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Variety is the Spice of Active Management

(as well as Life)

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Introduction

In general, the opportunity set for active management is defined by both market *volatility* and by the cross-sectional dispersion (i.e. "*variety*") of the security returns for the universe allowed under the strategy.

- You can think of tax aware strategies such as tax-loss harvesting as options, in which variety is the "volatility of the underlying."
- We will next consider how we can correct for the usual (but generally incorrect) presumption that the cross-section of returns is normally distributed.

We will introduce formulae that can be used to predict the magnitude of the cross-sectional dispersion measure and the ex-ante implications for classic equity factors such as "value" and "momentum".

- We will show how to calculate other useful information such as the average pairwise correlation across a universe of securities.
- Empirical data from late 2020 through late 2024 will be used to illustrate the various uses to which the variety concept can be put.

Asymmetric Beta from Market Timing

- Early research into "factor exposure variation" assumed that the only factor that mattered were market returns and that all time series variation in portfolio beta was the result of intentional market timing.

- Treynor and Mazuy (Harvard Business Review, 1966) proposed a variation of the CAPM to test for market timing, but we can use the same formulation for individual securities by redefining a few things.,

$$R_t = \alpha + \beta * (R_{mt} - R_{ft}) + c * (R_{mt} - R_{ft})^2 + \varepsilon_t$$

- The model estimates the coefficients by time series regression.
  - The estimated alpha of a stock estimated is α.
  - The expectation of additional return due to asymmetry in beta is the variance of the market premium times coefficient C
  - *In this construct in which time series **volatility** that matters.*

Alpha as Asymmetric Beta?

Another way that has been used to describe the expected magnitude of a security's market relative performance is to assert that the CAPM beta of the stock will have different values in up markets and down markets.

If the "up beta" is bigger than the "down beta" we have positive alpha

If the "up beta" is smaller than the "down beta" we have negative alpha

All expectations converge at zero when the expected return of the market portfolio is equal to the risk-free rate.

This concept is often applied at the level of actively managed portfolios and referred to as "market capture".

Estimation requires a sufficiently large numbers of observations such that the up and down beta values have small confidence intervals, so that the statistical significance of the difference between the two values can be determined.

High Concentration/Conviction Portfolio Research

There is a large and growing literature that reports that contrary to the efficient market hypothesis, the return/risk tradeoffs associated with concentrated active portfolios are generally superior.

- The underlying hypothesis was first proposed by Roll (JPM, 1992) suggesting that active managers who were *actually confident* in the forecasts would be willing to concentrate their portfolio more in order to accentuate their difference from the benchmark.

The basic argument is why be like the benchmark if you think you can beat it?

- Wilcox (JPM, 1994): *Why EAFE is for Wimps!* Shows you can be diversified in absolute terms but concentrated in benchmark relative by adopting equal country weights in international portfolios.

Academic papers include:

- Fulkerson and Riley (JEF, 2019)
- Choi, Fedenia, Skiba, and Sokolyk (JFE, 2017)
- Jordan and Riley (JFE, 2015)

Correlation is Key

- Assuming a one factor model Solnik and Roulet (2000) show that he correlation of any pair of two securities is given by:

$$P_{ij} = 1 / [(1 + s_e^2 / B_i^2 s_w^2)^{.5} * (1 + s_e^2 / B_j^2 s_w^2)^{.5}]$$

P_{ij} = the correlation of asset i with asset j

s_e = the residual standard deviation of the return for each asset

s_w = the standard deviation of the return for the world index

B_i = the beta of asset i with respect to the world index

- The comparable calculations for multifactor models can be found in Optimization with Composite Assets Using Implied Covariance Matrices

Defining *Variety*

- Lilo, Mantegna, Bouchard and Potters (2002) first used the term *variety* to describe cross-sectional dispersion of stock returns.
  - They also define the cross-sectional dispersion of CAPM alpha as *idiosyncratic variety* (noted as v(t)).

- Usefully, they find that the instantaneous estimate of average correlation between all possible pairs of stocks conditional on a single value of the market return (r_m) is:

$$C(t) = 1 / [1 + (v^2(t)/r_m^2(t)]$$

- Average correlation then becomes link between volatility (as observed in the squared market return) and variety.

