Risk Containment for Hedge Funds

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Raging Asset Class

- 30 fold growth in assets under management since 1990
- Estimate > 2000 new funds launched last year
- In US equities: 5% of assets, but 30% of trading volume
  (source: sec.gov)

- Premium for top funds, e.g. Caxton 3/30, Renaissance 5/44, SAC 50% of profits
Extensive Literature


Fundamental Idea

For both investors and managers, hedge funds (though they may be benchmarked to long-only or cash) are a totally different animal.

- Non-Gaussian return distributions
- Liquidity and leverage/credit considerations
- Dynamic investment strategies

Traditional measures of performance and risk – std dev, tracking error, $\beta$, $\alpha$, Sharpe ratio – are non-descriptive.
Part I: Complications for the Investor


Weisman 2002, “Informationless Investing And Hedge Fund Performance Measurement Bias”

“How to manufacture performance with no skill”
1. No Skill $\alpha$

From Lo 2001:

**Table 3. Capital Decimation Partners, LP, January 1992–December 1999**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>CDP</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly mean (%)</td>
<td>3.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Monthly standard deviation (%)</td>
<td>5.8</td>
<td>3.6</td>
</tr>
<tr>
<td>Minimum month (%)</td>
<td>-18.3</td>
<td>-8.9</td>
</tr>
<tr>
<td>Maximum month (%)</td>
<td>27.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Annual Sharpe ratio</td>
<td>1.94</td>
<td>0.98</td>
</tr>
<tr>
<td>Number of negative months (out of total)</td>
<td>6/96</td>
<td>36/96</td>
</tr>
<tr>
<td>Correlation with S&amp;P 500</td>
<td>59.9</td>
<td>100.0</td>
</tr>
<tr>
<td>Total return (%)</td>
<td>2,721.3</td>
<td>367.1</td>
</tr>
</tbody>
</table>
The Secret: Short Volatility
(selling insurance - risk is invisible until it happens)

- Writing options
  Lo’s example sells out of the money puts

- Writing synthetic options by $\Delta$ hedging
  (dynamically altering the mix of stock and cash)
  *Executed without owning derivatives*

- Issuing credit default swaps

- Betting that spreads return to typical levels
  e.g. LTCM, see Jorion 2000
Frequent Small Gains Exchanged for Infrequent Large Losses

$S(T)$

Probability
Option Writer Gain
Option Writer Loss
Performance of Short Vol Strategy

From Weisman 2002:
2. Estimated Prices for Illiquid Securities

- Value of infrequently traded securities is estimated
- Even operating in earnest, one is likely to undershoot both losses and gains
- Underestimate volatility
- Overestimate value after a series of losses
  i.e. exactly when positions must be liquidated
- Behavior evidenced by serial correlation in returns

- A separate phenomenon: Up returns are, in general, shrunk by performance fees. So, the return of the underlying investments (in particular, downside) is more volatile than indicated by reported returns
Suppose the estimate is a combination of present and past true returns:

\[ r_t^{\text{estimated}} = (1-w)r_t + wr_{t-1} \]
\[ \sigma^2_{\text{estimated}} = [(1-w)^2 + w^2] \sigma^2 \]

\[ \text{SR} = \frac{r-r_f}{\sigma} \]

\[ w = 50\% \quad \rightarrow \quad \text{Estimated SR} \uparrow 41\% \]
\[ 25\% \quad \rightarrow \quad \uparrow 26\% \]
\[ 10\% \quad \rightarrow \quad \uparrow 10\% \]
3. Increasing Bets After Loss

Weisman 2002 – St. Petersburg Investing

If you lose $1 on the first bet, wager $2 on the next. If you lose that bet, wager $4 on the next, etc.

Low probability of losing, but loss is extreme

Can happen inadvertently – $10 long, $10 short, $10 cash. Lose on the shorts: $10 long, $12 short, $10 cash. Size of bets jumps from 200% to 275%

($20 on net $10 → $22 on net $8)
Performance of St. Petersburg Strategy

From Weisman 2002:
Theory Meets Reality

LTCM
90% of return explained by monthly changes in credit spread
1/98 → 8/98, lost 52% of its value. Leverage jumped from 28:1 → 55:1

Nick Maounis, founder of Amaranth Advisors:
"In September, 2006, a series of unusual and unpredictable market events caused the fund's natural-gas positions, including spreads, to incur dramatic losses"

“We had not expected that we would be faced with a market that would move so aggressively against our positions without the market offering any ability to liquidate positions economically.”

