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Estimating the Probability of Equity Market Crashes

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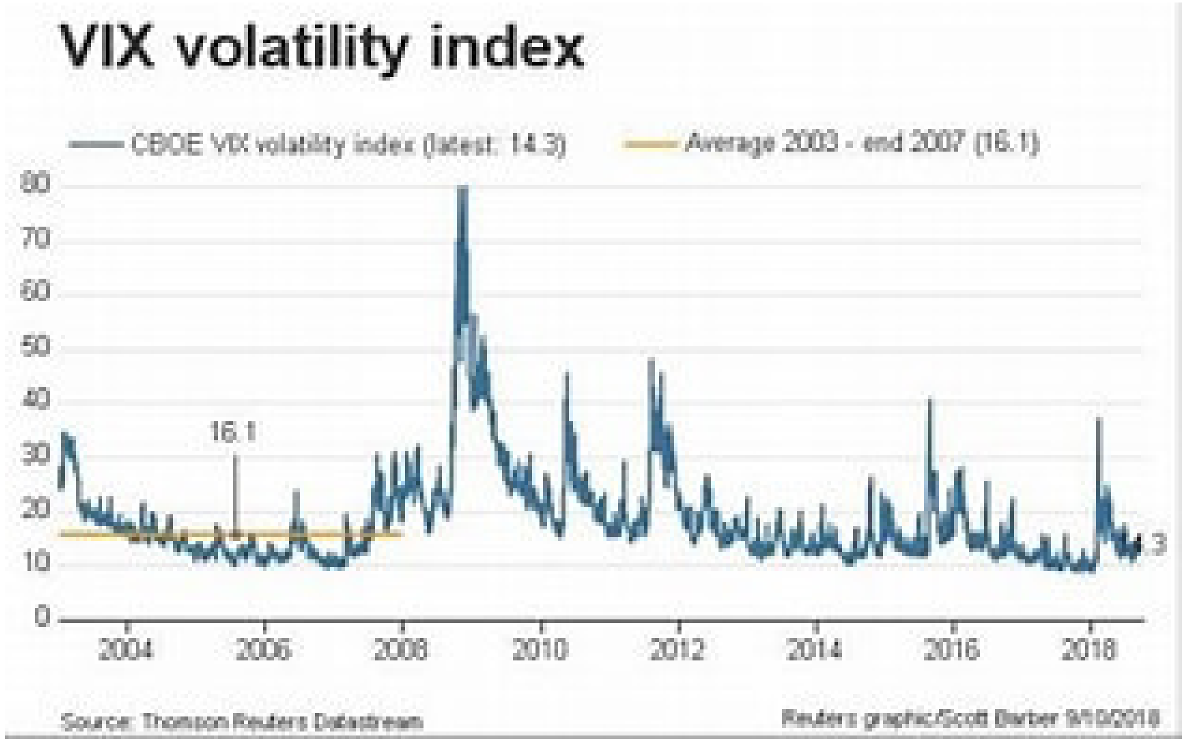
Webinar

July 2023

Motivation

In academic studies, US equity market volatility described by the VIX index is often referred to as the "markets fear gauge". We will argue this representation is poorly understood.

On July 5th, 2023 an article on the Fortune.com website quoted an interview that Jeremy Grantham did with a publication called Wealthtrack. The quoted suggested that Jeremy had reduced the likelihood of an equity market crash from 85% to 70%. Neither the magnitude of a "crash", nor the time horizon over which the given probability was applicable, were defined. Since a "crash" could be a minor decline over an infinite time horizon, this kind of *very vague assertion* is of no benefit to investors.



Outline

During rare periods of crisis, the VIX estimate of S&P 500 annualized volatility has exceeded 80% (four to five times the historical average level).

As investors fear loss rather than gain, using a symmetric measure is indirect at best.

In this presentation we will illustrate how to use the VIX to decompose equity market risk into two states *to directly address the probability of specified magnitude of loss.*

In the first state, the market continues with typical levels of annual volatility (e.g. 15-20%).

In the second state, we will assume the equity market crashes defined as a negative return that is an N (e.g. 2) standard deviation event relative to typical volatility levels.

