TAA With Pair-wise Strategies

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Outline

- Why pairs?
- Conventional approach
  - Tactical asset allocation
  - Active currency management
  - Sector strategies
- Pair-wise strategies
- TAA with pair-wise strategies
Conventional Approach

- US Stock Forecast
- US Bond Forecast
- UK Stock Forecast

- US Stock Position
- US Bond Position
- UK Stock Position
Conventional Approach

- Forecasting process
  - Individual time series model
  - Time series information coefficient (IC) \( IC = \rho(f_t, r_t) \)
- Portfolio construction
  - MV optimization
  - Sample covariance matrix
- Performance
  - Information ratio (IR) \( IR = \frac{\text{avg}(\alpha)}{\text{std}(\alpha)} \)
Problems With Conventional Approach

- Equity bias
  - Solution: de-mean the forecasts

- Three mysteries
  1) Steining doesn’t help?
  2) Additional models don’t help IR?
  3) Diagonal covariance is superior in back test?

- Why? Answer: pair-wise analysis
Pair-wise Strategies

- Risky asset versus risk-free asset or risky asset versus risky asset
- IC is only good for risky asset/cash pairs, it is no good for pairs between two risky assets

\[ w_1 = \lambda^{-1} f_1 \]
\[ w_0 = -\lambda^{-1} f_1 \]
\[ \alpha = w_1 r_1 = \lambda^{-1} f_1 r_1 \]
\[ \text{avg}(\alpha) \propto \text{corr}(f_1, r_1) \sigma(f_1) \sigma(r_1) \]
Two Risky Assets

- Pair-wise IC matters! PIC – correlation coefficient between the forecast premium and return premium

\[
\begin{align*}
    w_1 &= \lambda^{-1}(f_1 - f_2) \\
    w_2 &= \lambda^{-1}(f_2 - f_1) \\
    \begin{pmatrix}
        w_1 \\
        w_2
    \end{pmatrix} &= \lambda^{-1} \begin{pmatrix}
        1 & -1 \\
        -1 & 1
    \end{pmatrix} \begin{pmatrix}
        f_1 \\
        f_2
    \end{pmatrix} \\
    \alpha &= w_1 r_1 + w_2 r_2 = \lambda^{-1}(f_1 - f_2)(r_1 - r_2) \\
    \text{avg}(\alpha) &\propto \text{corr}(f_1 - f_2, r_1 - r_2) \sigma(f_1 - f_2) \sigma(r_1 - r_2)
\end{align*}
\]
Pair-wise IC (PIC)

- PIC is a combination of IC and cross IC

\[
\text{PIC} = \frac{\text{cov}(f_1 - f_2, r_1 - r_2)}{\sigma(f_1 - f_2)\sigma(r_1 - r_2)} = c_1 \rho(f_1, r_1) + c_2 \rho(f_2, r_2) - c_3 \rho(f_2, r_1) - c_4 \rho(f_1, r_2)
\]

- Positive IC is good for PIC, but positive cross IC is not
An Example

- **Domestic TAA**: stock/bond/cash
- **IC**: 0.2, 0.2; **Cross IC**: 0.2, 0.15
- **PIC**: 0.08!

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<th>Bond Return</th>
<th>Stock Forecast</th>
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Pair-wise IR

- Pair-wise tracking error

$$\text{std}(\alpha) \propto \sigma (f_1 - f_2) \sigma (r_1 - r_2)$$

- Pair-wise IR is approximately pair-wise IR

$$\text{avg}(\alpha) \propto \text{PIC} \cdot \sigma (f_1 - f_2) \sigma (r_1 - r_2)$$
Mystery Solved

- Pair-wise IC is part of the reason for (1) and (2).
- Steining increased model IC, but it also increased cross IC. The combining effect on PIC is zero.
- Additional models have good IC itself, but might have poor PIC when combined with other existing models.
Implications for Model Building

- When building separate models, be mindful of cross IC. Global or common factors often bring cross IC
- Focus on specific factors
- When possible (when N is small), build pair-wise premium models
- Always analyze and assess forecasts through pair-wise framework
TAA With Pair-wise Strategies

- We can prove MV optimization in the simplest form is equivalent to a combination pair-wise strategies
- Mathematics versus intuition
- We will demystify the black box
Pair-wise Trading With Stock/bond/cash

- Three pairs: (0 1), (0 2), (1 2)
- Three pair-wise bets
  \[ w_{1,0} = \lambda^{-1}(f_1) \quad w_{2,0} = \lambda^{-1}(f_2) \quad w_{1,2} = \lambda^{-1}(f_1 - f_2) \]
  \[ w_{0,1} = -\lambda^{-1}(f_1) \quad w_{0,2} = -\lambda^{-1}(f_2) \quad w_{2,1} = \lambda^{-1}(f_2 - f_1) \]
- Three pair-wise alphas
  \[ \alpha_{ij} = \lambda^{-1}(f_i - f_j)(r_i - r_j), \quad i, j = 0, 1, 2; \ i < j \]
Pair-wise Weights

- The only remaining decision is how to mix them.
- Pair-wise weights:
  \[ p_{01}, p_{02}, p_{12} \]
- Total alpha:
  \[
  \alpha = p_{01} \alpha_{01} + p_{02} \alpha_{02} + p_{12} \alpha_{12}
  = \lambda^{-1} \sum_{i<j} p_{ij} (f_i - f_j)(r_i - r_j)
  \]
TAA With Pair-wise Strategies

- Construct TAA with pair-wise strategies
  - Select pair-wise weight
  - Scale active weights in pairs by $p_{ij}$
  - Aggregate weight in all relevant pairs

