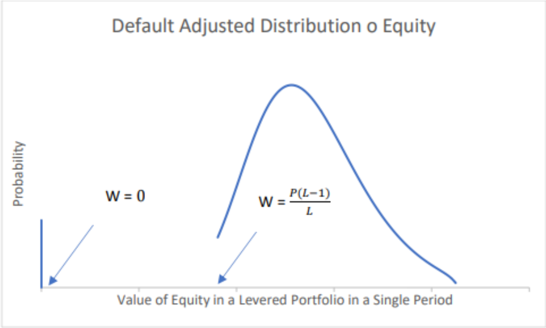
$$(1+\theta)\int_0^P (P-W) * f(W)dW = (1+\theta) * E_{Loss\_unlevered}(P-W) = E_{Loss}(P-W)$$

- Reflecting this in the prior EXPLO utility formula:

$$\boldsymbol{U_{EXPLO} = E_{Profit}(W_1 - P) - \lambda * (1+\theta) * E_{Loss\_unlevered}(P - W_1)}$$

Expected Loss Revisited

- The revised Expected Loss introduces a substantial impact of negative skew and kurtosis due to leverage:



- The ratio:

$$\theta = \frac{\int_0^{\frac{P(L-1)}{L}} (W) * f(W) dW}{\int_0^P (P - W) * f(W) dW}$$

can be significant in cases of high leverage and unlevered distributions with naturally high negative skew and kurtosis.

Revised Loss Aversion

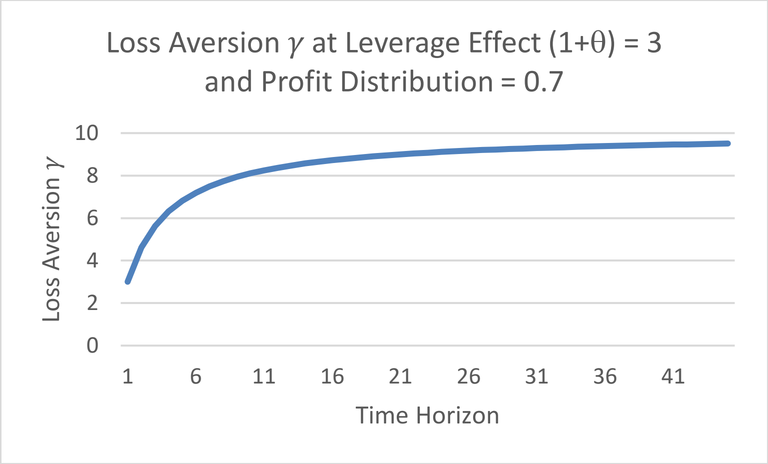
- Based on the latest formulation of the EXPLO Utility, we can express the revised loss aversion coefficient as:

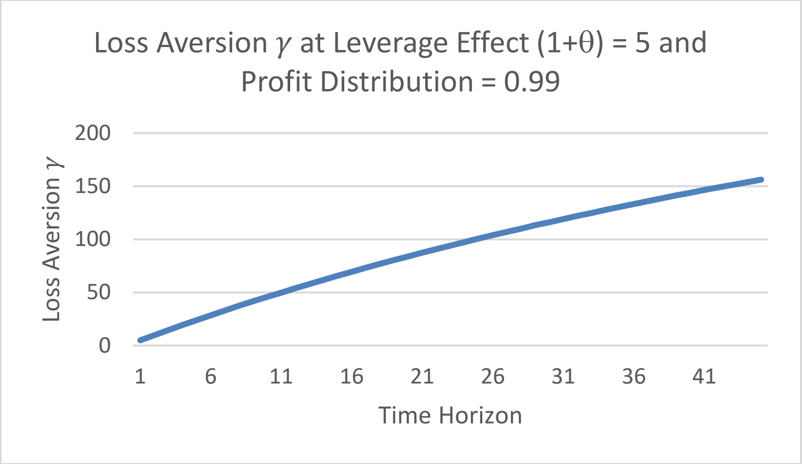
$$\gamma = \frac{(1+\theta)(\tau)}{\{1+(\tau-1)(1-\phi)\}}$$

