Multi-Period Optimization for Private Client Asset Allocation

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Today's Goals

- Create an understanding of why simply doing a traditional asset allocation, and changing that as conditions change isn't good enough
- Consider a "life planning' approach to integrating financial planning so as to allow for a projected asset allocation schedule
- Review various techniques for controlling asset class turnover efficiently
- Describe a method of multi-period optimization that does not require solution via dynamic programming

The Challenge of Private Clients

- Private clients are heterogeneous. They require a high degree of customization
  - Most investments are taxable, and taxes are a vastly bigger issue than the transaction costs that all investors face
  - Private investors will often have different pools of wealth set aside to fund specific consumption events. An intuitive approach, but inefficient
  - Investor preference functions evolve during a finite life span. The goals and objectives will be constantly changing
  - The desire to liquidate investment assets for consumption is less predictable than institutions

A Proposal

- To address the particular asset allocation needs of private clients we propose a multi-period optimization approach that includes three key elements
  - Provides appropriate integration of taxable and tax deferred investments, including taxes on distributions
  - Provides a “life balance” sheet approach to revising the investor’s risk tolerance through time to maximize the median of expected wealth accumulation
  - Includes a nearly exact solution to multi-period optimization without the need for complex dynamic programming

A Major Theoretical Concern with Traditional Asset Allocation

- Markowitz Mean-variance optimization
  - Its assumed that our forecasts of future returns and risks are exactly correct and good forever. Risk tolerance is presumed to constant across time
  - If its free to rebalance a portfolio, the single period assumption does no harm. When our market beliefs change, our portfolio changes with them. *Traditional methods are reliant on this view*
  - ***In the real world, changing asset allocations is very costly in fees and taxes. We need to think ahead to avoid unnecessary rebalancing costs***
  - Estimation errors are especially important as it's often expensive to rebalance taxable portfolios. We assume you already address this issue
  - For rational investors risk tolerance changes over time and with wealth in a predictable fashion

Traditional Asset Allocation Adapted

- The key issue in formulating investment policies is how aggressive or conservative an investor should be to maximize their long term wealth subject to a shortfall constraint (a floor on net worth)

  $$U = E\{ R * (1-T^*) - L S^2 (1-T^*)^2 / 2 \}$$

  - L is the ratio of total assets/net worth
  - In Northfield terminology RAP = 2/L
  - **T* is the effective tax rate which can vary by asset class**

- We derive the total assets and net worth from the assets and liabilities on an investor's "life balance sheet". This can be flexibly defined to include the present value of implied assets such as lifetime employment savings, and expected outlays such as retirement college tuition, charitable donations and estate taxes

Life Cycle Investing
Using the Life Balance Sheet

- The “life balance sheet” concept integrates changes in both age and wealth into a single determinant of optimal aggressiveness
- See the CFA handbook we wrote. Jarrod wrote this part
- We can use different discount rates to arrive at present value based on preferred certainty of the outcome.
  - I may want to be 99.9% sure of meeting my retirement goals, but am willing to live with a 75% chance of fulfilling a desired charitable donation. I discount retirement needs at the risk free rate, the charitable donation like a junk bond
  - I have 100% certainty of my current financial assets, but only 50% certainty of the inheritance that may go my evil twin brother. I discount my expected inheritance like a junk bond
- Using this procedure over time will maximize the median rather than the mean of log wealth in the long run. This is similar to the concept to Constant Proportion Portfolio Insurance

The Implications For Portfolio Turnover

- If we periodically derive risk tolerance as a function of an investor's personal "life balance sheet", it will evolve in predictable ways as time passes
  - We get closer to retirement
  - College expenses arise and are completed
  - Saving is reduced at retirement
- Our net worth and risk tolerance will also change as result of fluctuations in wealth arising from financial market returns being different than we have predicted
- Both influences will cause predictable degrees of turnover in our asset allocation over time

What Would We Really Like to Do?

