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Expressing Alpha on Non-Linear Derivatives

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Introduction

Traditional derivative pricing models such as the Black-Scholes model for options have a feature that many investors find unintuitive.

The investor's expectations about the expected return of the underlying asset (e.g. the "alpha" of a stock) are not part of the inputs to valuing options.

Common intuition might be that if the "alpha" on a stock was positive, that would make call options more attractive than put options, which should be reflected in the respective option prices.

In this presentation we will briefly dissect the assumptions under which derivative pricing models operate such that underlying alpha is presumed irrelevant.

We will also provide appropriate procedures for expressing the alpha on an option conditional on the alpha of the underlying security.

Black Scholes Model for Option Pricing

The option pricing model of Black and Scholes (JPE, 1973) is one of the key foundations of modern finance.

To calculate the value of a European (exercisable only at expiration) call or put option, the following parameters are needed.

- Current price of the underlying asset (e.g. a stock)
- The strike price of the option
- The time to expiration
- The risk-free interest rate
- The future volatility of the underlying asset

For American options (exercisable before expiration) various approximations have been developed that also require a representation of any income produced by the asset

For stocks this is the dividend yield

Black Scholes Assumptions

The BSM is derived as a partial differential equation that is closely related to PDE that describes how heat diffuses through a solid.

The elegance and conciseness of the BSM arises from various assumptions about the structure of financial assets that make the mathematical representation much more tractable.

- Asset prices move randomly in a geometric Brownian motion and are lognormally distributed for both individual assets and the market
- Transaction costs are zero, so "no arbitrage" conditions hold
- Markets are complete (there are always ways to hedge risk) and trading is continuous

Investors understand these assumptions to be heroic, so use the BSM in retrograde fashion solving for the implied future volatility input based on a market price.

The Dog That Didn't Bark (My Dog)

Many investors find the BSM unintuitive because the relative prices of calls (right to buy) and puts (right to sell) are independent of the expected return of the underlying asset.

- In the derivation of the BSM as a PDE, the "drift" term (return on the underlying asset) drops out of the math.

The assumption that *stock returns are perfectly random* clearly conflicts with the concept of active management, where there is the assumption that *the investor has some level of predictive power* for future asset returns.

It is therefore necessary to estimate return expectations for options in the construction of portfolios.

The Usual Case of Option Arbitrage Returns

Many trading strategies in option markets revolve around finding temporary violations of the "no arbitrage" assumption.

- For example: two call options for the same underlying asset and the same expiration date, but different strike prices.
- Using BSM to compute the implied volatility of the underlying asset, we find that the two implied volatility values are different for two options.

Since both options will exist for the same time period, the underlying asset cannot have two different volatilities under the BSM assumptions.

- The investor can earn an arbitrage profit by selling the option with the higher implied volatility and buying the option with the lower implied volatility.
- The alternative explanations are that asset returns are not random, or that the expected distribution of the asset price is not lognormal.

Implied Asset Return Distributions

There were many early papers on the idea of inferring the four moment return distribution of an asset from differences in option pricing across various options (calls, puts, different strikes)

- A good early one was Corrado and Su (*European Journal of Finance*, 1997)

A broad generalization of these concepts is described in the Recovery Theorem proposed by Steve Ross (*Journal of Finance*, 2015)

Instantaneous Option Return for Passive Managers

- The justifying concept of capitalization weighted index funds is that the market is efficient and operates in equilibrium under the CAPM.
- Under the CAPM, the expected return of an asset is

E[Ri] = Rf + Bi * (E[Rm-Rf])

- The *instantaneous* beta of an option is given by

B(option) = B(stock) * Delta * Stock Price/Option Price

E[Ro] = Rf + (Delta * Stock Price/Option Price) * (Bi * E[Rm-Rf])

Instantaneous Option Returns for Active Managers

The leverage effect of using options applies to alpha as well.

For an active manager we get:

$$E[Ri] = Rf + Bi * E[Rm-Rf] + ai$$

$$E[Ro] = Rf + (Delta * Stock\ Price/Option\ Price) * (Bi * E[Rm-Rf] + ai)$$

$$ao = Rf + (Delta * Stock\ Price/Option\ Price) * ((Bi-1) * E[Rm-Rf] + ai)$$

Adding in Time Decay

Due to time decay in the value of options, the beta and expected returns are constantly changing.