For a chosen crash magnitude, we will illustrate how to estimate the probability of occurrence within a relevant time horizon using three analytical steps.

Robertson and Fryer (1969)

Cornish and Fisher (1938)

diBartolomeo (*Investments and Wealth Monitor*, 2020)

The Flaw in the "Fear Gauge"

Over the history of the VIX since 1993, there have been two periods when the volatility index exceeded 60% and once over 80%

- Given the definition of "80% volatility" in *standard deviation of annual arithmetic return* units, this would imply there was a 10.57% probability of a market return greater than the expected return plus 100%, and a 10.57% probability of less than the expected return minus 100%.

Since returns of less than -100% are not possible, the lower tail of this distribution must be truncated.

- Satchell and Hall (Journal of Asset Management, 2013) show plausible investor utility functions are increasing in skew.
- The resultant ex-ante distribution would have positive skew, which investors should find favorable rather than fear.
- You can sidestep the math problem using log returns.

Our two-state model resolves this conflict for arithmetic returns and produces intuitive behaviors.

A Very Short Literature Review

The VIX is an implied volatility index from options.

There were many early papers on the idea of inferring the four moment return distribution of an asset from differences in pricing across various options (calls, puts, different strikes)

- A good early one was Corrado and Su (*European Journal of Finance*, 1997)

A broad generalization of these concepts is described in the Recovery Theorem proposed by Steve Ross (*Journal of Finance*, 2015)

We've Been Busy with Two State Modeling

diBartolomeo and Kantos (*Journal of Asset Management*, 2020) looks at how CAPM and factor returns change if investors assume that there will be rare but large crisis events (e.g. war, pandemic, crashes)

Blackburn, diBartolomeo and Zieff (*Global Commodity Applied Research Digest*, 2022) uses the two-state model to combine market risk and operational risk in crypto currencies.

diBartolomeo (*Journal of Performance Measurement*, 2023) describes required adjustments to investment performance measures when returns are not normal distributed and serially uncorrelated.

diBartolomeo (*Journal of Portfolio Management*, 2023) uses the two-state model to correct a large bias in the ex-ante use of manager information ratios

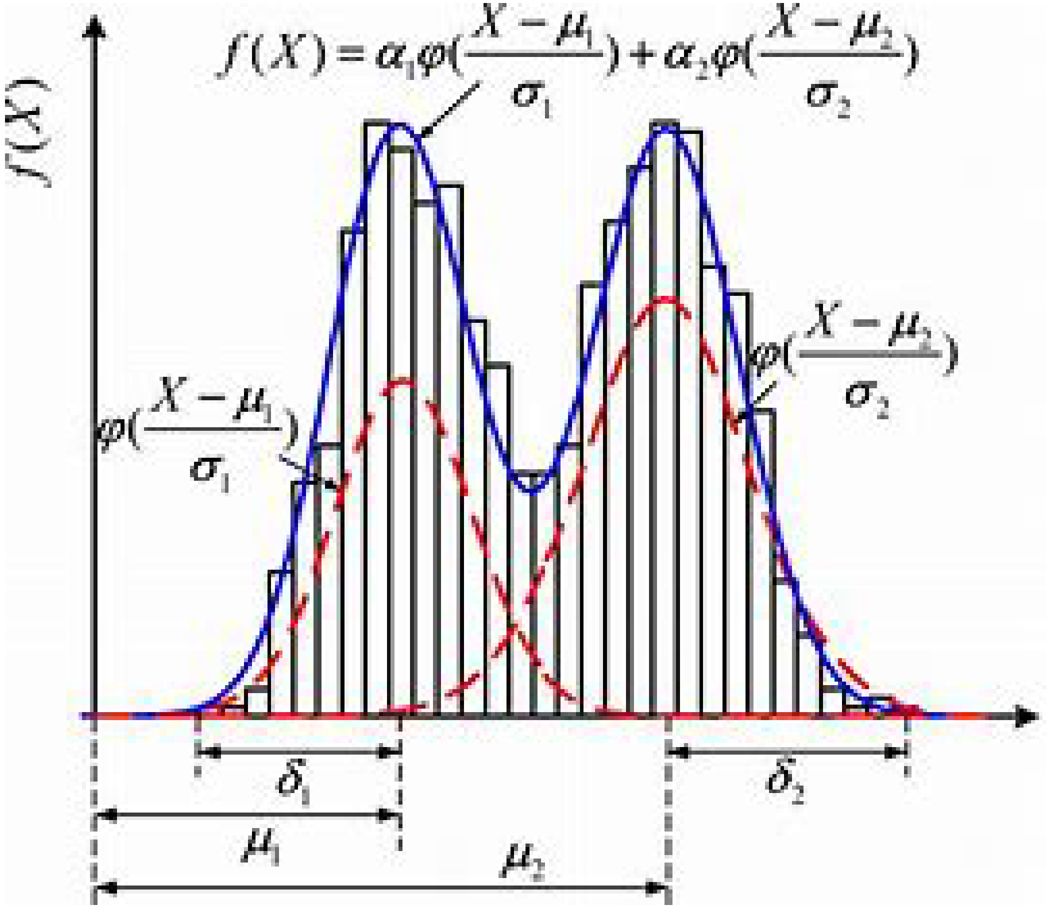
Two States: A Familiar Use of Mixture Distributions

