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Cointegration of Sector Returns

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Introduction

To a large extent changes in the profitability of publicly traded companies are economic transfers from one sector to another.

For example, increases in energy prices make energy producing companies more profitable, but make energy consuming companies less profitable.

Taken together, the total profits of the two sectors may be stable, while the profits of each sector viewed separately would be volatile.

To the extent that a combination of multiple time series has the statistical property of stationarity when the underlying series are not stationary is called *cointegration*.

We will illustrate a simple way to use cointegration methods to better forecast long term factor returns and define sector weights that have the maximum likelihood to outperform conventional equity indices over long horizons.

Active Management as Hedging

Many problems in finance are naturally structured as hedging problems, wherein the objective is to find some set of assets that will offset (hedge) the changes in value of a particular set of liabilities.

The hedging paradigm can also be applied to an actively managed portfolio process where the objective is to outperform some specified market index.

- Such framing of the active management is not only possible but has many desirable aspects.

For example, let us assume that we are active managers seeking to find the portfolio that will outperform the Standard & Poor's 500 index by 300 basis points per year.

This problem could easily be viewed in an asset-liability or hedging framework. It would be our goal to find the portfolio that would be a "successful hedge" against a liability whose value grows at a rate equal to the return on the Standard & Poor's 500 index plus 300 basis points per year.

Defining a Successful Hedge

- Within the hedging construct for active management, we can arbitrarily assign the initial value of the liability to be equal to the initial value of our investment assets.
- Hence we have an initial surplus (assets minus liabilities) of zero.
- Over time our hedge is successful as long as the dollar value of surplus never gets too large in magnitude, either positive or negative.
- The time series of the value of surplus would then be called *stationary*. If the value of surplus has this property, we know that the changes in asset values are adequately keeping up with changes in the value of the liability.
- If we can find a set of portfolio assets that result in a stationary surplus, we can say that the portfolio members are *cointegrated* with the liability.

A Test for Stationarity

- By stationary we mean that the time series has consistent mean, standard deviation and autocorrelation properties. We can formally define stationarity with equation (1).

$$X_t = aX_{t-1} + bt + c + e_t$$

X_t = the value of series X at time t
t = the increment count of time
e_t = error term at time t

- This equation can be estimated using typical regression analysis methods. Series X will be considered stationary if the absolute value of coefficient *a* is statistically significantly less than one.
- Since we must consider both the possibility that *a* is equal to greater than one and the possibility that *a* is less than or equal to negative one, *a regular t-statistic test for statistical significance is not appropriate.*
- Popular alternative test are Dickey-Fuller, and Engle-Granger statistics.

Cointegration of Assets and Liabilities

- Cointegration is the situation where we able to form a stationary time series from a linear combination of series that are not individually stationary.
  - For example, stock price time series are generally not stationary. If we could form a fixed weight portfolio of stocks such that the portfolio had a value time series that was stationary, we would say that those stocks formed a cointegrated set.
  - The list of the stock weights within the portfolio would be called the cointegrating vector. First, we define the surplus.

$$S_t = S_{i = 1 \text{ to } n}[w_i A_{it}] – L_t$$

S_t = the value of surplus at time t
w_i = the weight of asset i in the portfolio
A_{it} = the value of asset i at time t
L_t = the value of the liability at time t

- We can then test for stationarity of the surplus time series S_t

Cointegration Applied to Investing

A simple way to think about cointegration for investing, is to consider the time series of the cumulative returns of various assets.

- Two or more series are cointegrated if the *differences in the time series of cumulative returns* pass the appropriate statistical tests. Widely used for "pairs trading"

Unlike traditional MPT methods for portfolio analysis, cointegration methods work directly on portfolio values rather than returns and make no statistical assumptions about the distribution of the values.

Finding Cointegrating Portfolio Weight Vectors

- There are a number of ways that we can estimate the portfolio weights that will form the cointegrating vector, if such a vector exists.
  - If unconstrained *short positions are allowed, we can use multiple regression analysis* to find the set of asset weights such that the time series of portfolio values best fit the time series of liability values.

$$L_t = S_{i = 1 \text{ to } n}[w_i A_{it}] + e_t$$

w_i = the weight of asset i in the portfolio
A_{it} = the value of asset i at time t
L_t = the value of the liability at time t
e_t = error term at time t

- The vector w would be the resultant regression coefficients.

- The error time series (e_t) would represent the surplus (S_t) and we can test the error term series for stationarity.