Variety as the Opportunity Set for Security Selection

- deSilva, Sapra, and Thorley (2001) provides an extensive survey of the implications of cross-sectional dispersion for active management of equity portfolios.
  - Rather than use variety as a way of inferring the correlation between assets, they derive an expression for the cross-sectional variation of returns across a set of securities, which is used as a metric for the opportunity set available for active managers.
  - In the spirit of Lilo, et al. they derive the following expression for the cross-section variance conditional on a one period market return:

$$E\,[D^2\,] = s_B^{\,2}\,(r_m - r_f\,) + s_e^{\,2}$$

D = the cross-sectional standard deviation of asset returns during the single period

s_B = the cross-sectional standard deviation of beta values

r_m = the return on the market portfolio during the single observation period

r_f = the risk-free return during the single observation period

s_e = the residual standard deviation of the return for each asset

Variety and Equity Style

We previously described the variety concept in a book chapter: diBartolomeo, Dan. 2007. "Applications of Portfolio Variety", in J. Knight and S. Satchell Editors, Forecasting Volatility, Butterworth-Heinemann.

- An early manuscript of this chapter is available on our website at Volatility, Variety and the Active Manager.

Many active equity "styles" can be understood through the concept of variety as strategies that are sensitive to security prices replicate option payoffs.

- Variety is the volatility of these implicit options.
- Momentum strategies (e.g. CPPI) replicate being long a put, while "value" replicates being short a put.
- For momentum, securities that have gone up are expected to keep going up, while securities that have gone down are expected to keep going down. The variety of observed returns will increase with the length of the observation period **faster** than the square root of time.
- For value strategies, we expect mean reversion in returns. The variety of observed returns will increase with the length of the observation period **slower** than the square root of time.

Variety and Tax Loss Harvesting

It should be intuitive that the opportunity to efficiently offset capital gains and losses in the management of a portfolio depends on the extent to which some securities in the portfolio go up in value, while other securities go down in value (i.e. variety).

- The decision when to engage in tax loss harvesting activities can be thought of as a “tax timing option” as the investor can decide when they want to act.

There are several papers describing the details of the tax timing option for various sorts of securities.

- Kalotay (JPM, 2014), Kalotay (FAJ, 2016), Kang, Paradise, and Dickson (FAJ, 2021),
- Israelov and Lu (2022), Davis, Li, and Nemtchinov (JBIS, 2022)

In a 2008 conference presentation, we simulated these relationships and showed that the ratio of the expected market return premium and variety, determines the efficiency of tax loss strategies.

- Microsoft PowerPoint - 1 - diBartolomeo Variety of Security Returns.ppt

Ex-Ante Alpha Scaling (Grinold and Kahn)

Quant equity managers are familiar with the "alpha scaling rule of thumb" of Grinold (JPM, 1994)

- Alpha = Volatility * IC * Score (z)

Many alpha scaling procedures fail to notice the detailed notation of the derivation provided in Chapters 10 and 11 of Active Portfolio Management (Grinold and Kahn, 2nd Edition, 2000)

- The basic formulation is designed for time series data on a single asset (i.e. is now a good time to buy IBM relative to the usual attractiveness of IBM)
- There is a separate derivation for cross-sectional data when we want to compare the attractiveness of multiple securities based on data only from the current moment in time.
- You have to be careful about one assumption and to not confuse cross-sectional and time series inputs

See Shah (2007) for further details
Microsoft PowerPoint - 3 - Shah Alpha Scaling.ppt (northinfo.com)

- These scaling methods and others (e.g. Black Litterman) are available in our optimizer.

Some Empirical Data to Consider

- The table below provides summary data from our US Fundamental Risk Model from October of 2020 through November of 2024.
  - The model covers everything traded in the US, including all foreign companies traded as cross-listings or ADRs (i.e. most large firms globally)
  - Average volatility values were calculated over the full period for factor risk, idiosyncratic risk, total risk and R-squared on both equal weighted (focus on small cap) and capitalization weighted (focus on large cap firms).
  - Smaller firms have great volatility overall. Most of the incremental increase arises from greater idiosyncratic risk with our chosen factor structure expected to explain about a third of variance.
  - For larger capitalization firms, the factors naturally explain more at about half of the variance.   While there are no statistically significant trends in the large firm time series, the small firm time series shows a downward trend in explanatory power (i.e. the sky is no longer falling due to COVID)

N	Avg_spec	Avg_fact	Avg_abs	Avg_R2	Wt_Spec	Wt_fact	Wt_abs	Wt_R2
6700.20	48.97	30.14	59.26	0.32	26.80	25.74	37.83	0.50

The Abnormality of Normality

Unfortunately, everything I've discussed today embeds the simplifying assumption that the cross-sectional distribution of security returns is normal.

