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Beyond Black-Litterman

Capital Market Assumptions with Heterogeneous Investors

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Introduction

- The Black-Litterman model (*Journal of Fixed Income*, 1991) was an important milestone in providing investors with a practical approach blending their own views of asset class returns with a set of independently derived *prior beliefs*.

- *Internal views of capital markets are often presented as asset allocations rather than explicit numerical forecasts.* In the case of BL model, the assumption is that global financial markets are efficient, so that expected returns for various asset classes can be inferred for a given asset class covariance matrix.

- If we assume that the covariances of each asset class are sufficiently defined by the beta of the asset class to the world wealth portfolio, *we are employing the equilibrium inherent in CAPM, rather than International Capital Asset Pricing Model (ICAPM)* as first proposed in Solnik (1974). While ICAPM does capture global currency of denomination effects (i.e. home market bias), most of the simplifying assumptions of the original CAPM are left in place.

Motivation for Today's Presentation

- Assumptions inherent in CAPM and ICAPM omit many of practical aspects of the asset allocation problem across heterogeneous investors.
  - For example, the preferences of a tax-exempt defined benefit pension are likely to be very different than the preferences of family office of taxable investors.
  - *Among these points of likely distinction are liabilities to fund future consumption, risk tolerance, regulatory constraints, leverage constraints, taxation and the preference for income versus capital growth.*

- In this presentation, we will present a model which parallels Black-Litterman in most ways, but explicitly provides for heterogeneous preferences in portfolio formation.

A Very Brief Review of Black-Litterman

- The Black-Litterman model is a method of optimizing portfolios, typically for allocation across asset classes and/or geographies.
  - Combines two views: the global wealth portfolio is in equilibrium (i.e. is efficient in a CAPM sense), and another portfolio in which the investor assumes that returns are not in equilibrium, and therefore the investor chooses to pursue *multiple active strategies* across asset classes.
  - Building on CAPM, BL assumes that *covariances are known and are adequately expressed by the one factor beta of an asset to the world wealth portfolio.*

- A *subjective* input parameter τ (tau) expresses the relative confidence that the investor has in the return expectations that would arise from the two competing portfolios (global passive, active).

- The investor must also have a forecast for the return premium of the global market portfolio over the risk-free asset.

Formal Statement of Black Litterman (O'Toole, 2017)

- The BL "master equation":

$$r_{BL} = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

r_{BL} is the N x 1 vector of expected returns;

τ is a parameter that reflects the level of confidence in the equilibrium expected returns;

Σ is the N x N matrix of asset return covariances;

P is the K x N view matrix of asset portfolio weights for K active investment strategies;

Ω is the K x K diagonal matrix containing measures of confidence in each strategy;

Π is the N x 1 vector of equilibrium expected returns; and

Q is the K x 1 vector of views, which are the expected returns for the active strategies.

First Order Critiques and Concerns

- Most critiques of BL focus on the tau parameter being entirely subjective, reflecting the view of the investor without any economic or statistical justification (see Michaud, Michaud, and Esch, 2013).
  - See Idzorek (2004) and O'Toole (2017) for useful discussion
  - The Black-Litterman Model: Active Risk Targeting and the Parameter Tau (northinfo.com)

- The original description of BL applies a CAPM type equilibrium to the global wealth portfolio.
  - The global wealth portfolio would fit better with some version of the International Capital Asset Pricing Model as first proposed by Solnik (1974) in which investors are *compensated for both market risks and separately for risks arising from foreign currency exposure.*
  - The risks and returns associated with foreign currency exposure are different depending on the base currency of the investor.

