

# Using the Analytic Hierarchy Process to Assist Asset Allocation and Manager Search

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# How to Quantify a Qualitative Process

- ◆ Institutional asset managers and consultants face the task of selecting and assigning assets to money managers to satisfy the needs of the beneficiaries.
- ◆ Academic theory says that this is accomplished by using mean-variance analysis to maximize utility, usually a quadratic function of active return and risk.
- ◆ Investment practice is very different from theory in this case, and the process is much more qualitative than theory assumes.

# The Analytic Hierarchy Process: Background

- ◆ Thomas Saaty, a professor at the University of Pittsburgh, developed the AHP as a way to improve complex decision making and to identify and weight selection criteria.
- ◆ AHP is a methodology that arises from operations research literature. AHP is used as a non-parametric method for making complex, often qualitative decisions in a robust, consistent fashion.
- ◆ AHP provides a proven, effective means to deal with analyzing the data collected for the decision criteria and expediting the decision-making process.
- ◆ A wide body of literature indicates the AHP is useful when making complex decisions involving multiple criteria.

# Analytic Hierarchy Process: Mechanics

- ◆ For *each* evaluation criterion, usually expressed as a multiple choice question, the AHP creates a comparison matrix.
- ◆ The upper triangle holds the relative ratings (1-9, with 1 being best) of the alternatives: asset classes or fund managers.
- ◆ The diagonal of the matrix is ones – every fund compared with itself is a 1!
- ◆ The lower triangle is the reciprocal of the upper triangle:  $x(i, j) = 1 / x(j, i)$ 
  - If A is 9 times as good as B, then B is 1/9 as good as A

# Analytic Hierarchy Process: Mechanics

- ◆ When the comparison matrix has been filled, the matrix's first eigenvector will contain the weights to assign to each choice.
- ◆ For this application we use these weights as the asset class or manager allocation for that criterion.
- ◆ The portfolio weights for each criterion are then averaged using the weight for each criterion.
- ◆ It's a form of "importance weighted" average score.

# Literature: Using the AHP in Investment Management

- ◆ Bolster, Janjigian, and Trahan, "Determining Investor Suitability Using the Analytic Hierarchy Process," *Financial Analyst's Journal*, July/August 1995
- ◆ Saraoglu and Miranda Lam Detzler, "A Sensible Mutual Fund Selection Model," *Financial Analysts Journal*, May/June 2002
- ◆ Khaksari, Shahriar, Ravindra Kamath and Robin Grieves. "A New Approach To Determining Optimum Portfolio Mix," *Journal of Portfolio Management*, 1989, v15(3), 43-49.

# Institutional Asset Allocation

- ◆ If we used mean-variance optimization, we would:
  - Choose the appropriate liability (benchmark):
    - ❖ The CPI (to preserve spending power)
    - ❖ A bond of known duration
    - ❖ A model portfolio that represents typical peer group policy
  - Develop return expectations for each asset class relative to liabilities.
  - Estimate the co-variance between each asset class
  - Use optimization to determine the efficient frontier
  - Pick the position on the efficient frontier that fits the beneficiaries' risk tolerance relative to liabilities.

# Example from HBS Case Study of Harvard Management Company

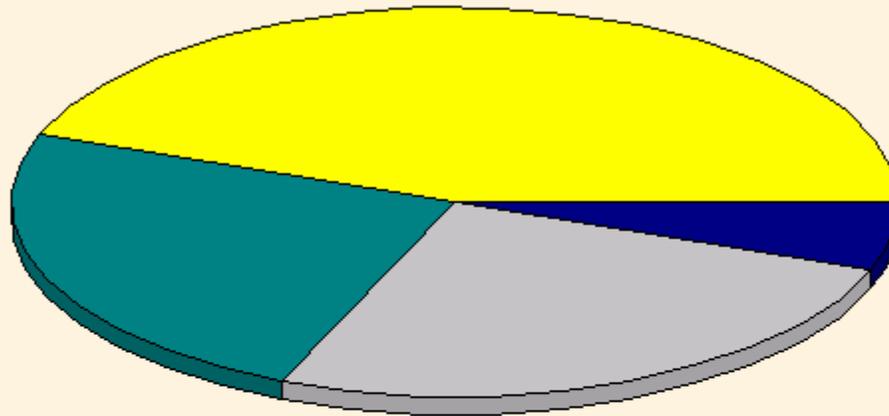
Asset Class	Policy Wt.	Real Return	Risk	r, US Stock	r, US Bond
Cash	-3%	3.0	1.0	0.10	0.10
Inflation Indexed Bonds	7%	3.6	3.0	0.10	0.40
Non US Bonds	4%	4.0	8.0	0.10	0.30
US Bonds	10%	4.0	7.0	0.35	1.00
Real Estate	7%	5.0	12.0	0.20	0.20
Commodities	6%	5.3	10.0	-0.15	-0.10
High Yield Bonds	3%	5.2	12.0	0.55	-0.40
Absolute Return Strategies	5%	5.3	12.0	0.70	0.25
Private Equity	15%	9.5	20.0	0.50	0.15
Emerging Market Bond & Stock	9%	8.5	20.0	0.50	0.05
Non US Stock	15%	6.5	17.0	0.60	0.15
US Stock	22%	6.5	16.0	1.00	0.35

# An Asset Allocation Example

- ◆ We have returns data on twelve reasonable asset class proxies that can model Harvard's asset allocation.
- ◆ We estimate returns:
  - Using historic returns and a Bayesian adjustment.
  - Using the returns from the case study in the previous slide
- ◆ We estimate the co-variance matrix using historic returns.
- ◆ We estimate the risk and return of the policy portfolio and compare it to the efficient frontier.