For any kind of "large event," we will introduce the forward-looking annual probability M and the conditional distribution of loss L, which has its own degree of uncertainty as to the severity.

- If we assume the probability of the negative event is M per year, then probability that conditions will remain stable, and the event will not occur is (1-M).

We now have two mutually exclusive states of the future, one in which conditions remain as they have been historically (the bad event has not happened yet), and one where the large, negative event takes place. Each of these two states can be represented as a normal distribution with some expectation of the mean u_l and standard deviation s_i.

- From a small amount of algebra, *we obtain the four moments of the ex-ante distribution.* We can use the Cornish-Fisher method (1938) *to calculate a volatility equivalent inclusive of the potential for a "large event" that has not yet taken place, conditional on the probability and likely range of severity of such an event.*



Robertson and Fryer (1969)

- To estimate the "volatility equivalent" appropriate to this situation, we will first combine the two distributions.

$$\mu = \Sigma_{i=1}^{2} m_i \mu_i$$

$$\sigma^2 = \Sigma_{i=1}^{2}[m_i(\sigma_i^2 + \mu_i^2) - \mu_i^2]$$

$$S = \frac{1}{\sigma^3}\{\Sigma_{i=1}^{2} m_i (\mu_i - \mu)[3\sigma_i^2 + (\mu_i - \mu)^2]\}$$

$$K = \frac{1}{\sigma^4}\{\Sigma_{i=1}^{2} m_i [3\sigma_i^4 + 6(\mu_i - \mu)^2\sigma_i^2 + (\mu_i - \mu)^4]\}$$

where

m_i = the probability of state i , μ = mean return of the combined distribution

σ = standard deviation of the combined distribution

μ_i = the expected return in state i , σ_i = the expected volatility in state i

S = skew of the combined distribution

K = kurtosis of the combined distribution (raw)

First Step

- For today, our Robertson and Fryer algebra requires five inputs:
  - M, the probability of no "event" (i.e. crash)
  - μ_1 the expected return in typical conditions, σ_1 the annual volatility in typical conditions
  - *μ_2, the expected return in a crash*, σ_2 the confidence interval around the expected return in a crash

- To parameterize the problem:
  - Use a typical market return expectation for μ_1 (e.g. T Bills plus 6%)
  - Use the expected market volatility from a risk model for σ_1
  - μ_2 is our chosen severity of a crash (e.g. -50%)
  - We assume the confidence interval σ_2 around μ_2 is symmetric and approximate it as