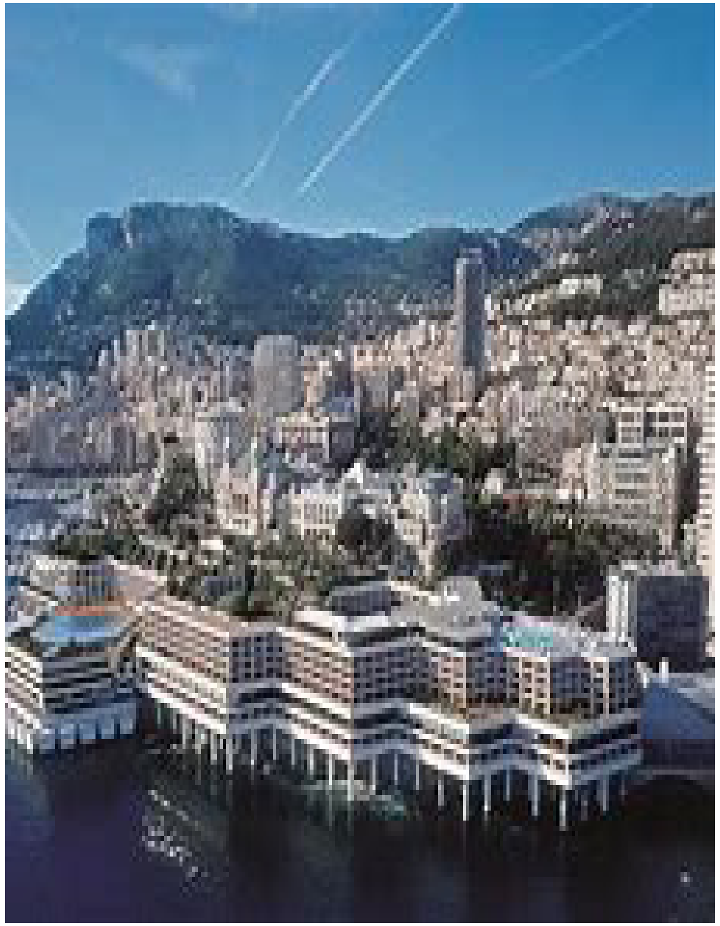
Long Only Cointegrating Portfolios

If we choose not to take short positions in the portfolio, we must have values for the weights that are subject to two constraints. Weights must be positive or zero, and sum to one.

- These constraints are also employed in returns-based style analysis, as developed by Sharpe (1992) requiring use of quadratic programming.
- An approximation to the standard error is presented in diBartolomeo and Lobosco (Financial Analyst Journal, 1997).

Another alternative for finding the cointegrating vector would be to use Monte-Carlo simulation to create a large number of portfolios of randomly arranged weights, subject to whatever constraints we care to apply.

- We could then apply our surplus equation directly and test each one to determine if it was cointegrating. *The final solution would be the average weight vector of those found to be cointegrating.*



Previous Research on Cointegration of Countries

Weighting countries within a broad global index has been well explored.

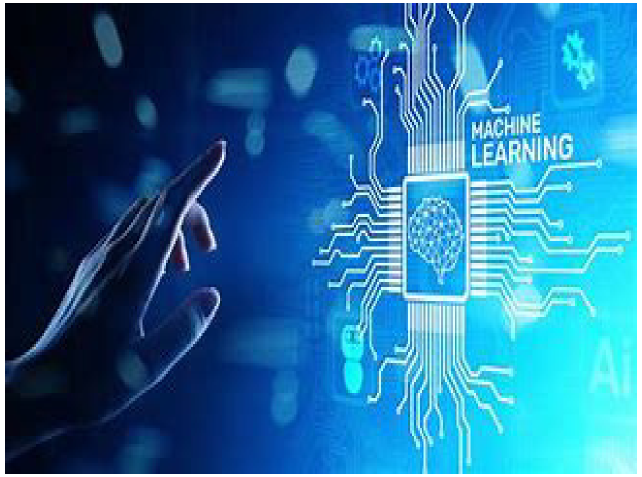
- Of particular interest is Wilcox ("Why EAFE is for Wimps", JPM, 1994) which found that a weight vector closer to equal weighting persistently outperformed traditional capitalization weighting to a significant degree.

