Northfield Information Services
Spring 2007 Investment Seminar – London
Thursday, June 21, 2007

Agenda

Improving Returns-Based Style Analysis
Daniel Mostovoy and Dan diBartolomeo - Northfield

Who should you listen to? A decomposition of stock returns by sector
Giuliano De-Rossi - UBS

Alpha Scaling Revisited
Anish Shah - Northfield

Portfolio Optimisation – A regression approach to Portfolio Construction
James Sefton - Imperial College

Distinguishing Between Being Unlucky and Unskillful
Dan diBartolomeo - Northfield

A Market Impact Model that Works
Dan diBartolomeo and Howard Hoffman - Northfield
Improving Returns-Based Style Analysis

Daniel Mostovoy
Summer 2007 Investment Seminar - London
June 2007
Main Points For Today

- Over the past 15 years, Returns-Based Style Analysis become a very widely used analytical method
- We’re going to review RBSA and discuss useful several improvements to the basic technique
  - Confidence Intervals
  - Testing for Regime Change
  - Kalman Filter / Exponential Weighting
  - Adjusting for Heteroscedasticity
- Recently, RBSA has gained a new usage in connection with hedge fund replication strategies
Basic RBSA

• First introduced as “Effective Asset Mix Analysis” by Sharpe (1988, 1992). Given a time series of returns of a fund, we try to find the mix of market indices that most closely fits the fund returns

\[ R_t = \sum_{j=1}^{N} W_j R_{jt} + \epsilon_t \]

- \( R_t \) is the return on the fund during period \( t \)
- \( W_j \) is the weight of index \( j \)
- \( R_{jt} \) is the return on index \( j \) during period \( t \)
- \( N \) is the number of indices
- \( \epsilon_t \) is the residual for period \( t \)

Sum of \( W_j = 1, \ 0 < W_j < 1 \)

Basically an OLS multiple regression with constrained coefficients
Refinement #1
Confidence Intervals

• Like any other estimate we need to know if our style weights results are meaningful
• A style weight estimate of “10% small cap value” isn’t very useful if it’s really “10% +/- 50%”
• We can only analyze a fund to the extent that the spanning indices are not linear combinations of each other
• Correlation between the spanning indices can frequently cause very large confidence intervals on style weights, as the constraints on the coefficients mask multicollinearity that would be observable in an OLS regression
Confidence Interval Problem Was Solved a While Ago


Oddly, most commercial style analysis software packages still do not incorporate any form of confidence interval on the results.
Refinement #2
Allowing Leverage

- Some portfolios such as hedge funds employ explicit leverage.
- Other funds have portfolio properties that are possibly outside the range of the spanning indices:
  - An equity portfolio with a beta higher than any available
  - A bond portfolio with maturities longer than any available index
- Solution is to include a cash equivalent among the spanning indices and allow negative weights, provided that the sum of weights is still constrained to one.
Refinement #3
Testing for Regime Changes

• Do we want to look at fund results over the last 3 years, 5 years, 32 months, etc. ?
• CUSUM is an optimum statistic to determine the change in the mean of a process
  – Was adapted for the purposes of monitoring external asset managers by the IBM pension fund
• Use CUSUM based methods to determine the optimal "look-back" period for the style analysis
  – Mathematically: What is the “look back” date such that the cumulative active return between then and now is least likely to have come about by random chance?
• Forthcoming paper by Bolster, diBartolomeo and Warrick summarized in our February 2005 newsletter
Refinement #4
Capturing Recent Influences

• Traditional style weights represent the fund average behavior over a time sample
  – What we should be worried about is a fund that has changed style recently, rendering “average” past information useless

• One Approach
  – Plot the absolute value of the residual against time during the sample
  – If the slope is positive, the fit is getting worse as we come forward in time. Exponentially weight observations until the slope is not statistically significantly positive

• Another way is to use Kalman filtering
  – Kalman filtering requires use of Markov Chain Monte-Carlo analysis if style weights are constrained to be positive
Refinement #5
Asking the Right Question

• An easy experiment
  – Nine year sample period from 1998 through 2006
  – Make up a monthly return stream for a hypothetical fund whose returns are equal to the S&P 500 for 1/3 of the 9 years, equal to the FTSE Europe for 1/3 of the 9 years, and equal to the Merrill Lynch Global High Yield for 1/3 of the 9 years
  – First intuition suggests style weights should be 1/3 S&P 500, 1/3 FTSE Europe and 1/3 MLGHY