$$\text{abs}(\mu_2 + 100)/3$$

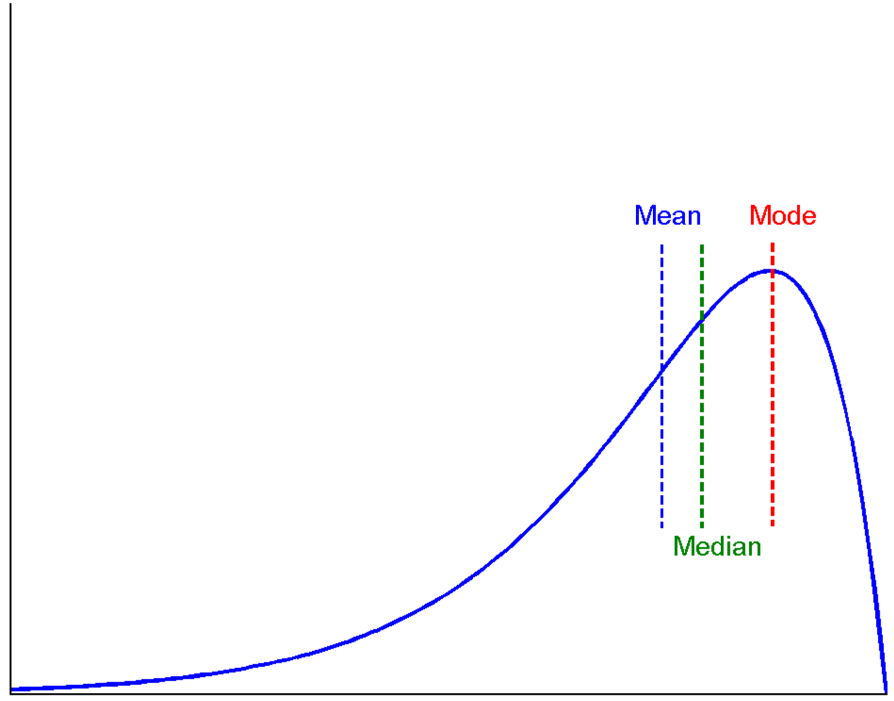
  - Pick any value for M, *so the probability of a crash of magnitude μ_2 is (1-M)*
  - The output will be a four-moment distribution with a mean, standard deviation, skew (S) and kurtosis (K)

Second Step using Cornish Fisher

We can put the parameters of our four-moment distribution into the Cornish-Fisher expansion and get the "volatility equivalent"

Iterate over the domain of M (probability of no crash)

If the "volatility equivalent" from CF is the equal to the current VIX value, we know the VIX implied combination of probability of a crash (1-M) for our selected magnitude μ_2



Cornish Fisher Expansion (1938) For Volatility Equivalence

$VIX \sim E[V_t] = \sigma W_p / Z_p$

Z_p is user-chosen, like a critical value for a confidence interval. A typical value is -2.

$$W_p = Z_p + (S/6) * (Z_p^2 - 1) + Z_p * ((K-3)/24) * (Z_p^2 - 3) - Z_p * (S^2/36) * (2Z_p^2 - 5)$$

W_p = the Cornish Fisher "tail weight" parameter

Note: The CF function is not monotonic for some values of Z. There are various fixes.

The problem is less of an issue for abs(Z) > $3^{.5}$. See diBartolomeo (Journal of Performance Measurement, 2023) and diBartolomeo (Journal of Portfolio Management, 2023)

Worked Example of Implied Crash Probability

Our input parameters are:

VIX = 60%

μ_1 the expected return in typical conditions = 8%

σ_1 the annual volatility in typical conditions = 15%

μ_2, *the expected return in a crash* scenario = -50%

σ_2 the confidence interval around the expected return in a crash = 16.67

Z = (-2)

Annual probability of no -50% crash = 92.6%

Annual probability of a -50% (or worse) crash = 7.4%

If I change the crash definition to -40% (or worse), the probability is 12.6%, *so the annual likelihood of an event between -40% and -50% is 5.2%*

Plausible Time Horizon

In diBartolomeo (*Investments and Wealth Monitor*, 2020) we propose a method for distinguishing when a spike in the VIX is perceived by investors as temporary as opposed to permanent.

- In the paper we used this method to infer that investors believed that the US equity market would regain stability in six to seven months after March of 2020, which is when the US market declined sharply in response to the COVID 19 pandemic.