For investors who want to estimate option returns over finite holding periods (e.g. one month), there are two tractable methods

One approach to is to use an expected average leverage over the horizon

- See A Heuristic Approach for Delta Hedging in Discrete Time (northinfo.com)

The other is to embed your stock alpha estimate into calculation of the option values at end of the horizon period.

- Rubinstein (JoF, 1984)
- O'Brien (JoF, 1986)

Example for Rubinstein (1984)

We are evaluating both a call option and a put option on X

- Option expires in one year
- Volatility of X is 55%
- Share Price of X is 40
- Strike price is 40
- Risk free rate is 2%
- Dividend yield of X is zero

Under the BSM

- Value of the call is $8.98, delta = .622
  - You need 1.61 units to be delta neutral to one share)
- Value of the put is $8.18, delta = -.378
  - You need -2.64 units to be delta neutral to one share

Rubinstein (JoF, 1984)

Under this method, we need the expected return on stock X, which I will express as the expected annual return on the market (7%) and an active alpha forecast of 2.5% annually over a forecast horizon of 30 days.

At the 30 day horizon, the expected values for stock price and strike price

- Stock price X is ($40 * (1 + (.095 * 30/365)) = $40.31
- Strike price is ($40 * (1 + (.02 * 30/365) = $40.07

BSM with at 365 day and 335 days to expiration with and without returns

- Initial $8.98, W/O return call price is $8.61, W/Return call prices is $8.77
- Initial $8.18, W/O return put price is $7.88, W/Return put price is $7.80
- Call delta shifts slightly from .622 to .617
- Put delta shifts slightly from -.378 to -.383

Profits, Losses Converted to Alpha

Our alpha is positive, so it makes sense to be long calls or short puts, all else equal.

Holding the one share of stock X for 30 days

- Produces a gain of $.31 on an investment of $40
- $.23 is the expected market return, $alpha = $.08
- Annual % alpha = 2.5%

Being long the delta equivalent of call option produces a loss

- $ -.22 * 1.61 = $-.35 on an investment of $14.37, $alpha = $-.58
- Annual % alpha = -49.1%

Being short the delta equivalent of the put option produce a gain

- $ -.38 * -2.64 = $1.00 on investment of $21.60, alpha = $.77
- Annual % alpha = 43.3%

O'Brien (Journal of Finance, 1986)

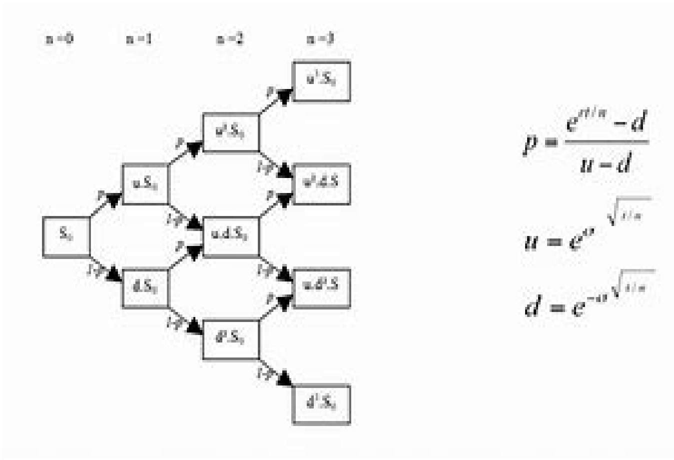
Since Black-Scholes requires asset prices to be lognormal and returns random in order to generate risk-neutral prices

- It must follow that if market returns are not lognormal, then the drift term won't drop out of option valuation and becomes expected return dependent.

O'Brien proposes a discrete time option valuation process (i.e. a binomial tree) where the expected return on the underlying asset is an input.

Some models used for valuating options embedded in bonds (e.g. issuer call options, prepayment of mortgages) use similar methods

- Ho and Lee (JoF, 1986)
- Hull and White (Journal of Derivatives, 1996)
- Kalotay, Williams, Fabozzi (FAJ, 1993)
- Black, Derman Toy (FAJ, 1990)



Conclusions

The assumptions that underly the BSM and other option pricing models are generally incompatible with the concepts that justify undertaking active management.

As such, academic literature has been sparse in terms of how to estimate expected returns for options conditional on the expected returns for underlying assets.

In this presentation, we demonstrated different methods for estimating returns on option positions.