Improving Returns-Based Style Analysis

Daniel Mostovoy

Summer 2007 Investment Seminar - London

June 2007

Main Points For Today

- Over the past 15 years, Returns-Based Style Analysis become a very widely used analytical method

- We're going to review RBSA and discuss useful several improvements to the basic technique
  - Confidence Intervals
  - Testing for Regime Change
  - Kalman Filter / Exponential Weighting
  - Adjusting for Heteroscedasticity

- Recently, RBSA has gained a new usage in connection with hedge fund replication strategies

Northfield
INFORMATION SERVICES, INC.

Basic RBSA

- First introduced as "Effective Asset Mix Analysis" by Sharpe (1988, 1992). Given a time series of returns of a fund, we try to find the mix of market indices that most closely fits the fund returns

$$R_t = \Sigma_{j = 1 \text{ to } n} W_j R_{jt} + \varepsilon_t$$

R_t is the return on the fund during period t
W_j is the weight of index j
R_{jt} is the return on index j during period t
N is the number of indices
ε_t is the residual for period t

Sum of $W_j = 1$, $0 < W_j < 1$

Basically an OLS multiple regression with constrained coefficients

3

Refinement #1
Confidence Intervals

- Like any other estimate we need to know if our style weights results are meaningful

- A style weight estimate of "10% small cap value" isn't very useful if its really "10% +/- 50%"

- We can only analyze a fund to the extent that the spanning indices are not linear combinations of each other

- Correlation between the spanning indices can frequently cause very large confidence intervals on style weights, as the constraints on the coefficients mask multicolinearity that would be observable in an OLS regression

4

Northfield
INFORMATION SERVICES, INC.

Confidence Interval Problem Was Solved a While Ago

Lobosco, Angelo and Dan DiBartolomeo. "Approximating The Confidence Intervals For Sharpe Style Weights," Financial Analyst Journal, 1997, v53(4,Jul/Aug), 80-85.

Oddly, most commercial style analysis software packages still do not incorporate any form of confidence interval on the results

Refinement #2
Allowing Leverage

- Some portfolios such as hedge funds employ explicit leverage
- Other funds have portfolio properties that are possibly outside the range of the spanning indices
  - An equity portfolio with a beta higher than any available
  - A bond portfolio with maturities longer than any available index
- Solution is to include a cash equivalent among the spanning indices and allow negative weights, provided that the sum of weights is still constrained to one.

Northfield
INFORMATION SERVICES, INC.

Refinement #3
Testing for Regime Changes

- Do we want to look at fund results over the last 3 years, 5 years, 32 months, etc. ?
- CUSUM is an optimum statistic to determine the change in the mean of a process
  - Was adapted for the purposes of monitoring external asset managers by the IBM pension fund
- Use CUSUM based methods to determine the optimal "look-back" period for the style analysis
  - Mathematically: What is the "look back" date such that the cumulative active return between then and now is least likely to have come about by random chance?
- Forthcoming paper by Bolster, diBartolomeo and Warrick summarized in our February 2005 newsletter
  - http://www.northinfo.com/documents/192.pdf

Northfield
INFORMATION SERVICES, INC.

Refinement #4
Capturing Recent Influences

- Traditional style weights represent the fund average behavior over a time sample
  - What we should be worried about is a fund that has changed style recently, rendering "average" past information useless
- One Approach
  - Plot the absolute value of the residual against time during the sample
  - If the slope is positive, the fit is getting worse as we come forward in time. Exponentially weight observations until the slope is not statistically significantly positive
- Another way is to use Kalman filtering
  - Swinkels, Laurens and Peter Van Der Sluis, "Returns based Style Analysis with Time Varying Exposures", ABP Working Paper, 2001
  - Kalman filtering requires use of Markov Chain Monte-Carlo analysis if style weights are constrained to be positive

Northfield
INFORMATION SERVICES, INC.