Approximate starting conditions in November 2019 (pre-COVID) for Gordon Shapiro DDM

- VIX = 20%
- My estimated of the required return on the equity market = 6% (Tbill rate 1.5 + 4.5% equity risk premium)
- Dividend Yield = 2%, Implied Growth Rate = 4%
- Price of $1 of dividends = $50 if the market is fairly valued = 1/(.06-.04)

COVID Crisis by March of 2020

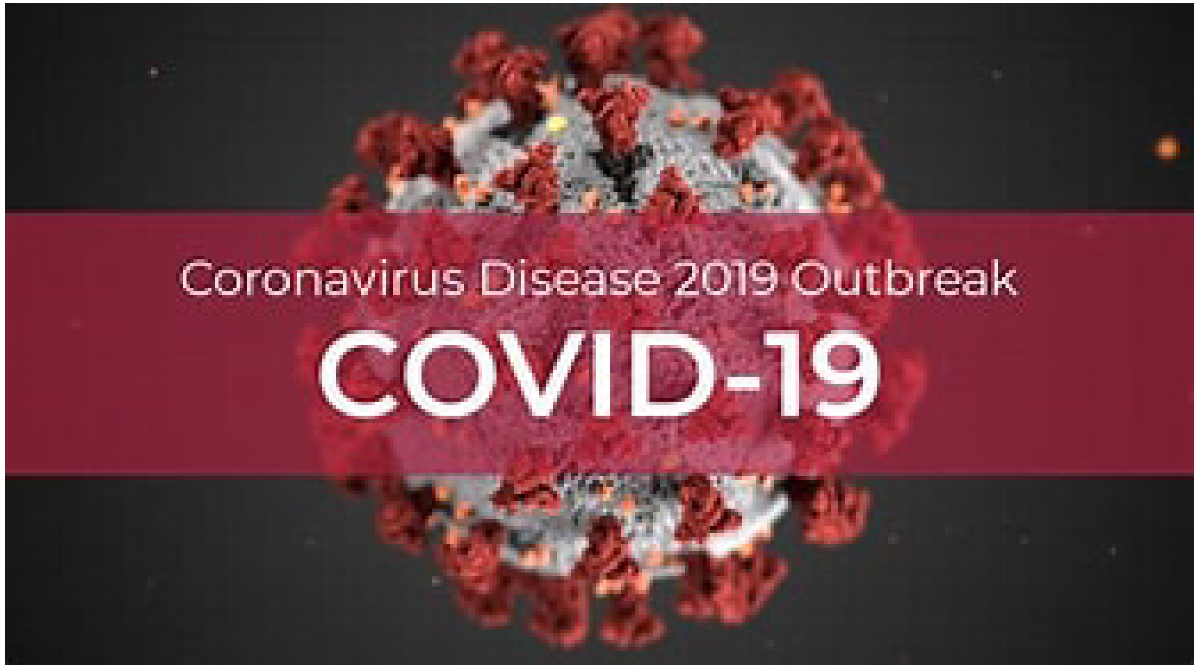
By March of 2020, conditions were much different

- VIX = 60%
- Increased required return 12.7% (see Litzenberger and Rubinstein, *Journal of Finance*, 1976) and [Estimating an Investor's Volatility/Return Tradeoff: The Answer is Always Six (northinfo.com)]
- New Price for a $1 of dividends = P (March 2020) = 1/(.127-.04) = $11.49, a decline of 78%

From November 2019 to March 2020, the S&P 500 went down 20% which makes sense if the permanent required rate of return increases from 6 to 6.5% under the Gordon Shapiro DDM.

To reconcile *the projected decline to the observed decline*, we assume that the average half-life of a company is 20 years, taken from diBartolomeo (*Journal of Investing*, 2010) and that variances are additive.

- The reconciliation inference was that the high volatility period would last six to seven months, put the timing of the "new normal" around September of 2020, which was approximately correct.



Conclusions

1

Investment prognostications about the potential for market declines are a popular, if usually very vague, way to describe the financial market return expectations.

2

Many investors track the VIX volatility index as way to estimate market risk, but fail to recognize that interpreted in an overly simplified way, investors should favor a high volatility environment rather than fear it.

3

To be actionable, a forecast of market declines must clearly identify the severity of a crash scenario, the probability of the crash scenario per unit time, and a plausible time horizon over which the cumulative probability of a crash can be estimated.

4

Our two-state model fulfills all the requirements using well vetted algebra and the VIX index as input.

- In the future, we will provide a similar process for individual stocks that can be included in our equity risk models.

Additional References

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