Risk Budgeting: Concept, Interpretation and Applications

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PANAGORA
The Concept

Risk Contribution

- **Risk contribution** – attribution of total risk to individual underlying components of a portfolio in percentage terms

- **Examples**
  - Fixed income portfolio: sector risk, yield curve risk, …
  - Equity portfolio: systematic risk (risk indices/industries), specific risk, …
  - Asset allocation: TAA risk (stock/bond, cap rotation, …), sleeve active risk, …
  - Manager selection, strategy allocation
  - Asset allocation portfolio: beta risk from stocks, bonds, commodities, …
    - Parity portfolios
The Concept

An Example

- **A simple illustration**
  - Two active strategies – strategy A at 1% active risk, strategy B at 2% active risk, two sources uncorrelated

- **Total active risk**
  - Equals \( \sqrt{1\%^2 + 2\%^2} = 2.24\% \)
  - What is the risk contribution from strategy A and B?
  - Answer – A contributes 20% and B at 80%
  - Contribution to variance
    - Strategy A \( \frac{1}{1+4} \), Strategy B \( \frac{4}{1+4} \)

- **Contribution based upon variances and covariances**
The Concept

Mathematical Definition

- Marginal contribution - $\frac{\partial \sigma}{\partial w_i}$
- Risk contribution - $w_i \frac{\partial \sigma}{\partial w_i}$
- VaR contribution - $w_i \frac{\partial \text{VaR}}{\partial w_i}$

- Well defined in financial engineering terms
- But objected by some financial economists for a lack of economic interpretation
The Concept

Objections

- Risk is not additive in terms of standard deviation or VaR
- You can only add risk when returns are uncorrelated
- A mathematical decomposition of risk is not itself a risk decomposition
- Risk budgeting only make sense if considered from mean-variance optimization perspective
- Marginal contribution to risk is sensible, but not risk contribution
**Interpretation**

**Loss Contribution**

- **Question**: For a given loss of a portfolio, what are the likely contribution from the underlying components?
- **Answer**: The contribution to loss is close to risk contribution

- Risk contribution can be interpreted as loss contribution
- Risk budgeting ~ loss budgeting
- *Risk budgets do add up*
Interpretation

An Asset Allocation Example

- **Balanced benchmark**
  - 60% stocks, 40% bonds
  - Is it really balanced?

- **Risk contribution**
  - Stock volatilities at 15%, bond volatilities at 5%
  - Stocks are actually *9 times riskier than bonds*
  - Stocks’ contribution is roughly 95% and bonds at 5%

\[
\frac{6^2 \cdot 3^2}{4^2 \cdot 1^2 + 6^2 \cdot 3^2} = 95\%
\]

- **The balanced fund – a misnomer**
Interpretation

Loss Contribution of a 60/40 Portfolio

<table>
<thead>
<tr>
<th>Loss &gt;</th>
<th>Stocks</th>
<th>Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2%</td>
<td>95.6%</td>
<td>4.4%</td>
</tr>
<tr>
<td>3%</td>
<td>100.1%</td>
<td>-0.1%</td>
</tr>
<tr>
<td>4%</td>
<td>101.9%</td>
<td>-1.9%</td>
</tr>
</tbody>
</table>

- Stocks’ contribution is almost 100%
- It is greater than the theoretical value of 95%, because stocks have greater tail risk
**Interpretation**

**Mathematical Proof**

- **Question:** For a given loss of a portfolio, what are the likely contribution from the underlying components?
- **Answer:** Conditional expectation

\[
\text{Total risk: } \sigma = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \rho w_1 w_2 \sigma_1 \sigma_2}
\]

\[
p_1 = \left( w_1 \frac{\partial \sigma}{\partial w_1} \right) / \sigma = \frac{w_1^2 \sigma_1^2 + \rho w_1 w_2 \sigma_1 \sigma_2}{\sigma^2}
\]

\[
p_2 = \left( w_2 \frac{\partial \sigma}{\partial w_2} \right) / \sigma = \frac{w_2^2 \sigma_2^2 + \rho w_1 w_2 \sigma_1 \sigma_2}{\sigma^2}
\]