• Generally WRONG. It depends on the order of events
## A Curious Result

<table>
<thead>
<tr>
<th></th>
<th>SE</th>
<th>$\alpha$</th>
<th>$R^2$</th>
<th>S&amp;P</th>
<th>FTSE</th>
<th>MLGHY</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTMLSP</td>
<td>6.63</td>
<td>2.87</td>
<td>0.67</td>
<td>11.65</td>
<td>28.66</td>
<td>59.69</td>
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<td>FTSPML</td>
<td>5.49</td>
<td>-3.43</td>
<td>0.85</td>
<td>39.9</td>
<td>43.44</td>
<td>16.65</td>
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<td>MLFTSP</td>
<td>6.61</td>
<td>-5.33</td>
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<td>5.54</td>
<td>58.01</td>
<td>36.44</td>
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<tr>
<td>MLSPFT</td>
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<td>23.85</td>
<td>37.59</td>
<td>38.56</td>
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<tr>
<td>SPFTML</td>
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<td>-0.74</td>
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<td>64.04</td>
<td>34.39</td>
<td>1.56</td>
</tr>
<tr>
<td>SPMLFT</td>
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<td>8.7</td>
<td>0.71</td>
<td>53.46</td>
<td>0</td>
<td>46.54</td>
</tr>
</tbody>
</table>

What’s Going On

• The results are order dependent
  – The style analysis process, like a regression is minimizing the sum of squared residuals
  – The volatility of markets is different across the three sub-periods, and the more volatile periods are counted more heavily
  – Not only do the weights vary across the different orders but goodness of fit changed a lot too

• Variation in alpha estimates ranged from -5.33 to +8.7
  – This huge difference in alpha arises from the “accidental” market timing arising from the ordering

• Averaged across all six possible orders we get our expected result of 1/3, 1/3, 1/3 for weights
Refinement #5
Asking the Right Question

- The more volatile periods do count for more in the returns experienced by investors
  - If we want to know what market returns influenced the returns of the fund, this is the right answer
  - This corresponds to Sharpe’s original concept of “Effective Asset Mix”
- But if we want to know whether a manager’s style was consistent with a prescribed strategy, we have to filter out the effects of heteroscedasticity within the sample period
  - For each time period calculate the “spanning dispersion”, the average absolute difference in return between all possible pairs within the spanning indices
  - Weight the observations inversely with the square root of the dispersion
Refinement #6
Volatility Based Spanning Indices

• Many hedge fund strategies are predicted on the level of market volatility, rather than expected returns
  – Purported uncorrelated with the direction of markets (e.g. writing option spreads)

• Fung and Hsieh (2002) suggest spanning indices that are volatility related
  – Relative returns between mortgage backed securities and coupon bonds are sensitive to interest rate volatility
  – Bondarenko (2004) constructs a index where the return is based on the difference between implied and realized OEX volatility
Using RBSA to Proxy Hedge Fund Holdings

• A common problem in the hedge fund industry is the need to analyze a hedge fund where the holdings of the fund are not disclosed
  – Create a proxy portfolio for risk management and asset allocation purposes
  – Hold the proxy portfolio as a “synthetic” version of the fund
• We will illustrate a procedure for estimating proxy holdings for a fund where the true underlying holdings are unknown.
  – Using a combination of returns based style analysis and portfolio optimization
• Our proxy portfolio is not meant to be a guess at the true underlying portfolio, but rather an efficient estimate of:
  – The typical style bets of the fund
  – The degree of portfolio concentration
  – The balance between asset specific and factor risks.
Selecting the Spanning Indices

- For each fund we need to select the right set of spanning indices
- Over a universe of funds, some indices will be significant to lots of funds, some indices will be significant to only a few funds
- Use what we know about the fund strategy to manually select a set of “likely suspects”
- Start with a large list of indices. Iteratively run the analysis dropping out the least statistically significant.
  - Easy to get fooled because T stats on indices improve as we drop correlated but less significant indices
- Start with a short list of indices representing major asset classes
  - Run analysis, drop insignificant asset classes. Replace remaining indices with sub-indices. Rerun analysis and again drop out insignificant indices
RBSA Analysis Output

By running the style analysis, we get three pieces of information:

- Observed volatility of the subject hedge fund during chosen sample period
- The "style" exposures of the subject fund (growth, value, short volatility, etc.) expressed as percentages of the different indices that best mimic the fund’s return behavior over time.
- The relative proportion of risk coming from style factors and from fund specific risk.
Now Let Us Start to Form Holdings

• Take the constituents of our spanning indices and form a portfolio of these constituents weighted by results of the style analysis.
• If our style analysis said the fund behaved like 50% the S&P 500 and 50% EAFE
• We would form a portfolio that was 50% the weighted constituents of the S&P 500 and 50% EAFE.
  – At this point, we should have a portfolio that has the right "style" exposures to match our fund
• However, these two indices together have about 1600 stocks.
• The resulting portfolio would be far too diversified to represent a typical hedge fund.
  – It is likely to have far lower risk than a real hedge fund portfolio.
Let’s Refine the Proxy Holdings

Now we’ll consider portfolio volatility

• Load the proxy portfolio into the Optimizer as both the benchmark and the starting portfolio.
  – In our example, our version starting portfolio/benchmark would have 1600 stocks.