# Asset Allocation: Optimal Portfolio

Optimal (Portfolio 006)



Fund	Init	Opt
Citi Treasury-InflationLkd	13.0	44.3 %
Citi ESBI Index USD!	3.0	24.3 %
NAREIT Equity!	7.0	25.7 %
Goldman Sachs Commodity!	6.0	5.7 %

**This does not seem to be a well diversified portfolio.**

**Returns were estimated using Bayesian adjustment over a ten-year sample period.**

## Portfolio # 6

Investment Horizon	5.0
5 Years Return	3.9
5 Years Std Dev	3.3
Single Year Return	3.9
Single Year Std Dev	7.4
Annual Yeild	-0.1

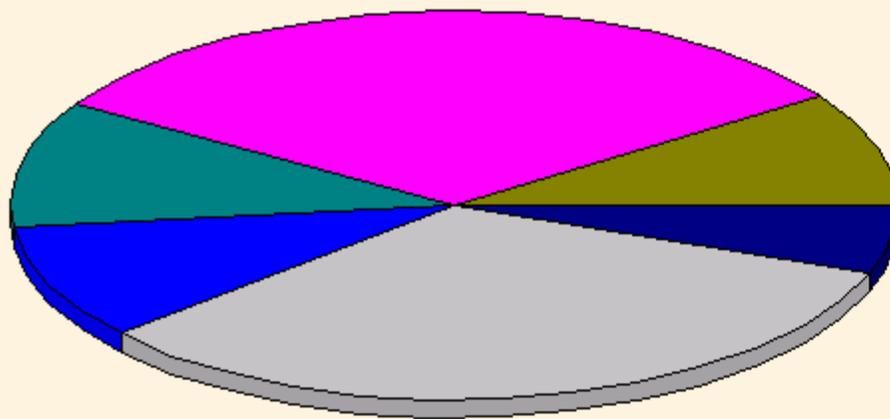
## Return

1 Year(+)	11.3
5 Years(+)	7.2
Expected	3.9
5 Years(-)	0.6
1 Year(-)	-3.5



# Asset Allocation: Optimal Portfolio

Optimal (Portfolio 006)



Fund	Init	Opt
Lehman High Yield(US Corp)	6.0	9.5 %
Citi CurHgd Non-USD WGBI	12.0	31.7 %
Citi ESBI Index USD!	3.0	10.6 %
FTSE All-Wld Adv Emerg USD!	3.0	9.7 %
NAREIT Equity I	7.0	32.9 %
Goldman Sachs Commodity I	6.0	5.6 %

**This is still not a well diversified portfolio.**

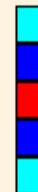
**Returns based on HBS case study**

## Portfolio # 6

Investment Horizon	5.0
5 Years Return	6.0
5 Years Std Dev	3.7
Single Year Return	6.0
Single Year Std Dev	8.4
Annual Yeild	-0.1

## Return

1 Year(+)	14.4
5 Years(+)	9.8
Expected	6.0
5 Years(-)	2.3
1 Year(-)	-2.3



# Mean Variance Results

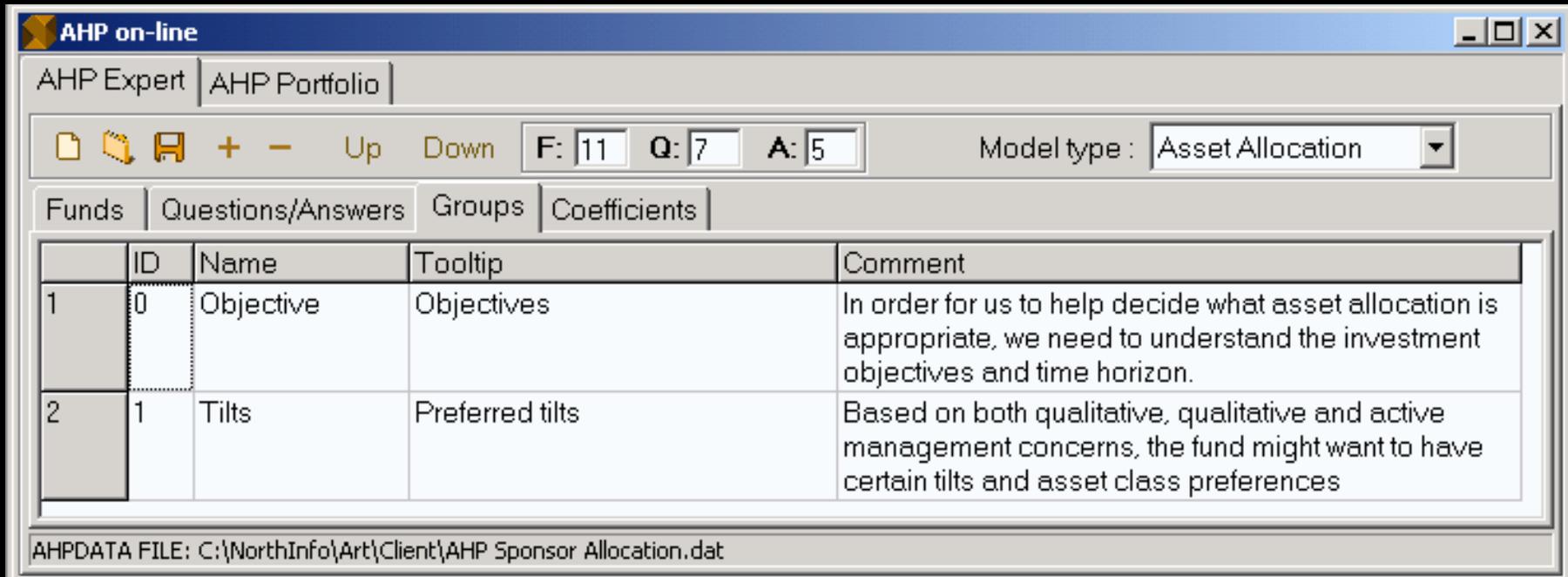
- ◆ Using Bayesian adjustment, the optimal portfolio gets only four of twelve asset classes.
- ◆ Two chosen asset classes, emerging market and TIPS bonds, are not typically given much weight in policy portfolios.
- ◆ The return estimates from the HBS case study are an improvement, but the portfolio is still “unusual” and poorly diversified.
- ◆ Clearly, there must be a better way to develop a reasonable strategic allocation.