"We viewed the probability of market movements such as those that took place in September as highly remote … But sometimes, even the highly improbable happens.”
Addressing Short Volatility

- Bondarenko 2004

- From options on futures, price a variance contract
  \[ \frac{dF_t}{F_t} = \sigma_t \, dW_t \]
  \[ d\log F_t = \frac{dF_t}{F_t} - \frac{1}{2} \sigma_t^2 \, dt \]

  \[ E_0 \left[ \int_0^T \sigma_t^2 \, dt \right] = \text{price of variance contract at time } 0 \]
  \[ = -2 E_0 [\log (F_T / F_0)] + E_0 [\int_0^T \frac{dF_t}{F_t}] \]
  \[ = -2 E_0 [\log (F_T / F_0)], \text{ calculated via option prices’ risk-neutral density} \]

- Over the interval, sample realized variance, \( \int_0^T \sigma_t^2 \, dt \)

- (Sampled – Priced) / Priced = the return to variance. Averaging over experiments yields the variance return premium
Empirical Value of Short Volatility

- The premium is negative, i.e. the market pays (above the value of the risk itself) to pass off variance

- Adding the time series of variance returns as a factor in style analysis
  1) reveals a fund’s exposure
  2) corrects estimated alpha to account for this source of return

- Bondarenko finds hedge funds as a group earn 6.5% annually from shorting volatility
Addressing Serial Correlation

Fit model that explicitly incorporates the structure of serial correlation

Getmansky et al 2004

\[ r_t^{\text{reported}} = \sum_k \theta_k r_{t-k} \]
\[ r_t = \mu + \beta m_t + \epsilon_t \]
\[ \text{mean } 0 \]
\[ \sum_{k=1..K} \theta_k = 1 \]
\[ \epsilon_t, m_t \sim \text{IID}, \]
\[ \text{var}(r_t) = \sigma^2 \]
Nonlinearities: Different Up and Down Market Sensitivities

From Lo 2001:
\[ r_t = \alpha + \beta^- m_t^- + \beta^+ m_t^+ + \varepsilon_t \]

Lo also provides a model to account for phase-locking behavior

e.g. correlations across assets rising during catastrophic markets

<table>
<thead>
<tr>
<th>Table 8. Nonlinearities in Hedge-Fund Index Returns: Monthly Data, January 1996–November 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Style Index</td>
</tr>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Currencies</td>
</tr>
<tr>
<td>ED—distress</td>
</tr>
<tr>
<td>ED—merger arb</td>
</tr>
<tr>
<td>EM—equity</td>
</tr>
<tr>
<td>EM</td>
</tr>
<tr>
<td>EM—fixed income</td>
</tr>
<tr>
<td>ED</td>
</tr>
<tr>
<td>Fund of funds</td>
</tr>
<tr>
<td>Futures trading</td>
</tr>
<tr>
<td>Growth</td>
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<tr>
<td>High yield</td>
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<tr>
<td>Macro</td>
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<tr>
<td>Opportunistic</td>
</tr>
<tr>
<td>Other</td>
</tr>
<tr>
<td>RV—convertible</td>
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<tr>
<td>RV—EQLS</td>
</tr>
<tr>
<td>RV—option arb</td>
</tr>
<tr>
<td>RV—other—stat arb</td>
</tr>
<tr>
<td>Short selling</td>
</tr>
<tr>
<td>Value</td>
</tr>
</tbody>
</table>

Note: Regression analysis of monthly hedge-fund index returns with positive and negative returns on the S&P 500 used as separate regressors. ED = event driven; arb = arbitrage; EM = emerging market; RV = relative value; EQLS = equity long/short; stat = statistical.