Other CAPM Versions Provide Different Risk Premiums

- Different versions of the CAPM result in different explanations and estimated magnitudes of the market risk premium
  - The Intertemporal CAPM (Merton, 1973) assumes that investors realize that risk may vary over time and will sometimes choose to hedge volatility to reduce fluctuation in "drawdown" risk.
  - The Consumption CAPM proposed by Lucas (1978) and Breeden (1979) follow Merton by redefining beta against their pattern of consumption spending rather than market value fluctuations.
  - *Assuming these versions of CAPM would allow BL some flexibility to reflect heterogeneous investor preferences.*

- Black himself (1974) proposes a modified CAPM where the "zero beta asset" is not risk free but has no correlation to the market portfolio.
  - See diBartolomeo and Kantos (Journal of Asset Management, 2020) for an extensive study of equity factor returns under a "zero beta" model when the expectation of the market return is not normally distributed.

An Alternative Approach

- An alterative representation of BL is proposed in Bertsimas, Gupta, and Pascalidis (2012) where instead of assuming a CAPM equilibrium, we *infer expected returns for the market portfolio and active portfolios*
  - Often referred to as "reverse optimization".

- Reverse optimization can only express the returns of different assets as scalar multiples of one another (e.g. Rx = 2 Ry)
  - To get numeric estimates we must specify a presumed level of risk aversion
    - We can't use the MV tangency slope of the efficient frontier (no returns)
  - Economic theory of how risk averse investors should be is presented in Litzenberger and Rubinstein (JoF, 1976), and Wilcox (JPM, 2000 and 2003)
  - An efficient heuristic is presented in Estimating an Investor's Volatility/Return Tradeoff: The Answer is Always Six (northinfo.com) *that can estimate heterogeneous risk aversion based on the volatility of an investor's current portfolio*

Investor Preferences Do Matter

- Equilibrium theories of asset pricing (CAPM, APT) embed a variety of assumptions into the process.
  - Single period, no transaction costs or taxes
  - Perfect liquidity including free flows of capital
  - Homogeneous preferences, no limitations on leverage
  - All investors have all information
- Many real-world examples *illustrate that investor preferences do matter* in asset pricing because these assumptions don't hold.
  - Yield differences between taxable and tax-exempt bonds
  - Big liquidity premium for private equity with a small liquidity premium for private placement bonds (bond holders hold to *known maturity date*)
  - Arbitrage opportunities in the presence of capital controls
    - Shanghai/Hong Kong equity price spreads persisted for decades

What if Investor Preferences Matter a Lot?

- An efficient market would be one where the global wealth portfolio reflected the aggregate Pareto optimal tradeoff between expected returns and heterogeneous investor disutility.
  - Taxed investors would may more for tax free securities (muni bonds) than non-taxable investors
  - Investors preferring income to capital gain would price assets differently.
  - *If you know who the global investors all are and what they dislike, you can use linear programming to solve for the expected returns of all assets.*
  - US Patent 5806049A to Petruzzi (1996), EXPLO method by Belev (2022), Valuation of Private Companies Using Risk, Growth, and Time (northinfo.com)

- An interesting implication is that very long term expected returns for asset classes are predictable because the demographic shifts driving the investor mix are predictable.
  - See Arnott and Chavez (2012) and Northfield News-September 2016 (northinfo.com)

BL for Fully Customized Investor Preferences

- The logic of BL says that we want to blend equilibrium (passive) beliefs about expected returns with active views of expected returns.
- If we drop CAPM and adopt the "reverse optimization" concept of formulating a BL problem, we can set the return blending problem for cases *where the active portfolio composition is motivated by investor preferences or constraints unrelated to expected returns*
  - Preference for income/capital growth
  - Liquidity constraints, regulatory constraints
  - Taxation of legacy positions
- For liquid assets, there should always be a clearing price for each asset
  - We're not actually formulating investor specific expected returns (all investors will experience the same future returns), but we can use this information to quantify the cost of constraints, taxes, and other non-return preferences.

A Non-Parametric Black-Litterman?