Refinement #5
Asking the Right Question

- An easy experiment
  - Nine year sample period from 1998 through 2006
  - Make up a monthly return stream for a hypothetical fund whose returns are equal to the S&P 500 for 1/3 of the 9 years, equal to the FTSE Europe for 1/3 of the 9 years, and equal to the Merrill Lynch Global High Yield for 1/3 of the 9 years
  - First intuition suggests style weights should be 1/3 S&P 500, 1/3 FTSE Europe and 1/3 MLGHY
- Generally WRONG. It depends on the order of events

Northfield
INFORMATION SERVICES, INC.

A Curious Result

	SE	α	R^2	S&P	FTSE	MLGHY
FTMLSP	6.63	2.87	0.67	11.65	28.66	59.69
FTSPML	5.49	-3.43	0.85	39.9	43.44	16.65
MLFTSP	6.61	-5.33	0.75	5.54	58.01	36.44
MLSPFT	6.89	-2.16	0.71	23.85	37.59	38.56
SPFTML	5.79	-0.74	0.87	64.04	34.39	1.56
SPMLFT	7.29	8.7	0.71	53.46	0	46.54

Northfield
INFORMATION SERVICES, INC.

What's Going On

- The results are order dependent
  - The style analysis process, like a regression is minimizing the sum of squared residuals
  - The volatility of markets is different across the three sub-periods, and the more volatile periods are counted more heavily
  - Not only do the weights vary across the different orders but goodness of fit changed a lot too
- Variation in alpha estimates ranged from -5.33 to + 8.7
  - This huge difference in alpha arises from the "accidental" market timing arising from the ordering
- *Averaged across all six possible orders we get our expected result of 1/3, 1/3, 1/3 for weights*

Northfield
INFORMATION SERVICES, INC.

Refinement #5
Asking the Right Question

- The more volatile periods do count for more in the returns experienced by investors
  - If we want to know what market returns influenced the returns of the fund, this is the right answer
  - This corresponds to Sharpe's original concept of "Effective Asset Mix"
- But if we want to know whether a manager's style was consistent with a prescribed strategy, we have to filter out the effects of heteroscedasticity within the sample period
  - For each time period calculate the "spanning dispersion", the average absolute difference in return between all possible pairs within the spanning indices
  - Weight the observations inversely with the square root of the dispersion

Northfield
INFORMATION SERVICES, INC.

Refinement #6
Volatility Based Spanning Indices

- Many hedge fund strategies are predicted on the level of market volatility, rather than expected returns
  - Purported uncorrelated with the direction of markets (e.g. writing option spreads)
- Fung and Hsieh (2002) suggest spanning indices that are volatility related
  - Relative returns between mortgage backed securities and coupon bonds are sensitive to interest rate volatility
  - Bondarenko (2004) constructs a index where the return is based on the difference between implied and realized OEX volatility
  - diBartolomeo (2006) reviews literature on dispersion of security returns within asset classes, http://www.northinfo.com/documents/234.pdf

Northfield
INFORMATION SERVICES, INC.

Using RBSA to Proxy Hedge Fund Holdings

- A common problem in the hedge fund industry is the need to analyze a hedge fund where the holdings of the fund are not disclosed
  - Create a proxy portfolio for risk management and asset allocation purposes
  - Hold the proxy portfolio as a "synthetic" version of the fund
- We will illustrate a procedure for estimating proxy holdings for a fund where the true underlying holdings are unknown.
  - Using a combination of returns based style analysis and portfolio optimization
- Our proxy portfolio is not meant to be a guess at the true underlying portfolio, but rather an efficient estimate of:
  - The typical style bets of the fund
  - The degree of portfolio concentration
  - The balance between asset specific and factor risks.

Northfield
INFORMATION SERVICES, INC.

Selecting the Spanning Indices

- For each fund we need to select the right set of spanning indices
- Over a universe of funds, some indices will be significant lots of funds, some indices will be significant to only a few funds
- Use what we know about the fund strategy to manually select a set of "likely suspects"
- Start with a large list of indices. Iteratively run the analysis dropping out the least statistically significant.
  - Easy to get fooled because T stats on indices improve as we drop correlated but less significant indices
- Start with a short list of indices representing major asset classes
  - Run analysis, drop insignificant asset classes. Replace remaining indices with sub-indices. Rerun analysis and again drop out insignificant indices

Northfield
INFORMATION SERVICES, INC.