- We'd like to plan the evolution of our portfolio asset allocation to take into account changing financial circumstances and changing conditions in the financial markets
- If changing isn't free, then what we have to do now is dependent on what we think we're going to have to do in the future
- A lot of academic work on this kind of process:
  - Samuelson, Paul A. "Lifetime Portfolio Selection By Dynamic Stochastic Programming," Review of Economics and Statistics, 1969, v51(3), 239-246.
  - Computationally very messy and expensive as problems get to realistic numbers of assets
  - Current work by Stanford Professor Gerd Infanger

Multi-period Optimization

- Nobody really does the full process. Parameter estimation error is the killer here
  - In practice, investors have enough difficulty estimating long run risk and return parameters as of “now”
  - *For full multi-period optimization we must estimate now what the return distribution parameters will be for all future periods, period by period*
- But what if we assume return distributions are stable (as we do in traditional asset allocation) but that allocation changes arise from changes in investor risk tolerance over time, and fluctuations in market returns
  - The degree of turnover is predictable to a high degree
  - It took one of my staff two pages of integral calculus that you don’t really want to see. At least I didn’t.
  - This will come in very handy later

Geometric Versus Linear Tradeoffs

- For small transaction costs, arithmetic amortization is sufficient, but if costs are large we need to consider compounding
- Assume a trade with 20% trading cost and an expected holding period of one year.
  - We can get an expected return improvement of 20%. But if we give up 20% of our money now, and invest at 20%, we only end up with 96% of the money we have now.
- Solution is to adjust the amortization rate to reflect the correct geometric rate

Probability of Realization Measure

- In the traditional single period assumption our expectations are always forecasts of the population statistics, not forecasts of the sample statistics over a finite horizon
- If we can predict portfolio turnover through the life cycle approach, we can adjust the MVO process to incorporate the likelihood that one portfolio will outperform another over a finite horizon
- We define the probability of realization, P, like a one-tailed T test

$$P = N\,(((U_o - U_i) / TE_{io}) * (1/ A)^{.5})$$

- N(x) is the cumulative normal function:

$$N(x) = \frac{1}{\sqrt{2p}} \int_{-\infty}^{x} e^{-u^2/2}\, du$$

The Realization Probability

- The numerator is the improvement in risk adjusted return between the optimal and initial portfolios
- The denominator is the tracking error between the optimal and initial portfolios. Essentially it's the standard error on the expected improvement in utility
  - If there is no tracking error between the initial and optimal portfolios, P approaches 100%. Consider "optimizing a portfolio" by getting the manager to cut fees. The improvement in utility is certain no matter how short the time horizon.
  - *Not something to which we usually pay attention*
- If turnover is very low, A will approach zero, so P will approach 100%. For long time horizons, we have the classical case that assumes certainty

Implementing the Fix

- Even if we are amortizing our costs sensibly, we are still maximizing the objective function to directly trade a unit of risk adjusted return for a unit of amortized cost per unit time.
  - This is only appropriate if we are certain to realize the economic benefit of the improvement in risk adjusted return, which is only true over an infinite time horizon
  - We propose to adjust the amortization rate to reflect the probability of actually realizing the improvement in utility over the expected time horizon, and the investor's aversion to the uncertainty of realization

$$U = R - S^2 / T - (C \times \Gamma)$$

$$\Gamma = A / (1 - Q * (1-P)), \quad T = 1/L, \quad Q = 1 - (T/200)$$

P is the probability of realizing the improvement in risk adjusted return over the expected time horizon and Q is the range of (0,1)

The Operational Recipe

1. Start with your usual estimates of risk return and correlation

2. Put together the investor's "life balance" sheet to get values for an initial L value

3. Run a traditional MVO with risk aversion L_t for each period of the projected asset allocation schedule with some initial assumption for portfolio turnover

4. Calculate expected turnover required including both risk aversion changes and drifts in asset weights

5. Go back to step #3 with an adjusted amortization rate that reflects the refined expectation of turnover. Keep repeating step 3, 4 and 5 until internally consistent

Conclusions

- Traditional fixed asset allocation schemes ignore both investor life cycle and important aspects of the costs of portfolio rebalancing
- The single period assumption in MVO implies that trading costs and improvements in utility can be traded as if both are certain. In addition to other sources of estimation error, finite holding periods imply that the improvement in utility is uncertain and the way we trade utility improvements and costs must reflect this
- Our proposed method allows for the creating of an optimal asset allocation schedule that incorporates important aspects of multi-period mean variance optimization

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