- To illustrate "investor specific" BL we will formulate an investor's asset allocation using the non-parametric method (no explicit returns, risks, or utility function) known as the Analytic Hierarchy Process. See Saaty (1980)
  - AHP is widely used in industrial decision theory
  - *Very robust to conflicting inputs or outliers in inputs*

- Asset allocation proposed in Khaksari, Kamath, and Grieves (1989)
  - AHP bases asset allocation on investor preferences expressed through responses to a set of multiple-choice questions inclusive of "investor suitability" rules or other constraints.
  - *The set of available assets are mapped to the question responses via a set of mathematical matrices that express investor preferences and constraints.*
  - Computations are related to both Bayes's Theorem and principal components analysis (allocation weights are a function of inverse squared values).

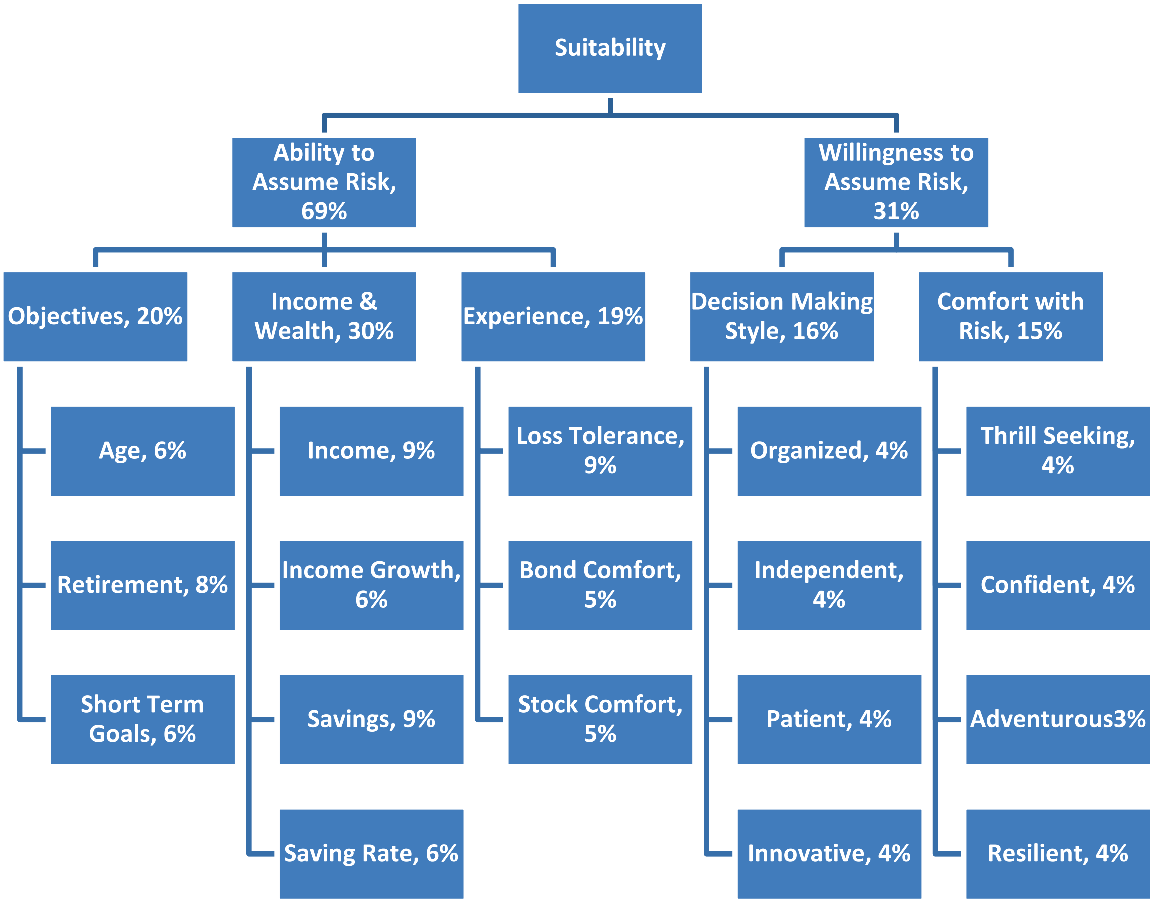
AHP Can Play a Dual Role

- If we bring an AHP portfolio into our version of the BL method, it can play either role.