# The Analytical Hierarchy Process

## First Steps

1. Develop question categories to help focus the client on the purpose of this group of questions.
2. Develop a number of questions for each category.
3. Split the responses for each question into levels, five being typical.
4. Assign weights to each question.
5. Select the asset classes that will be appropriate for the investor, in this case we use the ones in the HBS case study.

# Step 1: Develop Question Categories



The screenshot shows the 'AHP on-line' application window. The title bar reads 'AHP on-line'. Below the title bar, there are tabs for 'AHP Expert' and 'AHP Portfolio'. A toolbar contains icons for file operations and navigation, along with input fields for 'F: 11', 'Q: 7', and 'A: 5'. A dropdown menu for 'Model type' is set to 'Asset Allocation'. Below the toolbar are tabs for 'Funds', 'Questions/Answers', 'Groups', and 'Coefficients'. The 'Questions/Answers' tab is active, displaying a table with the following data:

	ID	Name	Tooltip	Comment
1	0	Objective	Objectives	In order for us to help decide what asset allocation is appropriate, we need to understand the investment objectives and time horizon.
2	1	Tilts	Preferred tilts	Based on both qualitative, quantitative and active management concerns, the fund might want to have certain tilts and asset class preferences

At the bottom of the window, the status bar displays the file path: 'AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat'.

## Step 2: Develop the Questions for Each Category

**AHP on-line** [AHP Expert | AHP Portfolio]

Model type: Asset Allocation

F: 11 Q: 7 A: 5

Group: All

Question N6:							Commodities Exposure			
Question	Group	H/V	Rap	Weight	Comment	Answer	Hidden	DefA		
1	Duration of Liabilities	0	0	0.0100	0.2000	What is the duration of your liabilities?	1	No Exposure	<input type="checkbox"/>	<input type="checkbox"/>
2	Spending Requirements	0	0	0.0010	0.2000	What are your current spending requirements?	2	Low Exposure	<input type="checkbox"/>	<input type="checkbox"/>
3	Inflation Protection	0	0	0.0100	0.2000	How important is inflation protection for your spending requirements?	3	Moderate Exposure	<input type="checkbox"/>	<input checked="" type="checkbox"/>
4	Domestic tilt	1	0	0.0100	0.1000	What level of domestic/foreign tilt is appropriate?	4	Overweight	<input type="checkbox"/>	<input type="checkbox"/>
5	Real Estate Exposure	1	0	0.0100	0.1000	What level of real estate exposure do you want?	5	Maximum	<input type="checkbox"/>	<input type="checkbox"/>
6	Commodities Exposure	1	0	0.0100	0.1000	What level of commodities exposure do you want?				
7	Emerging Markets	1	0	0.0100	0.1000	What level of emerging market exposure is appropriate?				

AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat

# Step 3: Selection of Asset Class Proxies

AHP on-line

AHP Expert | AHP Portfolio

ReCalc StartDate: 1985/01 EndDate: 2004/07

	Code	Name	Proxy Fund	Fund Name
1		Cash	115291	Citi 3 Month CD !
2		Investment Grade Bonds	114401	Lehman Aggregate !
3		High Yield Bonds	150339	Lehman High Yield(US Corp)
4	IDB	TIPS	115770	Citi Treasury-InflationLkd
5		Non US Bonds: Developed	115321	Citi CurHgd Non-USD WGBI
6	IIB	Non US Bonds: Emerging	106823	Citi ESBI Index USD!
7	IDE	US Equities	112302	Russell 3000 TR !
8	IFT	Non US Equity: Developed	122738	FTSE World Ex USA USD!
9	IFT	Non US Equity: Emerging	102012	FTSE All-Wld Adv Emerg USD!
10	IDE	Real Estate	117203	NAREIT Equity !
11	IHF	Commodities	117280	Goldman Sachs Commodity !