Inspired by Wilcox, we undertook two efforts to apply cointegration methods for further improve outcomes

- We wrote a short paper on the EAFE case in 1999 from which a lot of today's material is drawn Active Returns from Passive Management: Cointegration of Country Indices in EAFE (northinfo.com)
- The paper was updated in 2005, Cointegration - Dan.PDF (northinfo.com)
- A second update appeared in 2013, 573.pdf (northinfo.com)

All three efforts showed that portfolios could be found to reliably outperform EAFE, and that the significant predictive power was very long lasting.

Old School Machine Learning



The 1999 experimental design of the use of cointegration for country weights in EAFE is an early example of what is now called "machine learning"

We used Monte Carlo simulation to find forty cointegrating vectors of country weights for EAFE + D where D was various levels of incremental monthly returns (e.g. 20 bps).

- The cointegrating vectors were then averaged to form a final vector which was then held over various "out of sample" test periods.

A key finding was that of the forty cointegrating vectors analyzed for each value of D, the percentage of the forty that outperformed EAFE out of sample was increasing in D.

- It's like training for a runner to jump over hurdles on a track. If they practice on higher hurdles, the likelihood of failing on lower hurdles in deceased.

Cointegration of Global Sectors

Given the preceding analogy to "hurdles," our experiments with global sector returns were designed to be much tougher in terms of the ability of cointegration to add significant value.

Our global index was changed from EAFE to the Northfield internal index used to represent the market portfolio in our Global equity risk model.

- This index is square root of capitalization weighted, which should already capture much of the benefit of "close to equal weighting" found by Wilcox, presumably leaving less opportunity for cointegration to add value.

We used the six broad sector definitions from our Everything, Everywhere model.

- With only six sectors the "degrees of freedom" associated with the changing portfolio weights is far less than the seventeen countries represented in the EAFE studies.

Experimental Design with Six Global Sectors

Like the EAFE analyses, we used Monte Carlo simulation to obtain cointegrating vectors with various levels of monthly incremental returns (e.g. 10 bps)

To make it even more difficult for cointegration to add value, we calculated a fully equal weighted by sector index, to capture the potential benefit of full equal weighting as compared to the "square root" weighting of the Northfield index.

- Cointegrating vectors were built off the equal weighted sector portfolios
- Monthly incremental returns ranged from 5 to 20 bps.

To address the lower number of sectors compared to countries, the number of cointegrating vectors obtained for each level of incremental return was increased from forty to one hundred.

- In a live investment process, we would increase this to several hundred, as running times are not excessive for modest values of the return increment.

The overall sample period was 208 months from December 31, 2004 through March 31st, 2022 in the previous work, which has now been updated through February 29, 2024

Empirical Results One Year Holding Periods -2022

				Annualized Returns						
	Start Month In Sample	1	13	25	37	49	61	73	Average	Alpha
	End Month In Sample	120	132	144	156	168	180	192		EW
	Start Month Out of Sample	121	133	145	157	169	181	193		
	End Month Out of Sample	132	144	156	168	180	192	204		
Monthly	0.05	1.08	9.94	19.71	-8.46	21.53	17.11	13.49	10.63	0.90
Target	0.10	2.29	8.65	18.15	-7.60	20.97	14.08	14.28	10.12	0.39
Alpha	0.15	0.60	15.44	14.99	-8.20	21.10	21.81	14.71	11.49	1.77
	0.20	3.96	15.10	14.95	-9.15	20.45	15.69	14.47	10.78	1.06
	Equal	-0.44	10.82	17.25	-10.29	21.16	15.21	14.36	9.72	
Coint	Average	1.98	12.28	16.95	-8.35	21.01	17.17	14.24	10.76	
	NIS GLB	9.03	6.45	13.70	-8.90	14.40	10.70	14.35	8.53	
	Alpha EW	2.42	1.46	-0.30	1.94	-0.15	1.96	-0.12	**1.03**	
	Alpha GLB	-7.05	5.83	3.25	0.55	6.61	6.47	-0.11	**2.22**	

Empirical Results Three Year Holding Period-2022

	Annualized Returns					
Start Month In Sample	1	13	25	37	Average	Alpha
End Month In Sample	120	132	144	156		EW
Start Month Out of Sample	121	133	145	157		
End Month Out of Sample	156	168	180	192		
0.05	9.98	6.30	9.46	9.79	8.88	1.28
0.10	10.08	4.72	10.39	7.12	8.08	0.48
0.15	10.04	5.51	8.37	7.59	7.88	0.28
0.20	9.69	8.15	8.36	6.06	8.07	0.46
Equal	8.96	5.24	8.42	7.78	7.60	
Average	9.95	6.17	9.15	7.64	8.23	
NIS GLB	9.79	4.04	4.81	4.74	5.85	
Alpha EW	0.99	0.93	0.73	-0.14	0.63	
Alpha GLB	0.16	2.13	4.34	2.90	2.38	