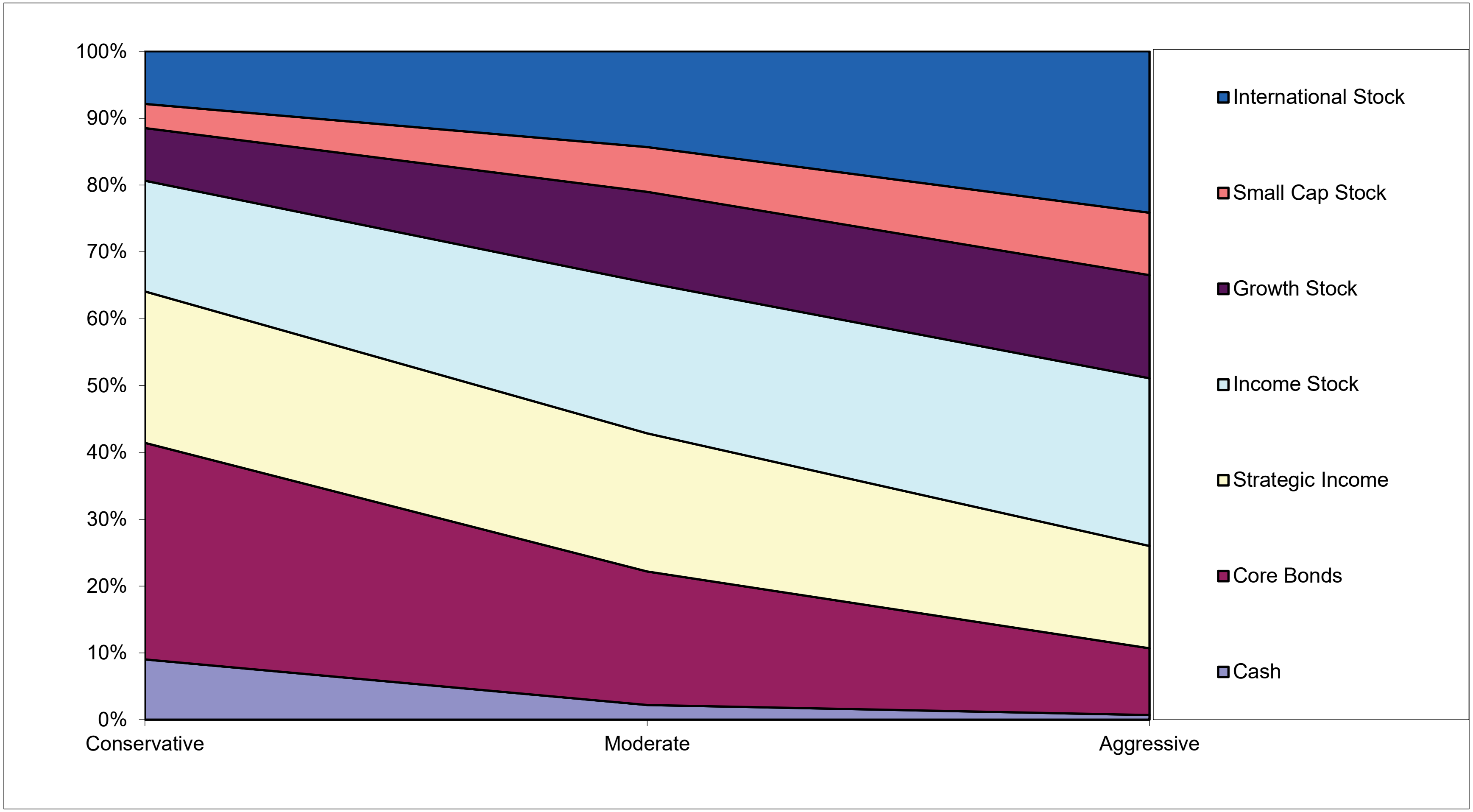
- We can treat the AHP portfolio as the “active portfolio” (as being different from the market portfolio) and use the global market portfolio as the prior.
  - *This approach allows the BL process to be customized to different investors.*