AHPDATA FILE: C:\NorthInfo\Art\Client\AHP Sponsor Allocation.dat

# Now for the Hard Part

- ◆ For each combination of asset class and question response level, we assign a suitability ranking.
- ◆ The suitability ranking is an integer ranging from 1 (most suitable) to some chosen upper limit. Normally the upper limit is 9, but sometimes we use 99 to ensure minimal exposure.
- ◆ For twelve asset classes, five response levels and seven questions, we have:
  - Ratings =  $12 \cdot 5 \cdot 7 = 420$  suitability judgments

# Suitability Judgments

AHP on-line

AHP Expert | AHP Portfolio

F: 11 Q: 7 A: 5 Model type: Asset Allocation

Funds | Questions/Answers | Groups | Coefficients

Sectors		Fund	US Equities	Question:	Emerging Markets	A1 A2 A3 A4 A5				
1	Cash	1	Duration of Liabilities			7	3	1.5	1	1
2	Investment Grade Bonds	2	Spending Requirements			7	3	1.5	1	1
3	High Yield Bonds	3	Inflation Protection			5	5	5	5	5
4	TIPS	4	Domestic tilt			1	3	5	7	9
5	Non US Bonds: Developed	5	Real Estate Exposure			9	9	9	9	9
6	Non US Bonds: Emerging	6	Commodities Exposure			9	9	9	9	9
7	US Equities	7	Emerging Markets			9	9	9	9	9
8	Non US Equity: Developed									
9	Non US Equity: Emerging									
10	Real Estate									
11	Commodities									

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# Questions to Determine Objectives

**AHP on-line**

AHP Expert | AHP Portfolio

Questionnaire | Portfolio

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Objectives | Preferred tilts

**In order for us to help decide what asset allocation is appropriate, we need to understand the investment objectives and time horizon.**

Duration of Liabilities  
What is the duration of your liabilities?

Less than 2 Years     2 to to 5 Years     5 to 10 Years     10 to 15 Years     More than 15 Years

Spending Requirements  
What are your current spending requirements?

More than 5%     Less than 5%     Less than 4% of assets     Less than 3%     Less than 1%

Inflation Protection  
How important is inflation protection for your spending requirements?

No Inflation Protection     Minimal Protection     Moderate Protection     Significant Inflation     Maximal Protection

## Questions to Assess Desired Tilts

**AHP on-line** [AHP Expert | AHP Portfolio]

Questionnaire | Portfolio

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Objectives | Preferred tilts

**Based on both qualitative, quantitative and active management concerns, the fund might want to have certain tilts and asset class preferences**

**Domestic tilt**  
What level of domestic/foreign tilt is appropriate?

Maximum US exposure    Strong US tilt    Moderate US tilt    Moderate Non US tilt    Strong Non US tilt

**Real Estate Exposure**  
What level of real estate exposure do you want?

Low exposure    Moderate Exposure    Typical Exposure    High Exposure    Very high exposure

**Commodities Exposure**  
What level of commodities exposure do you want?

No Exposure    Low Exposure    Moderate Exposure    Overweight    Maximum

**Emerging Markets**  
What level of emerging market exposure is appropriate?

No Exposure    Low Exposure    Typical    Above Average    High Exposure

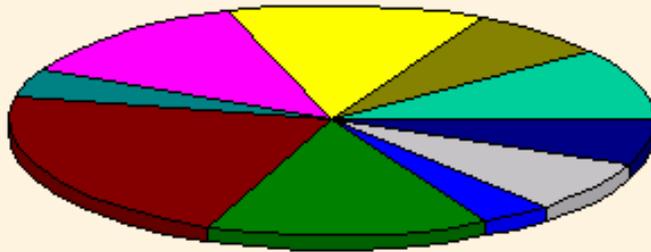
# What Does the AHP Do?

- ◆ Let's assume that we have a plan sponsor that has average liability duration and spending requirements and no desired tilts away from a reasonable policy portfolio.
- ◆ What is the asset allocation? Our sample approximates the HBS case study.
- ◆ What is the portfolio's expected return and risk?
- ◆ How does the AHP portfolio compare to the efficient frontier?

# Policy Portfolio

## Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)



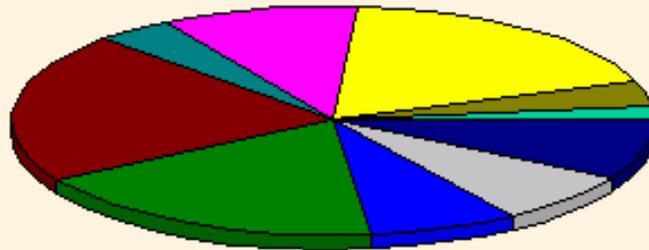
Fund	Init	Opt
Citi 3 Month CD !	0.2	0.0 %
Lehman Aggregate !	10.2	1.8 %
Lehman High Yield(US Corp)	6.9	0.0 %
Citi Treasury-InflationLkd	13.0	0.0 %
Citi CurHgd Non-USD WGBI	12.4	42.3 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR!	22.1	12.5 %
FTSE World Ex USA USD!	14.2	23.3 %
FTSE All-Wld Adv Emerg USD!	3.6	6.4 %
NAREIT Equity!	7.3	13.7 %
Goldman Sachs Commodity!	6.2	0.0 %

**Returns estimated by averaging ICAPM and Black Litterman estimations**

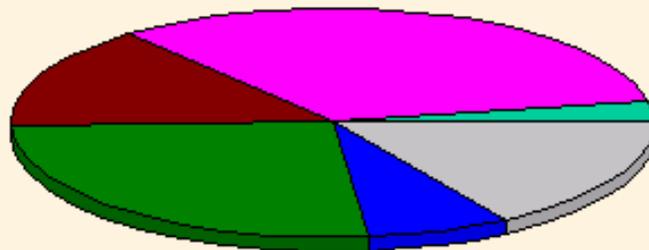
# Low Spending, High Duration and Maximum Inflation Protection