Source: AlphaSimplex Group.
Part II: Complications for the Manager

- Chow & Kritzman 2002, “Value at Risk for Portfolios with Short Positions”

Recall Usual Brownian Motion Model for Stock Price Movement

\[
\frac{dS}{S} = \mu \, dt + \sigma \, dW \\
\frac{d\log S}{S} = (\mu - \frac{1}{2} \sigma^2) \, dt + \sigma \, dW
\]

Although instantaneous return is normal, 
(1 + return) over time is lognormal:

\[
\frac{S_T}{S_0} = e^{\left[(\mu - \frac{1}{2} \sigma^2)T + \sigma W_T\right]}
\]

Sum of lognormal ≠ lognormal
Lognormal has positive skew, limited downside

- Positive skew in returns...
- ...becomes a long left tail for short positions.

- A separate issue:
  - Long and wrong → exposure decreases
  - Short and wrong → exposure increases
Lognormal portfolio approximation, ok for long only, breaks down with long/short

From Van Royen, Kritzman, Chow 2001:
A Better Framework for Long/Short Risk

Model *each side* of a long/short portfolio as a geometric Brownian motion

\[
\begin{align*}
dL/L &= \mu_L \, dt + \sigma_L \, dW_L \\
dS/S &= \mu_S \, dt + \sigma_S \, dW_S \\
\end{align*}
\]

\[dW_L dW_S = \rho \, dt\]

Dynamics of \(L - S\) describe behavior of long/short portfolio

Answer quantitative and qualitative questions (Winston 2006)
“What is the expected time to hit drawdown?”
“What is the probability the portfolio is > $110 in 1 year without falling below a drawdown of $80 in the interim?”
“How does increasing short-side volatility affect the probability of ruin?”

\(L - S\) is not a geometric Brownian motion

See mathematical literature for options on spreads
Ways to tame the non-GBM, L – S

Approximate L-S by a Brownian motion with the same mean and variance at time T

Look at ratio, \( f = L / S \)
\[
df = \frac{dL}{S} - \frac{dL}{S^2} + \frac{L}{S^3} \, d\langle S \rangle - \frac{1}{S^2} \, d\langle S, L \rangle
\]
\[
df/f = [\mu_L - \mu_S + \sigma_S^2 - \rho \sigma_L \sigma_S] \, dt + \sigma_L dW_L - \sigma_S dW_S \quad \rightarrow \quad f \text{ is GBM}
\]

Kirk approximation (used in Winston 2006)
Interested in \( P(L – S < \text{critical } k) = P(L/[S+k] < 1) \)
let \( g(L, S) = L/[S+k] \)
will be approximating \( S/(S+k) \) by \( S_0/(S_0+k) \)

\[
\begin{align*}
dg &= \frac{dL}{S+k} - \frac{dL}{(S+k)^2} + \frac{L}{(S+k)^3} \, d\langle S \rangle - \frac{1}{(S+k)^2} \, d\langle S, L \rangle \\
dg/g &= \frac{dL}{L} - \frac{dS}{S} \left[ \frac{S}{(S+k)} \right] + \sigma_S^2 \left[ \frac{S}{(S+k)} \right]^2 \, dt - \rho \sigma_L \sigma_S \left[ \frac{S}{(S+k)} \right] \, dt \\
&\approx \frac{dL}{L} - \frac{dS}{S} \left[ \frac{S_0}{(S_0+k)} \right] + \sigma_S^2 \left[ \frac{S_0}{(S_0+k)} \right]^2 \, dt - \rho \sigma_L \sigma_S \left[ \frac{S_0}{(S_0+k)} \right] \, dt \\
&\quad \text{which is BM}
\end{align*}
\]
Applications

Success and failure surfaces from Winston 2006:

- Initial leverage=3 (long=2, short=1)
- 1 year
- 300bps skill on long side, 200bps skill on short side
- .5 correlation
- 90% drawdown absorbing barrier
Summary

Hedge funds offer investment strategies poorly described by traditional tools and measures.

If investors aren’t aware of the hidden risks, surely they will select for them. e.g. 4:00 mile is fast, 3:30 mile = a goat?

Managers of long/short portfolios are exposed to phenomena not present in long-only. Avoiding a blow-up requires extra vigilance.