RBSA Analysis Output

By running the style analysis, we get three pieces of information:

- Observed volatility of the subject hedge fund during chosen sample period

- The "style" exposures of the subject fund (growth, value, short volatility, etc.) expressed as percentages of the different indices that best mimic the fund's return behavior over time.

- The relative proportion of risk coming from style factors and from fund specific risk.

Northfield
INFORMATION SERVICES, INC.

Now Let Us Start to Form Holdings

- Take the constituents of our spanning indices and form a portfolio of these constituents weighted by results of the style analysis.
- If our style analysis said the fund behaved like 50% the S&P 500 and 50% EAFE
- We would form a portfolio that was 50% the weighted constituents of the S&P 500 and 50% EAFE.
  - At this point, we should have a portfolio that has the right "style" exposures to match our fund
- However, these two indices together have about 1600 stocks.
- The resulting portfolio would be far too diversified to represent a typical hedge fund.
  - It is likely to have far lower risk than a real hedge fund portfolio.

Northfield
INFORMATION SERVICES, INC.

Let's Refine the Proxy Holdings

Now we'll consider portfolio volatility

- Load the proxy portfolio into the Optimizer as both the benchmark and the starting portfolio.
  - In our example, our version starting portfolio/benchmark would have 1600 stocks.
- We must reduce the number of positions such that the overall risk of the portfolio approximates the observed risk of the subject fund.
  - We can do this by running an optimization while using the "Maximum number of Assets" parameter.
- With a little trial and error, we can find the portfolio that matches the benchmark (and the subject fund) in style.
  - We reduce the diversification to the point where the expected volatility of the proxy portfolio matches the observed volatility of the subject hedge fund.

Northfield
INFORMATION SERVICES, INC.

Check the Balance Between Factor and Asset Specific Risks

- We now load the refined (reduced number of positions) proxy portfolio into the Optimizer as the portfolio with a cash benchmark.

- By running a risk report, we can determine how much of:

  - the expected risk of the refined proxy portfolio arises from factor bets

  - Arises from asset specific risk.

  - If this is a reasonable match to the subject fund (from the style analysis) we're done.

Changing the Balance Between Factor and Asset Specific Risks

- If we find we don't have the appropriate balance between factor and asset specific risks
  - Repeat the process of "refining" the proxy portfolio
  - In addition to defining the Max Assets parameter, we can change the Optimizer's degree of risk tolerance for factor and asset specific risk
  - Again with a little trial and error, we can find risk acceptance parameter values that bring the relative proportions of factor risk and asset specific risk into line with our analysis of the subject fund
- We now have our proxy portfolio to hold or use as a composite asset in other analyses

20

Northfield
INFORMATION SERVICES, INC.

Conclusions

- The effectiveness of Returns Based Style Analysis can be enhanced in a number of important ways

- These enhancements are particularly important in analyzing funds where substantial shifts in strategy may be expected over time

Northfield
INFORMATION SERVICES, INC.

Empirical Example 100 HF

- Ran 100 HF through a set of 14 spanning indices, retained TValues
- Reduced number of independent variables by adding them in order of decreasing abs(TValue) & Rerunning
  - For each fund, dropped all independent variables with abs(TValue) < .5

Northfield
INFORMATION SERVICES, INC.

Style Analysis Results

- R2 between .005 and .9, averaging about .36

- Interesting trend: the greater Cash Allocation, the lower the R2:

    - The more "hedged" the fund, the less information there is for the style analysis to pick up on…

Northfield
INFORMATION SERVICES, INC.