- We can treat the AHP portfolio as the prior and use the investor’s choice of active portfolio to express their expectations about the returns of active strategies
  - *In this approach we take advantage of the natural robustness of AHP portfolio solutions which will illustrate in an upcoming slide.*

Bolster and Warrick (2008) Example Questionnaire



The Goal of BL Was of Adding Robustness to MV



AHP Scoring Matrices

AHP on-line

AHP Expert | AHP Portfolio

F: 8 Q: 11 A: 5 Model type : Asset Allocation

Funds | Questions/Answers | Groups | Coefficients

Questions (S x A)

	Name
1	Age
2	Time to Retirement
3	Time to Major Expense
4	Household Income
5	Income Growth
6	Savings Rates
7	Savings Totals
8	Stocks and Equity Mutual Funds
9	Fixed Income
10	Tolerance for Loss
11	Liquidity

Fund High Yield Bonds **Question:** Household Income **Answer:** Less than $150,000

	Sectors	A1	A2	A3	A4	A5
1	Money Market Taxable	2	6	9	33	99
2	High Quality Bonds	1	1	2	3	9
3	High Yield Bonds	9	4	3	4	7
4	Global Bonds	9	6	4	5	8
5	Income	9	4	2	1	1
6	Growth	33	8	4	2	1
7	Small Cap. Stocks	33	8	6	4	3
8	International Stocks	9	7	5	3	1

Non-Parametric Allocation Connected to BL

- *The idea of bringing a non-parametric formulation of the BL active portfolio was first proposed in diBartolomeo (2014) due to the intrinsically robust nature of AHP frontiers.*

- In theory, we could simply use AHP methods to create customized investor specific portfolios.
  - We could then use the BL assumption that risk is represented by a known covariance matrix.
  - We use the "rule of six" derived from Litzenberger and Rubinstein to infer risk tolerance of an AHP investor.
  - Now we apply the "reverse optimization" approach to BL to obtain the vector of expected returns of the "active portfolio," although the "active" aspects of the portfolio are determined non-parametrically for reasons other than return.

Solving the Tau Puzzle

- As noted earlier, one of the major criticisms of BL is that the tau parameter is arbitrary assigned by the investor.

- Bolster and Warrick (2008) provide a procedure to define the distance between the AHP "efficient frontier" and a conventional MV mean-variance efficient frontier (conditional on a given set of capital market assumptions).
  - *If the AHP frontier and the MV frontier are not statistically significantly different, we can assume that we are equally confident in expected returns that could be inferred as being applicable to the preferences of a specific investor, so tau implies equal weighting.*
  - If the frontiers are statistically significantly different, we can modify the AHP matrices until the differences are no longer significant.
  - The more we need to "bend the AHP decision" tree, we can set tau accordingly.

Conclusions

- While the Black-Litterman approach to making asset allocation optimization more robust has been widely implemented by practitioners, *there is a lot of room for potential enhancement.*
  - Use of different versions of CAPM that reflect investor base currency, time varying risk, and expected timing of consumption expenditures.
  - Rather than use a CAPM equilibrium approach, we can estimate investor risk aversion heterogeneously, then apply "reverse optimization" to obtain expected returns.
  - We can formulate "active portfolios" whose composition is based on a broad range of considerations, rather than just expected returns using the AHP non-parametric method.
  - Use of AHP inherently anchors the BL efficient frontier to a frontier of portfolios that is *both highly robust and is be fully customized to the different preferences of each investor.*