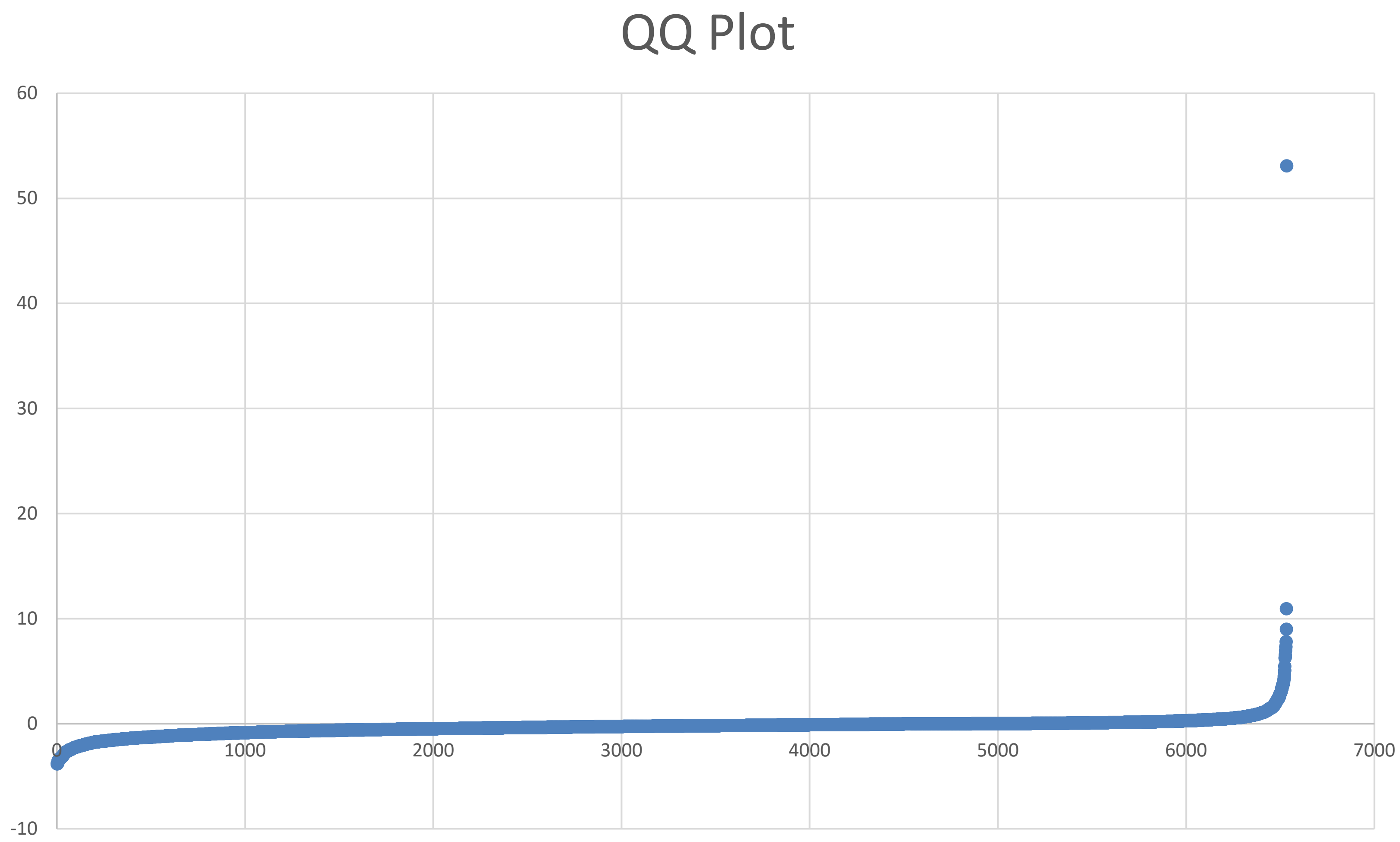
- **That's almost always wrong.**
- Typically, the cross-sectional distribution is very fat tailed (positive excess kurtosis)

Active managers **must** believe that their strategies will create superior performance.