%Cash Allocation vs Style R2

R2 = .465

Northfield
INFORMATION SERVICES, INC.
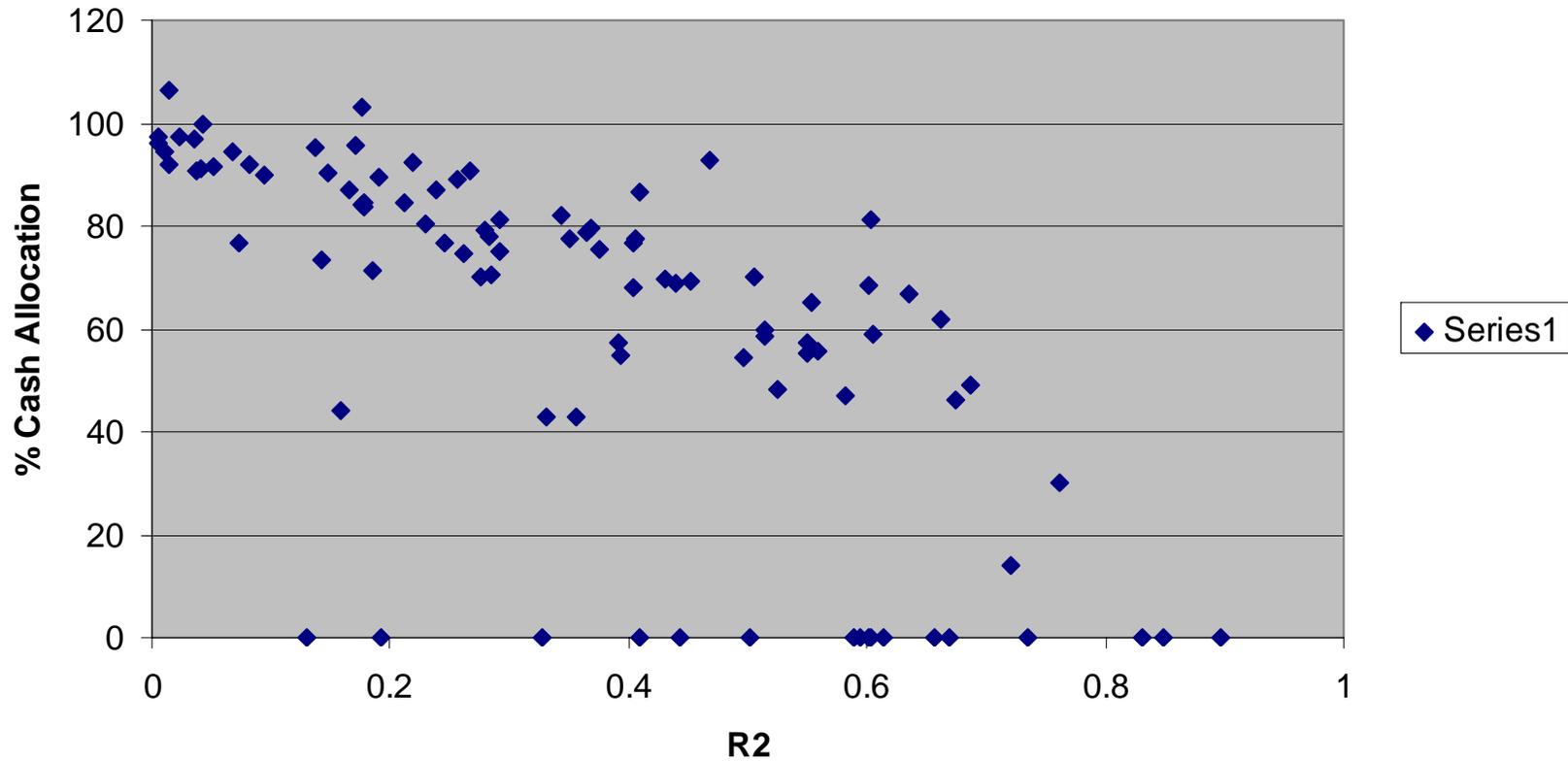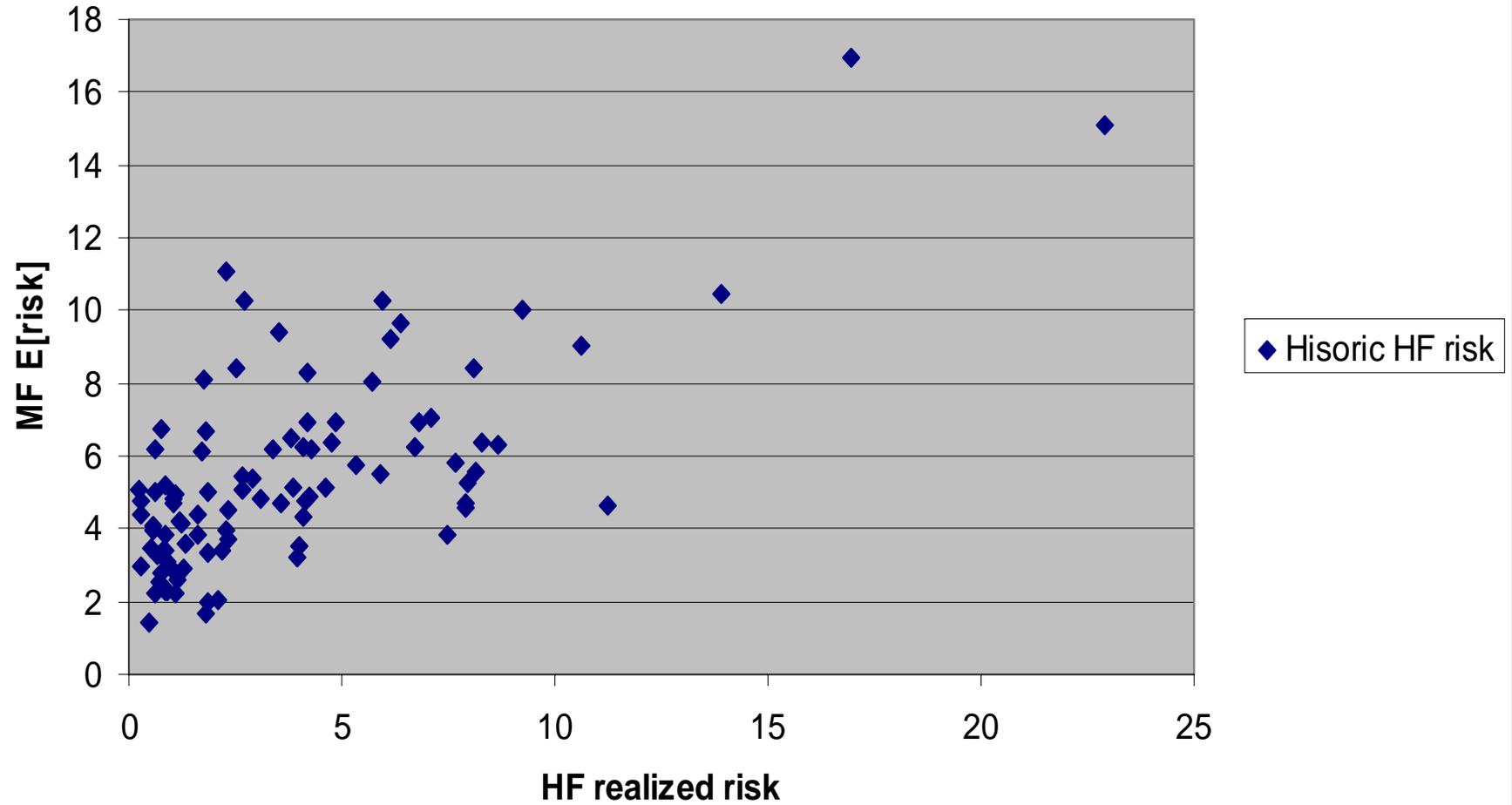
Next Step (Empirical Ex. Cont)