**Risk contribution:**

- Loss contribution? \[ c_i = E \left( w_i r_i \mid w_1 r_1 + w_2 r_2 = L \right) / L \]
Interpretation

Mathematical Proof

- The loss contribution is approximately risk contribution
- Cases where they are identical
  - Zero expected returns
  - Large losses
  - Mean-variance optimal weights

\[
c_1 = p_1 + \frac{p_2 w_1 \mu_1 - p_1 w_2 \mu_2}{L} \quad p_1 + \frac{D_1}{L}
\]

\[
c_2 = p_2 + \frac{p_1 w_2 \mu_2 - p_2 w_1 \mu_1}{L} \quad p_2 + \frac{D_2}{L}
\]
**Interpretation**

**Mean-variance Optimal**

- Risk contribution is equivalent to loss contribution
- Risk contribution is also equivalent to expected return contribution

\[
\frac{w_1 \mu_1}{p_1} = \frac{w_2 \mu_2}{p_2} = \lambda
\]

- The interpretation is valid and very close even if the portfolio is not MV optimal
Interpretation

Loss Contribution

Figure 1  The value of $D_1$ over standard deviation for asset allocation portfolios

- The difference between risk contribution and loss contribution is quite small
Application

Loss Contribution of a 60/40 Portfolio

<table>
<thead>
<tr>
<th>Loss</th>
<th>Predicted $c_1$</th>
<th>Realized $c_1$</th>
<th>N</th>
<th>Predicted Std</th>
<th>Realized Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4% to -3%</td>
<td>93.5%</td>
<td>89.8%</td>
<td>45</td>
<td>28.0%</td>
<td>26.1%</td>
</tr>
<tr>
<td>-5% to -4%</td>
<td>92.8%</td>
<td>92.7%</td>
<td>23</td>
<td>21.0%</td>
<td>20.7%</td>
</tr>
<tr>
<td>-6% to -5%</td>
<td>92.3%</td>
<td>88.1%</td>
<td>11</td>
<td>16.8%</td>
<td>16.1%</td>
</tr>
<tr>
<td>-7% to -6%</td>
<td>92.0%</td>
<td>99.5%</td>
<td>9</td>
<td>14.0%</td>
<td>18.7%</td>
</tr>
<tr>
<td>-8% to -7%</td>
<td>91.8%</td>
<td>90.1%</td>
<td>8</td>
<td>12.0%</td>
<td>18.6%</td>
</tr>
<tr>
<td>-19% to -8%</td>
<td>91.3%</td>
<td>102.4%</td>
<td>12</td>
<td>10.5%</td>
<td>12.3%</td>
</tr>
</tbody>
</table>

- Realized loss contribution from stocks increases as losses grow
- It could be due to higher tail risks of stocks
Application

Fat Tails

Risk contribution in terms of standard deviation assumes normal distribution

We need to extend the risk contribution to non-normal return portfolio
  • Hedge funds

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>US LT Gvt</th>
<th>60/40 Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg Return</td>
<td>0.98%</td>
<td>0.46%</td>
<td>0.78%</td>
</tr>
<tr>
<td>Stdev</td>
<td>5.61%</td>
<td>2.27%</td>
<td>3.61%</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.39</td>
<td>0.66</td>
<td>0.40</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.58</td>
<td>5.09</td>
<td>7.64</td>
</tr>
<tr>
<td>Corr w/ S&amp;P 500</td>
<td>1.00</td>
<td>0.14</td>
<td>0.97</td>
</tr>
</tbody>
</table>