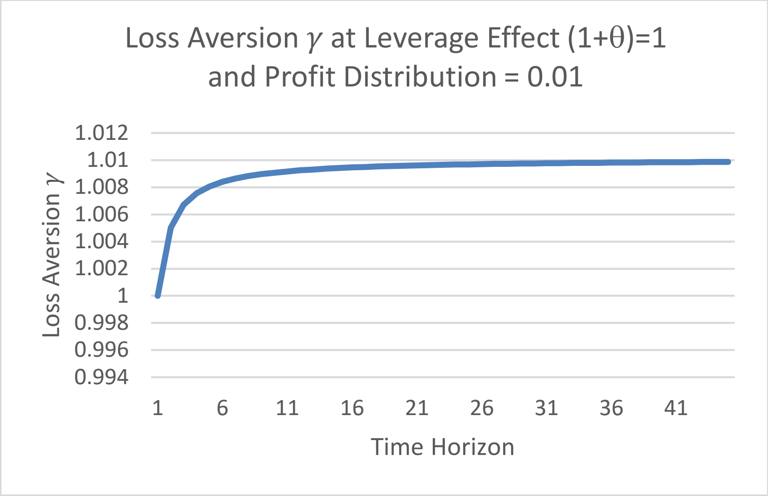
which now incorporates the effect of leverage.

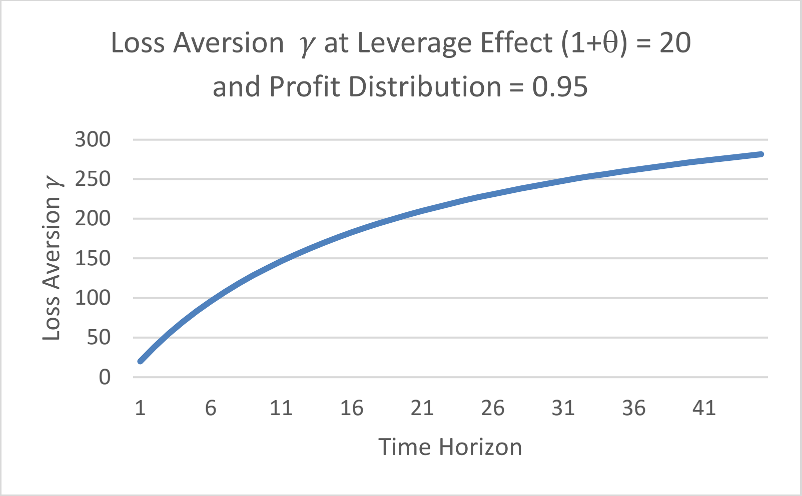
- The following graphs demonstrate how γ looks under different values of the variables on which loss aversion depends

Loss Aversion γ Behavior









Applications

- Valuation
  - We demonstrated the model's utilization for this purpose. It does not depend on the choice of a subjective discount rate premium, a single period model "beta", or make any distributional assumptions
- Optimization
  - A separate volume of this work is specifically dedicated on optimization of the utility form herein, inclusive of higher moments
- Liquidity Premium Estimation
  - The time horizon to full liquidation τ and the required amount of discretionary periodic consumptions are specific to investor profiles that results in different fair value to two different investors with distinct liquidity needs and horizons, a difference which is effectively equal to the liquidity premium
- Estimation of Real World Probabilities from Risk-Neutral Probabilities
  - The "stretch" of outcomes in the EXPLO loss region is a one-to-one reassignment of probabilities from higher to lower outcome values

Conclusions

- The logarithmic wealth utility model has a profound economic rationale
- Building on prior work in this area we introduce additional variables that make the model reflect reality better
- We derive a model that in its simplest form is intuitive and transparent
- The model form is tested empirically on three popular stock market investments and shows convincing results
- The utility function suggested by the model has a number of other useful applications in the investment practice

Questions, Comments, Feedback

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