One Year Rebalance Results to Today

					Annualized Returns								
	Start Month In Sample		1	13	25	37	49	61	73	85	97	Average	Alpha
	End Month In Sample		120	132	144	156	168	180	192	204	216		EW
	Start Month Out of Sample		121	133	145	157	169	181	193	205	217		
	End Month Out of Sample		132	144	156	168	180	192	204	216	228		
Monthly		0.05	1.08	9.94	19.71	-8.46	21.53	17.11	13.49	-11.77	16.58	8.80	1.11
Target		0.10	2.29	8.65	18.15	-7.60	20.97	14.08	14.28	-11.60	12.31	7.95	0.26
Alpha		0.15	0.60	15.44	14.99	-8.20	21.10	21.81	14.71	-10.83	13.10	9.19	1.50
		0.20	3.96	15.10	14.95	-9.15	20.45	15.69	14.47	-6.92	12.34	8.99	1.30
	Equal		-0.44	10.82	17.25	-10.29	21.16	15.21	14.36	-11.71	12.82	7.69	
Coint	Average		1.98	12.28	16.95	-8.35	21.01	17.17	14.24	-10.28	13.58	8.73	
	NIS GLB		9.03	6.45	13.70	-8.90	14.40	10.70	14.35	-16.41	11.65	6.11	
	Alpha EW		2.42	1.46	-0.30	1.94	-0.15	1.96	-0.12	1.43	0.76	1.05	
	Alpha GLB		-7.05	5.83	3.25	0.55	6.61	6.47	-0.11	6.13	1.93	2.62	

Three Year Rebalance through Today

		Annualized Returns							
	Start Month In Sample	1	13	25	37	49	61	Average	Alpha
	End Month In Sample	120	132	144	156	168	180		EW
	Start Month Out of Sample	121	133	145	157	169	181		
	End Month Out of Sample	156	168	180	192	204	216		
Monthly	0.05	9.98	6.30	9.46	9.79	12.98	4.77	8.88	0.14
Target	0.10	10.08	4.72	10.39	7.12	17.74	6.68	9.46	0.72
Alpha	0.15	10.04	5.51	8.37	7.59	15.86	5.19	8.76	0.02
	0.20	9.69	8.15	8.36	6.06	15.98	4.87	8.85	0.11
	Equal	8.96	5.24	8.42	7.78	16.87	5.17	8.74	
Coint	Average	9.95	6.17	9.15	7.64	15.64	5.38	8.99	
	NIS GLB	9.79	4.04	4.81	4.74	13.49	4.76	6.94	
	Alpha EW	0.99	0.93	0.73	-0.14	-1.23	0.21	0.25	
	Alpha GLB	0.16	2.13	4.34	2.90	2.15	0.62	2.05	

Empirical Results Discussion

Using a 120 month in-sample period, successive 12 month out of sample periods showed an *annual outperformance of 1.02%* (2022) and 1.05% (2024) compared to equal weighted.

- As expected, the equal weighted index outperformed the "square root" weighted Northfield index substantially. *The annual outperformance of the cointegrating portfolio relative to the Northfield index was 2.22% (2022) and 2.62% (2024)*

Maintaining the 120 month in-sample period, but increasing the out of sample period to 36 months, the cointegrating portfolios *outperformed the equal weighted index by .63% per annum (2022) and .25 (2024 and the Northfield "square root" index by 2.38% (2022) and 2.05% (2024).*

The benefit of cointegration was highly statistically significant as established by a Komologorov-Smirnoff Type 2 test (KS2)

Given the relatively large outperformance relative to the Northfield index, the benefit would be expected to be economically material even after transaction costs.

Conclusions

Cointegration analysis is a powerful statistical technique for finding asset allocations that may be able act as a hedge against a variety of liabilities.

It can be used in a hedging framework to pursue active management returns.

Previous empirical tests of cointegration on country index data provided encouragement that such techniques may provide avenues to earn returns in excess of market indices.

Recent analysis of sector returns confirms the benefit of cointegration despite additional measures intended to distinguish the benefit of cointegration from “closer to equal” weighting.