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)

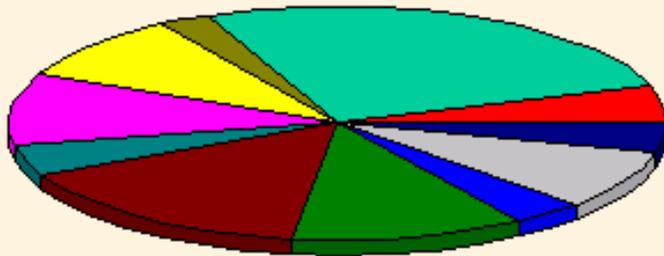


Fund	Init	Opt
Citi 3 Month CD !	0.1	0.0 %
Lehman Aggregate !	1.9	3.2 %
Lehman High Yield(US Corp)	3.5	0.0 %
Citi Treasury-InflationLkd	18.2	0.0 %
Citi CurHgd Non-USD WGBI	9.8	32.8 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR !	20.9	14.5 %
FTSE World Ex USA USD!	18.4	26.4 %
FTSE All-World Adv Emerg USD!	7.5	7.3 %
NAREIT Equity !	7.3	15.9 %
Goldman Sachs Commodity !	8.2	0.0 %

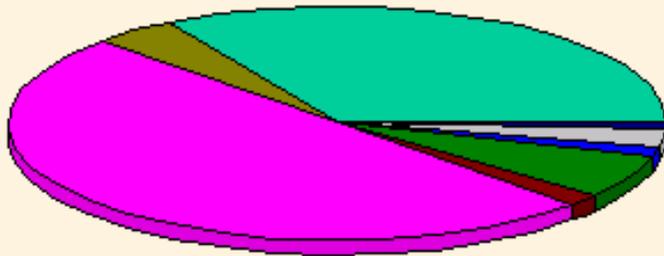
# High Spending, Low Duration and Minimum Inflation Protection

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)

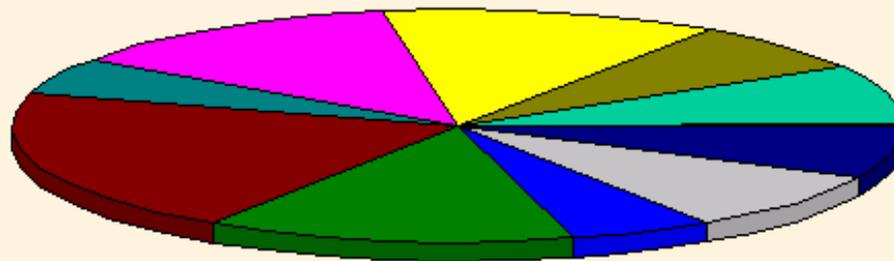


Fund	Init	Opt
Citi 3 Month CD !	5.1	0.0 %
Lehman Aggregate !	26.3	33.5 %
Lehman High Yield(US Corp)	2.7	4.2 %
Citi Treasury-InflationLkd	9.1	0.0 %
Citi CurHgd Non-USD WGBI	10.0	50.0 %
Citi ESBI Index USD!	4.0	0.0 %
Russell 3000 TR !	15.6	1.6 %
FTSE World Ex USA USD!	11.5	6.0 %
FTSE All-Wld Adv Emerg USD!	3.6	1.3 %
NAREIT Equity !	8.1	2.6 %
Goldman Sachs Commodity !	4.0	0.8 %

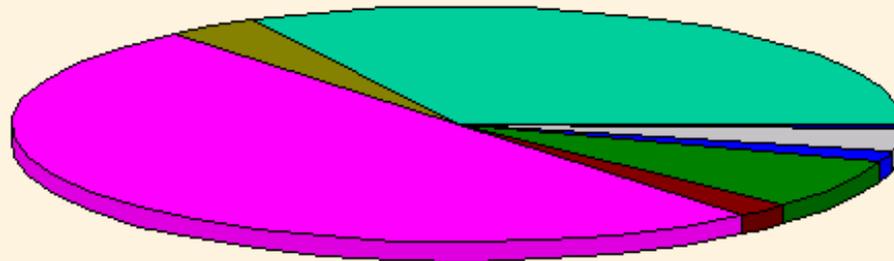
# Non US, Real Estate, Commodity and Emerging Markets Tilt

Optimal vs. Initial Weights

Initial Portfolio



Optimal (Portfolio 006)



Fund	Init	Opt
Citi 3 Month CD !	0.1	0.0 %
Lehman Aggregate !	8.4	32.6 %
Lehman High Yield(US Corp)	6.9	3.4 %
Citi Treasury-InflationLkd	12.3	0.0 %
Citi CurHgd Non-USD WGBI	12.4	50.0 %
Citi ESBI Index USD!	5.3	0.0 %
Russell 3000 TR !	20.3	2.0 %
FTSE World Ex USA USD!	13.4	6.7 %
FTSE All-Wld Adv Emerg USD!	5.3	1.6 %
NAREIT Equity !	8.3	3.0 %
Goldman Sachs Commodity !	7.2	0.6 %