- Combined spanning index constituents according to style weights, used result as benchmark, optimized, max 500 assets.

- Harvested resultant expected standard deviation of returns.

- Calculated historic standard deviation of HF returns.

Modelled Vs Realized Risk

Scatter plot with y-axis labeled "MF E[risk]" ranging from 0 to 18, and x-axis labeled "HF realized risk" ranging from 0 to 25. Legend: ♦ Hisoric HF risk

Northfield
INFORMATION SERVICES, INC.

Results

- Slope = .978
- R2 = .486
- 85% of the modeled portfolio risks were smaller than the respective observed HF risks.

Northfield
INFORMATION SERVICES, INC.

Final Step (Empirical ex. Cont)

- Adjust expected benchmark risk to match historic hedge fund risk by iteratively adding or deleting cash from the portfolio:
  - Factor exposures will change in magnitude, but not relative proportion to one another
- Adjust factor variance to total variance ratio to match style analysis R^2 through iterative optimizations adjusting sysRAP and unSysRAP parameters, e.g.
  - $U = a - (sysRAP * FactorVariance + unSysRap * residualVariance)$

28

Modeled vs Historic Variance
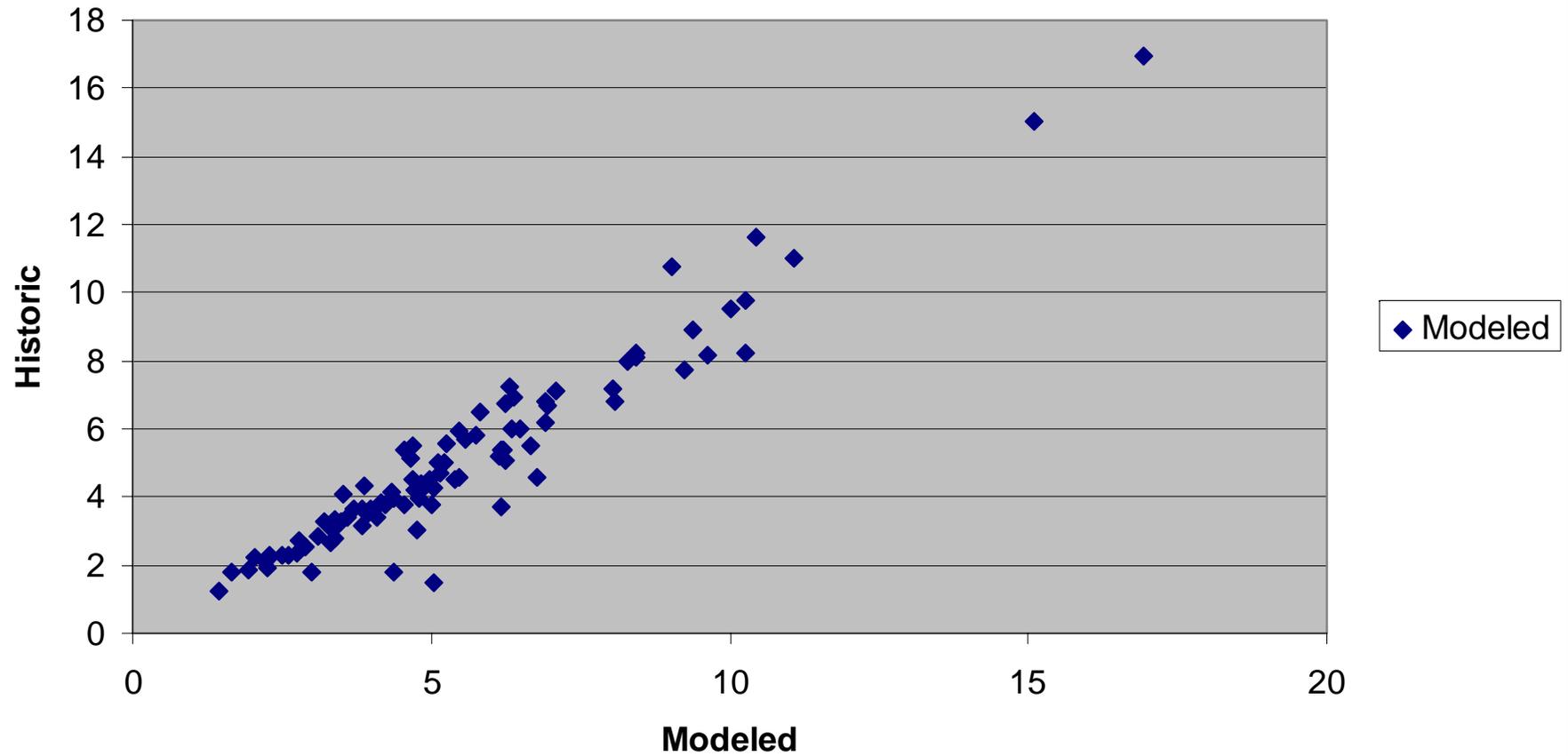
R^2 = .92
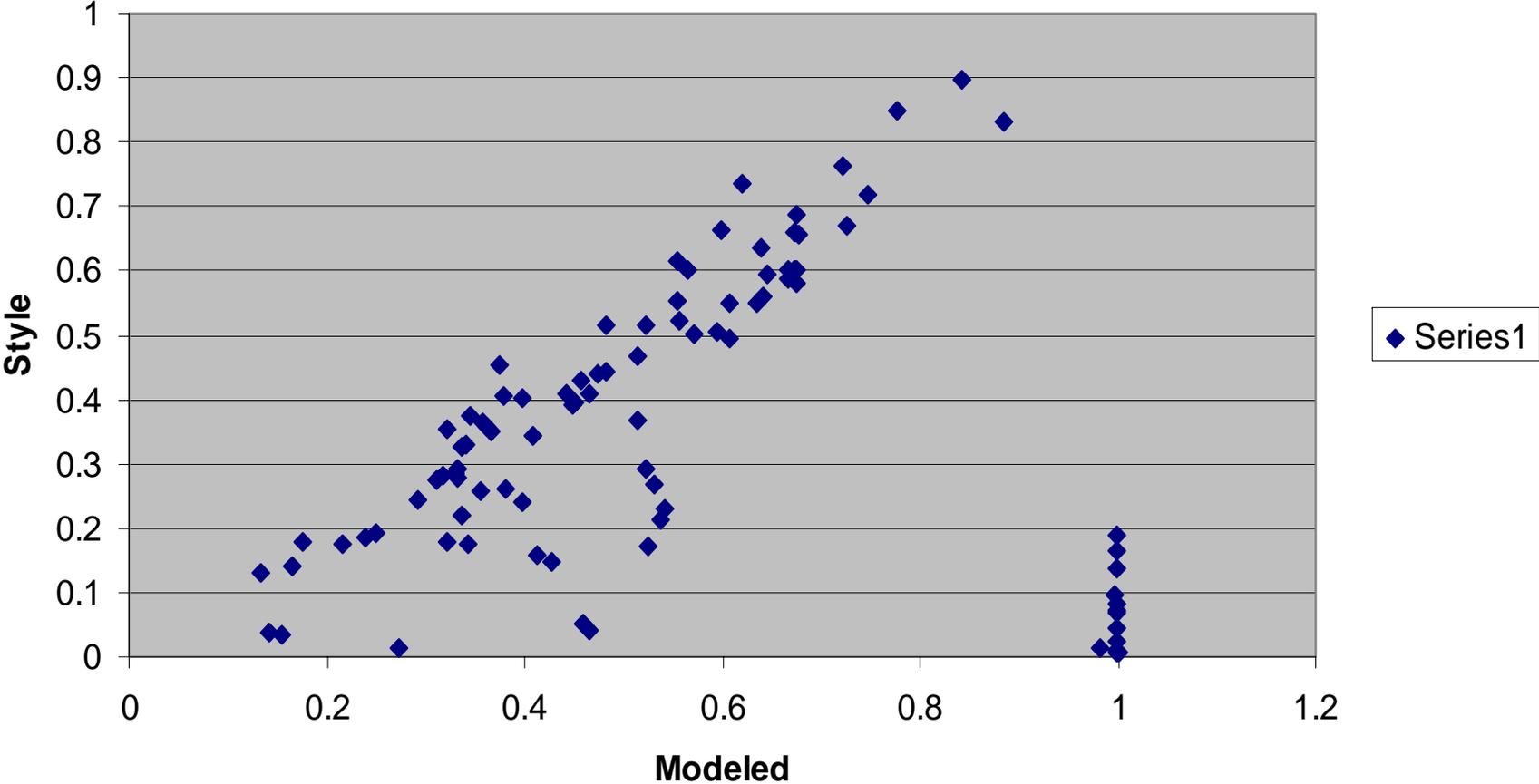
R^2 = .829

Empirical Conclusions

- The Style Analysis does a good job of modeling a Hedge Fund's Factor Variance.

- A further adjustment of stock specific is required to beef up the Expected Risks to fit.
  - This can be done by iteratively adjusting:
    - Cash in Benchmark
    - UnsysRAP vs SysRAP ratio

Northfield
INFORMATION SERVICES, INC.