Active Returns from Passive Management: Portfolio Formation through Cointegration

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Topics for Today

• Basics of Cointegration

• Mathematics of Cointegration

• Literature Review

• An Empirical Example
  - Country Weights in a Global Equity Portfolio

• Conclusions
Motivation

• A small but increasing number of asset management mandates for pension funds are now being measured directly against the accrual of the liability, rather than market benchmarks.

• At least one hedge fund uses the country weighting strategy from our 1999 working paper. They represent that they developed it independently and we believe them.

• Academic literature on using cointegration for portfolio formation has gotten much richer.

• Research shows that returns on hedge funds may seem uncorrelated with financial markets but are actually cointegrated.
Basics of Cointegration

• Cointegration analysis is a widely used with economic times series
• Cointegration is the situation where we can form a linear combination of time series that is stationary from a series of times that are not individually stationary
• By stationary we mean that the time series has consistent mean, standard deviation and autocorrelation properties. We can formally define:

\[ X_t = aX_{t-1} + bt + c + et \]

- \( X_t \) = the value of series X at time t
- \( t \) = the increment count of time
- \( et \) = error term at time t

• Series X is stationary if the absolute value of coefficient a is statistically significantly less than one using the Dickey-Fuller statistic
More Cointegration Basics

- Most “level” based financial times series (prices, index values) are not stationary
- The cointegrating vector is the set of coefficients (i.e. weights) applied to each input times series to form the stationary combined series

\[ S_t = \sum_{i=1}^{n} w_i A_{it} \]

- If series S is stationary and the input series are not, then W is the cointegrating vector
- See Hamilton (1994) for math details
Financial Markets Rationale

• Cointegration methods look directly at financial asset prices, rather than returns
  - Financial assets are way of demarcating the capital provided to enterprises by investors
  - If one asset class produces much higher returns than another over time, eventually one has to look “expensive” and the other “cheap”
  - The relative value judgments of investors keep the relative prices of financial assets within rational bounds
Literature Review

• In finance, cointegration methods have been widely used to test theories of market efficiency
More Literature

- Others have looked more generally at finding economic linkages between asset markets
  - Kanas, Angelos. 
Interesting Recent Studies

• Larger hedge funds may look uncorrelated to major stock financial markets are actually cointegrated

• Another study has formalized and extended our 1999 paper. They find cointegration methods to track an index is equally efficient as traditional MV optimization based on historic data, but is much better for enhanced strategies (index +) based on statistical arbitrage
A Side Note

• Similar mechanisms for forming portfolios weights are around
Using Cointegration for Portfolio Formation

• Form a hedging problem where we seek to have the surplus between assets and liabilities be stationary
• Work in units of value, not returns

\[ S_t = \sum_{i=1}^{n} w_i A_{it} - L_t \]

- \( S_t \) = the value of surplus at time \( t \)
- \( w_i \) = the weight of asset \( i \) in the portfolio
- \( A_{it} \) = the value of asset \( i \) at time \( t \)
- \( L_t \) = the value of the liability at time \( t \)

• We can specify the “liability” as we wish
  - Actual liabilities accruing to a pension fund over time
  - The liability grows with the total return to a market index benchmark
  - The liability grows with the total return of market index plus a return premium (e.g. S&P 500 + 2% per annum)
Finding the Cointegrating Vector

• If we use cointegration methods to form portfolio weights it is analogous to traditional optimization in Markowitz (1959) using historical data
  – But cointegration methods do not assume that the return times series are stationary, or that returns normally distributed, nor that correlations amongst returns are stable

• Assuming unlimited shorting we can use a special form of regression to find the “best” cointegrating vector

• Without shorting, we employ a constrained regression, like returns-based analysis developed in Sharpe (1992)
  – Requires approximation of the confidence intervals on the coefficients as derived in Lobosco and diBartolomeo (1997)
Cointegration for Dummies

• Use Monte Carlo simulation to create lots of different asset weight vectors with whatever constraints you have

• Test each one for cointegration against your “liability”

• Keep going until you have a good sample of cointegrating portfolios (i.e. more than 40)

• Average your cointegrating vectors

• Select the particular cointegrating vector closest in weights to the average of the cointegrating vectors, based on the summation of the absolute differences in the asset weights
Our 1999 Empirical Test

• Take all seventeen countries in EAFE that had data from 1970 through 1998. All data in US$
• 180 months of in-sample data from January 1970 through December 1984
• 168 months of out of sample data from January 1985 through December 1998
• Use Monte-Carlo method to find 40 cointegrating vectors with all positive weights during the in sample period
• During the out of sample period all weights are rebalanced monthly
• Compare the out of sample results of cointegrated portfolios to non-cointegrated portfolios
• Test five levels of return premium from zero to 20 basis points per month
## Out of Sample Results 1985 - 1999

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<th>Monthly Return Premium</th>
<th>Mean Coint Return</th>
<th>Annual Target Return</th>
<th>Annual EAFE Return</th>
<th>% Coint &gt; Target</th>
<th>% Coint &gt; EAFE</th>
<th>% Not Coint &gt; Target</th>
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Update the Experiment

• Extend the return times series with FTSE data through April 2005
  - We don’t use much MSCI data anymore
  - Keep the old MSCI data through 1998
  - Replace EAFE with the FTSE Developed World ex-US
  - Replace the MSCI country indices with the corresponding FTSE indices

• Now analyze the longer out of sample period based on the original 1970 through 1984 sample period
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Discussion of Out of Sample Results

- Due to the Monte-Carlo portfolio formation method, all the portfolios are relatively close to equal weighted.
- The vast majority of portfolios both cointegrated and non-cointegrated consistently produced returns superior to the capitalization weighted index.
  - Previously described in Wilcox (1994).
- The cointegrated portfolios have a statistically significantly higher chance of meeting the target returns.
- The average returns on the cointegrated portfolios increase as the target is raised.
- The certainty of outperforming the basic index improves as the target is raised.
An Alternative Specification

• Use the equal weighted country indices plus a return premium as the target
  - This will remove the issue of equal weight versus capitalization weighted performance across countries
  - Reduces processing time dramatically

• Compute the active weights of the cointegrated portfolios versus the equal weighted country index

• Use these active weights in running against the traditional capitalization weighted benchmarks
  - Same as running a long/short based on the active weights and using futures or ETFs to get the capitalization weighted core component
Conclusions

• Cointegration methods attempt to detect economic linkages between the values of financial assets that are persistent in time.

• Cointegration methods provide a methodology for forming optimal portfolios without explicit return forecasting.

• Empirical evidence suggests that for “index plus” cointegration strategies, the higher the target return premium, the greater the likelihood of achieving at least some positive index relative return.
References