# Portfolio Construction

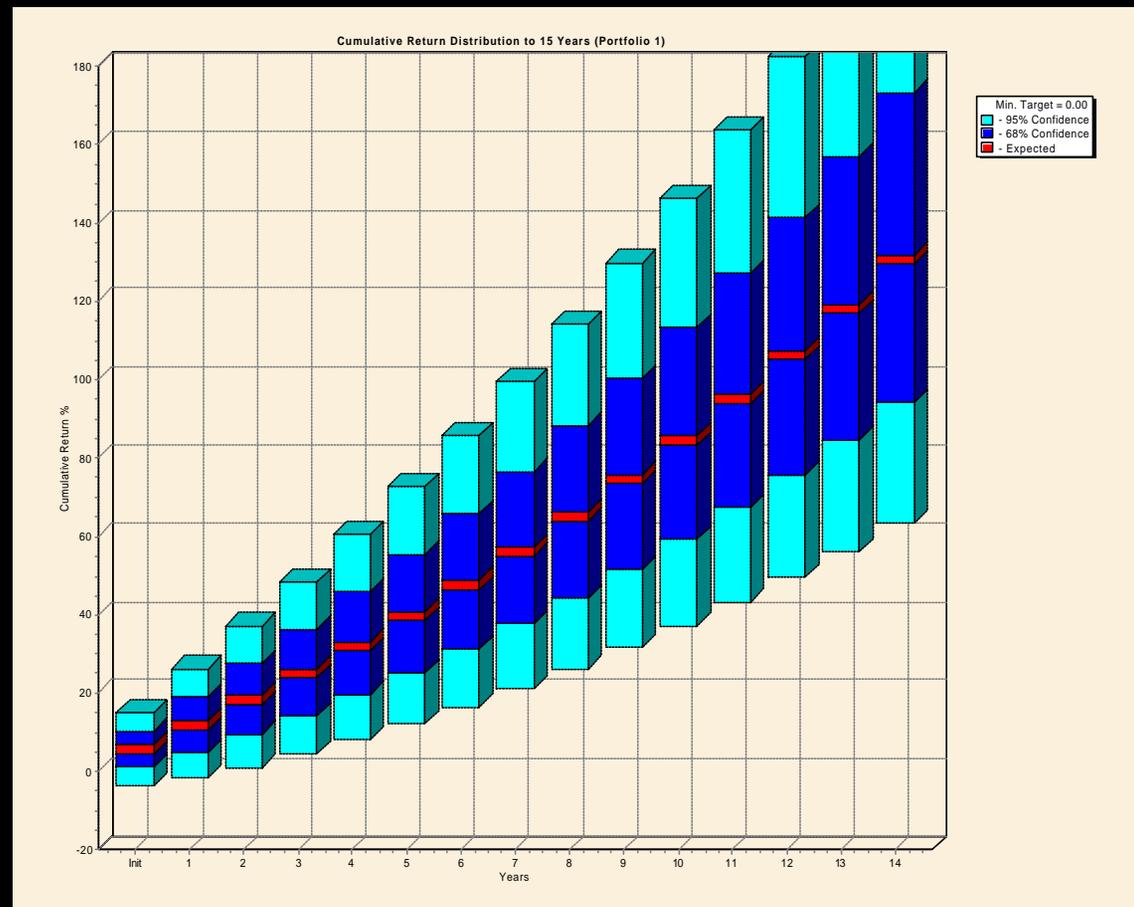
- ◆ Using “best guess” return assumptions, the suitable portfolios are within 15 to 25 b.p. of the efficient frontier.
- ◆ Next step: Choose asset managers accounts to implement the portfolio.
- ◆ Use optimization to minimize the funds’ tracking error vs. the asset allocation
  - This does not require developing expected returns for the “implementation portfolio”

# Portfolio Construction, Continued

- ◆ During the optimization process, sensible constraints (such as minimum and maximum holdings) can be used.
- ◆ After portfolio construction, return assumptions can be developed using historic averages, Bayesian adjustment, CAPM estimation or implied returns (Black-Litterman)

# Portfolio Construction, Completed

- ◆ After creating return expectations for the portfolio, we can create portfolio cumulative return expectations and confidence intervals.



# Traditional Manager Selection

- ◆ Let's assume that we have ten managers.
- ◆ How would we assign them weights in the portfolio?
- ◆ If we only used mean-variance optimization, we would:
  - Determine an appropriate benchmark, which could be either actuarial liabilities or a model portfolio.
  - Develop benchmark relative expected returns for each manager
  - Estimate the co-variance between each manager pair.
  - Use optimization to determine the efficient frontier
  - Pick the position on the efficient frontier that fits the beneficiary's risk tolerance.