- They do this by increasing the likelihood of chosen securities having returns in the upper tail of the cross-section and decreasing the likelihood of returns in the lower tail.
- This implies that their expectation of the distribution of active returns will have positive skew, which is often wrong (i.e. not all managers can outperform).

Like many other aspects of portfolio management that incorrectly assume normal distributions, we can correct the estimation of variety for higher moments.

US Universe Returns for October 2023



October 2023 QQ Plot with 5% Tails Removed

Tail Probability Representation of Alpha

I treat the cross-section of returns as three regions, and adjust to mean relative

- Upper 5%: Mean 40, CSD = 33, probability M_u
- Middle 90%: Mean 0, CSD = 9.5 probability M_{90}
- Lower 5%: Mean -40, CSD = 13 with probability M_l

- $M_u + M_{90} + M_l = 1$

I assume that the probability of stock J of landing in one of the tails is proportional to idiosyncratic risk which is the "volatility" input in the time series version of the Grinold formula.

- Probability of J being in a tail is

  $10\% * [\sigma_j / \text{mean}(\sigma)]$

A Simple Example

- We will start with a typical alpha estimate for Stock J = + 3%, $\sigma_j = 20$
- Assume mean value of σ = 16

20/16* 10% = 12.5% likelihood of being in one of the two tails

- The weighted mean of the three states must equal my alpha

$$3 = (M_u * 40) + (.875 * 0) + ((.125 - M_u) * (-40))$$

$$M_u = .1$$

$$M_l = .025$$

- My alpha estimate of 3% implies that the *odds of being in the upper tail are 4 times as great as being the lower tail, implying positive skew in the distribution of active return.*

Robertson and Fryer (1969)

- To obtain the four moments of our mixture distribution with three states

$$\mu = \Sigma_{i=1}^{3} m_i \mu_i$$

$$\sigma^2 = \Sigma_{i=1}^{3}[m_i \left(\sigma_i^2 + \mu_i^2\right) - \mu_i^2]$$

$$S = \frac{1}{\sigma^3} \{\Sigma_{i=1}^{3} m_i (\mu_i - \mu)[3\sigma_i^2 + (\mu_i - \mu)^2]\}$$

$$K = \frac{1}{\sigma^4} \{\Sigma_{i=1}^{3} m_i [3\sigma_i^4 + 6(\mu_i - \mu)^2 \sigma_i^2 + (\mu_i - \mu)^4]\}$$

where

m_i = the probability of state *i* , μ = mean return of the combined distribution

σ = standard deviation of the combined distribution

μ_i = the expected return in state *i* , σ_i = the expected volatility in state *i*

S = skew of the combined distribution, K = kurtosis of the combined distribution (raw)

Use CF to Adjust Variety for Higher Moments

- After we have computed the ex-ante distribution of active return using the Robertson and Fryer formulas, we can convert the four-moment distribution into the economically equivalent two moment distribution where the standard deviation value is *variety*
  - We rescale σ by the Cornish Fisher tail weight parameter to get the new estimate of *variety* V

$$V = \sigma W / Z$$

W = CF “tail weight” parameter = $Z + (S/6) * (Z^2-1) + Z * ((K-3)/24) * (Z^2 – 3)- Z*(S^2/36) * (2Z^2 -5)$

Z = user choice of Z-score for the preferred “best fit” confidence interval (e.g. 1.645 = 95%)

We need to be careful of dW/dZ going negative. W should be monotonically increasing in Z.

Conclusions

The effectiveness of all non-passive investment strategies are jointly dependent on the volatility returns (over time) and the variety of returns within each time period observed.

- The mathematical linkage between volatility and variety is the correlation of returns across securities.
- Security selection strategies, equity style strategies, and tax loss harvesting are all explicitly dependent on the variety of returns.

There is a useful set of simple formulae to describe and forecast variety for both single factor and multi-factor models which are then inputs to many other tasks (e.g. ex-ante alpha scaling)

- Unfortunately, most of the formulae make the convenient but incorrect assumption that the relevant distribution is normal.
- We use our usual methods to correct the estimation of variety for higher moments.