• Ibbotson 1929 - 2004
### Hedge Fund Returns

<table>
<thead>
<tr>
<th></th>
<th>Convertible Arbitrage</th>
<th>Dedicated Short</th>
<th>Emerging Markets</th>
<th>Equity Mkt Neutral</th>
<th>Distressed</th>
<th>E.D. Multi-Strategy</th>
<th>Risk Arbitrage</th>
<th>Fixed Income Arb</th>
<th>Global Macro</th>
<th>Long/Short Equity</th>
<th>Managed Futures</th>
<th>Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>8.94</td>
<td>-0.87</td>
<td>8.83</td>
<td>9.80</td>
<td>13.00</td>
<td>10.31</td>
<td>7.82</td>
<td>6.64</td>
<td>13.70</td>
<td>11.77</td>
<td>6.89</td>
<td>9.13</td>
</tr>
<tr>
<td>Stdev</td>
<td>4.70</td>
<td>17.66</td>
<td>16.88</td>
<td>2.99</td>
<td>6.64</td>
<td>6.13</td>
<td>4.30</td>
<td>3.79</td>
<td>11.46</td>
<td>10.52</td>
<td>12.20</td>
<td>4.35</td>
</tr>
<tr>
<td>IR</td>
<td>1.90</td>
<td>-0.05</td>
<td>0.52</td>
<td>3.28</td>
<td>1.96</td>
<td>1.68</td>
<td>1.82</td>
<td>1.75</td>
<td>1.20</td>
<td>1.12</td>
<td>0.57</td>
<td>2.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>-1.36</td>
<td>3.38</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.86</td>
<td>2.03</td>
</tr>
<tr>
<td>IR</td>
<td>-0.62</td>
<td>4.30</td>
</tr>
</tbody>
</table>

- CSFB Tremont 01/1994 – 03/2005

- Some strategies have high IR, but also high negative skewness and high positive kurtosis
Value at Risk
- Maximum loss at a given probability
- 95% VaR, 99% VaR

Contribution to VaR
- Interpretation: the expected contribution to a portfolio loss equaling the size of VaR
- It changes with different level of VaR
- It is not easy to calculate because analytic formula is rarely available for VaR
Calculating VaR Contribution

Cornish-Fisher Approximation

- Cornish-Fisher approximation for VaR

\[ \text{VaR} = \mu + \tilde{z}_\alpha \sigma \]

\[ \tilde{z}_\alpha \approx z_\alpha + \frac{1}{6} (z_\alpha^2 - 1) s + \frac{1}{24} (z_\alpha^3 - 3z_\alpha) k - \frac{1}{36} (2z_\alpha^3 - 5z_\alpha) s^2 \]

- Example
  - 99% VaR for the 60/40 portfolio
  - Normal assumption VaR at 99% is -7.6%, \( z_\alpha = -2.33 \)
  - Approximate VaR is -13%, \( \tilde{z}_\alpha = -3.81 \)
Calculating VaR Contribution

Cornish-Fisher Approximation

- Approximation for VaR contribution
- We have an analytic expression of VaR
- Algebraic function, linear, quadratic, cubic and quartic polynomials
- We can then calculate $w_i \frac{\partial \text{VaR}}{\partial w_i}$
Calculating VaR Contribution

Application to 60/40 Portfolio

<table>
<thead>
<tr>
<th>Loss</th>
<th>Predicted VaR %</th>
<th>Predicted c_f</th>
<th>Realized c_f</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.50%</td>
<td>84.90%</td>
<td>93.5%</td>
<td>89.8%</td>
</tr>
<tr>
<td>-4.50%</td>
<td>90.50%</td>
<td>92.8%</td>
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- Stocks’ VaR contribution increases as VaR increases
- Capture the high tail risks
- Contribution to standard deviation shows declining instead
Calculating VaR Contribution

Other Applications

- Need to calculate skewness and kurtosis from historical returns
- Risk management and risk budgeting
  - Asset allocation with traditional assets and hedge funds
    - High moments strategies: event driven, distressed securities, fixed income arbitrage, etc
  - Strategy allocation among managers with non-normal alphas
Summary

Risk Budgets Do Add Up

- Financial interpretation of risk contribution
  - Loss contribution
  - It applies to both standard deviation and VaR
  - Cornish-Fisher approximation can be used to calculate VaR contribution
  - It should clear up some doubts about the concept

- Applications
  - Risk budgeting for active risks
  - Risk budgeting for beta portfolios – parity portfolios
  - Efficient method for risk budgeting of hedge funds