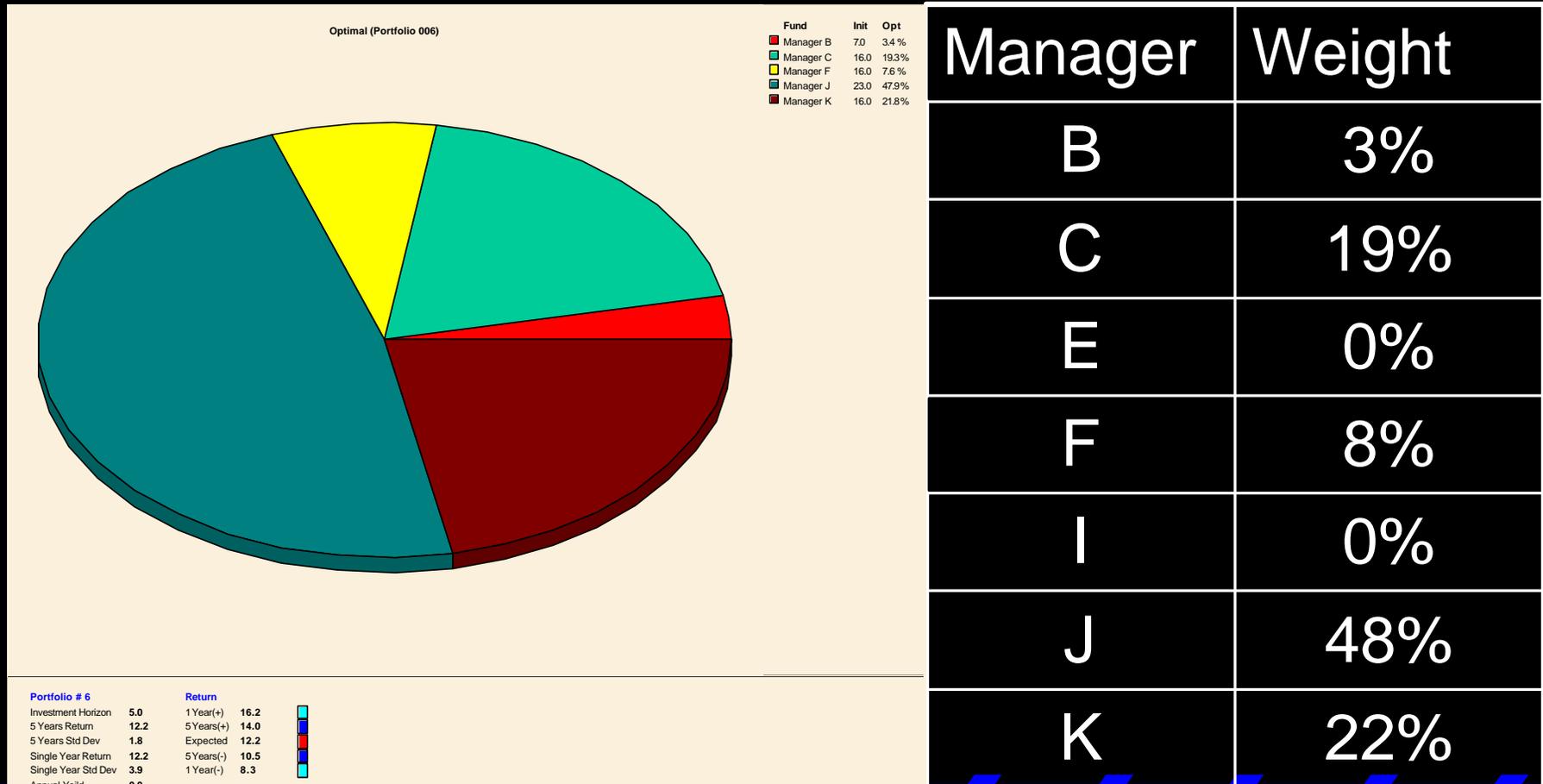
# A Manager Allocation Example

- ◆ We have returns data on seven managers that a consultant wants to evaluate and assign assets to manage.
- ◆ All managers are using an “absolute return strategy” and have been identified as “good to excellent.”
- ◆ We estimate returns using the Bayesian adjustment.
- ◆ We estimate the co-variance matrix using historic returns.
- ◆ We optimize and pick a point on the efficient frontier whose risk is similar to an equally weighted portfolio.

# Mean Variance Results

- ◆ Two “good to excellent managers” get no allocation.
- ◆ Two other managers get to share 10% of the allocation.
- ◆ Two managers share 40% of the allocation.
- ◆ One manager gets almost half of the allocation
- ◆ If all of these managers are “good to excellent,” this allocation is not reasonable.

# An Example: Manager Selection



# Let's Try the Analytic Hierarchy Process

- ◆ First we need a set of criteria on which to judge managers.
- ◆ Saraoglu and Detzler propose a set of criteria for selecting mutual funds, but we want something more applicable to institutional manager selection.
- ◆ At [www.ennisknupp.com](http://www.ennisknupp.com) (EK) we find a set of criteria for choosing asset managers.

# Manager Selection Ratings

AHP on-line

AHP Expert | AHP Portfolio

Managers | Ranking

+ - [Icons] C:\NorthInfo\Art\Client\Projects\Kate\AHP\ahpmanqa.qa View: Q/A Panel

	Code	Name	Comment
1	99001		
2	99002		
3	99003		
4	99004		
5	99005		

Selected: **Manager N1**

Fund Selection

**This sample questionnaire is based on a paper by Saraoglu and Detzler "A Sensible Mutual Fund Selection Model," for the May/June 1992 Financial Analysts Journal**

**Long Term Performance**  
The long term (10 to 15 year) total return of the fund measures how well the fund does over a few market cycles.

Poor     Below Average     Average     Above Average     Excellent

**Fees**  
The lower the fee, the higher the future returns are likely to be.

High Fee     Above Average Fee     Average     Below Average Fee     Low Fee

**Manager Tenure**  
A longer manager tenure is likely to be related to higher skill and better performance in different market conditions.