- The role of optimization
  - Optimization is one way to select pair-wise weights
  - Optimization gives a set of implied pair-wise weights
The Advantage of Pairs

- Each pair is a “security”
  - Expected return (alpha), risk (tracking error)
  - Correlation matrix among pairs
- TAA is a portfolio of pair-wise “securities”
  - Given pair-wise weights, we can compute IR
  - We can find the optimal pair-wise weights
  - In practice, we can choose pair-wise weights to
    - Treat pairs differently
    - Balance the risks of pairs
    - Trade pairs separately
MV Optimization and Pairs

- Two risky assets, one risk-free asset (stock/bond/cash) (2/1/0)
- No constraint, three pairs (0 1); (0 2); (1 2)
- MV optimization
  - No correlation
  - With correlation
MV Optimization and Pairs

- We can write the alpha from MV optimization in terms of three pair-wise alphas.

\[ S = \begin{pmatrix} \frac{1}{s_1^2} & \frac{\zeta}{s_1 s_2} \\ \frac{\zeta}{s_1 s_2} & \frac{1}{s_2^2} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = S^{-1} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \]

\[ \alpha = w_1 r_1 + w_2 r_2 \]

\[ \alpha_t \propto \left( \frac{1}{s_1^2} - \frac{\zeta}{s_1 s_2} \right) r_1 f_1 + \left( \frac{1}{s_2^2} - \frac{\zeta}{s_1 s_2} \right) r_2 f_2 + \left( \frac{\zeta}{s_1 s_2} \right) (f_2 - f_1)(r_2 - r_1) \]

\[ p_{01} \quad p_{02} \quad p_{12} \]
MV Optimization and Pairs

- MV optimization implies a set of weights
- These weights are a function of the covariance matrix
  - No correlation
    - Only two pairs between the risky assets and the risk-free asset
    - No bet between the two risky assets
  - Correlation
    - All three pairs
Another Mystery Solved

- If the pair between the two risky assets is inferior to the two other pairs, then using a full covariance matrix rather than a diagonal one leads to inferior performance.
TAA Performance

• Expected IR \( \alpha_t = \lambda^{-1} \sum_{i,j=1}^{N} p_{ij} \alpha_{ij} \)
  – Expected return of each pair \( \overline{\alpha}_{ij} \)
  – Covariance matrix of pairs \( \text{cov}(\alpha_{ij}, \alpha_{kl}) \)

• Given these results, we can obtain the expected alpha of TAA and expected tracking error using traditional portfolio theory

• We can also obtain the optimal pair-wise weights that give the TAA with the highest IR
Summary

- Beware of MV optimization
  - What are the implied pair-wise weights?
  - Are they consistent with pair-wise IC (PIC)?
- The role of covariance matrix
  - It is implicitly assigning pair-wise weights
  - Sample estimate
    - Good for long-term strategic purpose
    - Good for single-period risk management
    - Not a good choice for multi-period tactical asset allocation
Summary

- Advantage of pair-wise framework
  - Apply to a variety of macro quantitative strategies
  - Simplify modeling process
  - Balance risk contribution of pairs
  - Easy use of trading concept
  - Calculate expected performance
  - Attain optimal information for given set of forecasts
Appendix

Mathematical Proof
Active MV Optimization

- Objective function
  \[ G(\vec{A}_t) = \vec{A}'_t \cdot \vec{f}_t - \frac{1}{2} \lambda (\vec{A}'_t \cdot \vec{S} \cdot \vec{A}_t) \]

- Possible constraint
  \[ \vec{A}'_t \cdot \vec{i} = 0 \quad \text{No risk-free asset} \]

- Solution
  \[ \vec{A}_t = \lambda^{-1}(\vec{S}^{-1} \cdot \vec{f}_t) \quad \text{No constraint} \]

  \[ \vec{A}_t = \lambda^{-1}(\vec{P} \cdot \vec{f}_t), \vec{P} \cdot \vec{i} = 0 \quad \text{No risk-free asset} \]

- Alpha
  \[ \alpha_t = \lambda^{-1}(\vec{f}'_t \cdot \vec{P} \cdot \vec{r}_t) \]
MV Optimization – A Linear Combination of Pairs

\[ \bar{A}_t = \lambda^{-1}(P \cdot \bar{f}_t) \quad P \cdot \bar{i} = 0 \]

\[ P = \begin{pmatrix} \sum_{i \neq 1} p_{1i} & -p_{12} & \cdots & -p_{1N} \\ -p_{21} & \sum_{i \neq 2} p_{2i} & \cdots & -p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -p_{N1} & -p_{N2} & \cdots & \sum_{i \neq N} p_{Ni} \end{pmatrix} \]

\[ Q_{ij} = \begin{pmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 1 & -1 & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & -1 & 1 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & & & 0 \end{pmatrix} \]

\[ P = \sum_{i,j=1}^{N} p_{ij} Q_{ij} \]
MV Optimization – A Linear Combination of Pairs

$$\alpha_t = \lambda^{-1} \left( \tilde{f}_t' \cdot P \cdot \tilde{r}_t \right)$$

$$\tilde{f}_t' \cdot Q_{ij} \cdot \tilde{r}_t = (f_i - f_j)(r_i - r_j)$$

$$\rightarrow \alpha_t = \lambda^{-1} \sum_{\substack{i, j = 1 \\ i < j}}^{N} p_{ij} (f_i - f_j)(r_i - r_j)$$
MV Optimization – A Linear Combination of Pairs

- This proves that MV optimization with the constraint is equivalent to a linear combination of pair-wise strategies between risky assets.
- For unconstrained optimization, in addition to these pairs, it also includes pairs between all assets and the risk-free asset.