Short Tenure     Below Average     Average     Above Average     Long Tenure

**Risk Adjusted Performance**  
Treyrnor ratio or Sharpe ratio measure the fund performance relative to its risk.

Poor     Below Average     Average     Above Average     Excellent

# Let's Re-do the Manager Selection using the AHP Weights

- ◆ We estimate the Sharpe ratio for each of the 7 managers and assign performance of each of the managers for the last 3 years (the time history for this database).
- ◆ We do a long-short style analysis and observe the alpha, tracking error, style drifts and CUSUM statistics.

# Manager Allocation using AHP

- ◆ Based on the Sharpe ratio and other statistics, we rate the managers fair to excellent on the Performance and perceived skill.
- ◆ We leave the other rankings at “average” since we don’t have the information to make these judgments.

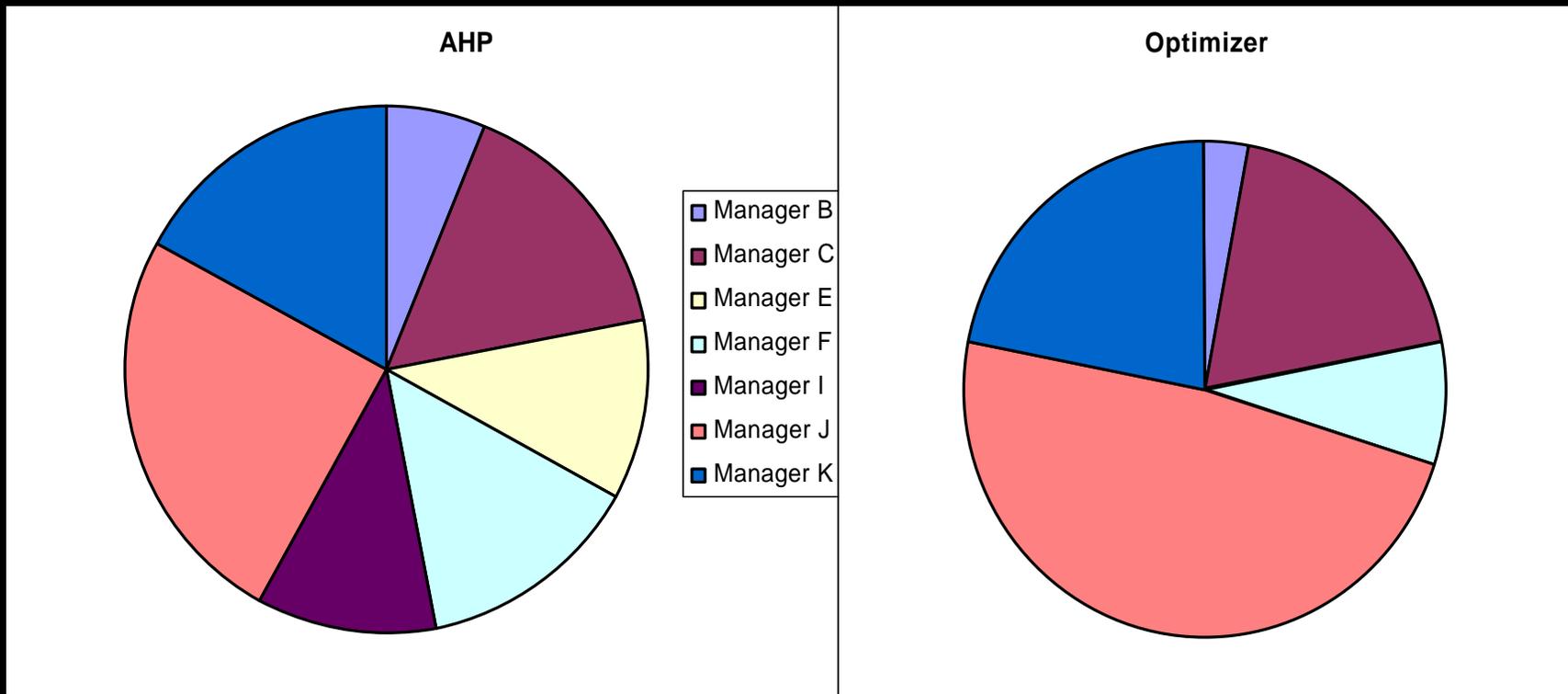
# Manager Allocation using AHP

- ◆ We then work out a new allocation, and then estimate the expected return and risk of the new allocation.
- ◆ We also estimate the implied returns for each manager using the AHP allocation and an estimation of the appropriate risk tolerance for the AHP portfolio.

# Comparison of the AHP and MV Optimal Portfolios

Fund Name	AHP Weight	Optimal Weight	Expected Return	Historic Return	Implied Return	Risk
Manager B	6	3	6.0	4.5	13.3	16.2
Manager C	16	19	9.3	9.7	9.8	5.3
Manager E	11	0	6.6	5.5	9.8	3.6
Manager F	14	8	16.3	20.8	23.5	16.2
Manager I	11	0	7.7	7.2	10.6	4.0
Manager J	25	48	14.5	18.0	11.5	6.7
Manager K	17	22	9.5	10.0	9.2	3.9

# Comparison of AHP and Optimizer Manager Allocations



# Conclusions

- ◆ AHP is a methodology that arises from operations research literature that is used as a non-parametric method for making complex, often qualitative decisions in a robust, consistent fashion.
- ◆ AHP has now been adapted as a tool in the selection of, and the allocation of capital to, investment managers.
- ◆ We think AHP is the way to go for many problems in investment decision making where quantitative and qualitative criteria must both play a role.