• We must reduce the number of positions such that the overall risk of the portfolio approximates the observed risk of the subject fund.
  – We can do this by running an optimization while using the "Maximum number of Assets" parameter.

• With a little trial and error, we can find the portfolio that matches the benchmark (and the subject fund) in style.
  – We reduce the diversification to the point where the expected volatility of the proxy portfolio matches the observed volatility of the subject hedge fund.
Check the Balance Between Factor and Asset Specific Risks

• We now load the refined (reduced number of positions) proxy portfolio into the Optimizer as the portfolio with a cash benchmark.

• By running a risk report, we can determine how much of:
  – the expected risk of the refined proxy portfolio arises from factor bets
  – Arises from asset specific risk.
  – If this is a reasonable match to the subject fund (from the style analysis) we're done.
Changing the Balance Between Factor and Asset Specific Risks

• If we find we don’t have the appropriate balance between factor and asset specific risks
  – Repeat the process of “refining” the proxy portfolio
  – In addition to defining the Max Assets parameter, we can change the Optimizer’s degree of risk tolerance for factor and asset specific risk
  – Again with a little trial and error, we can find risk acceptance parameter values that bring the relative proportions of factor risk and asset specific risk into line with our analysis of the subject fund

• We now have our proxy portfolio to hold or use as a composite asset in other analyses
Conclusions

• The effectiveness of Returns Based Style Analysis can be enhanced in a number of important ways
• These enhancements are particularly important in analyzing funds where substantial shifts in strategy may be expected over time
Empirical Example 100 HF

- Ran 100 HF through a set of 14 spanning indices, retained TVValues
- Reduced number of independent variables by adding them in order of decreasing abs(TValue) & Rerunning
  - For each fund, dropped all independent variables with abs(TValue) < .5
Style Analysis Results

• R2 between .005 and .9, averaging about .36

• Interesting trend: the greater Cash Allocation, the lower the R2:
  – The more “hedged” the fund, the less information there is for the style analysis to pick up on…
\( R^2 = .465 \)
Next Step (Empirical Ex. Cont)

- Combined spanning index constituents according to style weights, used result as benchmark, optimized, max 500 assets.
- Harvested resultant expected standard deviation of returns.
- Calculated historic standard deviation of HF returns.
Modelled Vs Realized Risk

MF E[risk] vs HF realized risk

Historic HF risk

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Results

- Slope = .978
- R2 = .486
- 85% of the modeled portfolio risks were smaller than the respective observed HF risks.
Final Step (Empirical ex. Cont)

- Adjust expected benchmark risk to match historic hedge fund risk by iteratively adding or deleting cash from the portfolio:
  - Factor exposures will change in magnitude, but not relative proportion to one another

- Adjust factor variance to total variance ratio to match style analysis $R^2$ through iterative optimizations adjusting sysRAP and unSysRAP parameters, e.g.
  - $U = a - (\text{sysRAP} \times \text{FactorVariance} + \text{unSysRap} \times \text{residualVariance})$
$R^2 = .92$
$R^2 = .829$
Empirical Conclusions

- The Style Analysis does a good job of modeling a Hedge Fund’s Factor Variance.
- A further adjustment of stock specific is required to beef up the Expected Risks to fit.
  - This can be done by iteratively adjusting:
    - Cash in Benchmark
    - UnsysRAP vs SysRAP ratio
Who should you listen to?
A sector decomposition of surprise stock returns

Giuliano De Rossi, Quantitative Analyst
James Sefton, Senior Quantitative Analyst
UBS Quantitative Research (Tel: +44 20 756 81873/79882)

13 March 2007
A preview: Performance of analysts’ earnings forecasts

Performance of a long-short strategy based on UBS analysts’ earnings upgrades forecasts

Source: UBS estimates
Outline of the talk

♦ We decompose return surprises into the amount that can be attributed to either
  a) Cash-flow news or
  b) Discount rate news.

♦ We discuss the results with respect to a global universe highlighting
  a) Sector differences,
  b) Links to similar results from a residual income model,
  c) Correlations in the returns portfolios constructed on basis of a value screen or a growth screen by sector.

♦ We discuss the results with respect to a US universe, estimated on quarterly data and augmented with macroeconomic factors, highlighting
  a) those sectors driven by cash flow news at the sector level and those where the majority of news is at a stock level.
  b) those sectors driven by news in the macroeconomy.

♦ We discuss the implications of the research for alpha generation
  a) Demonstrating the sectors in which a quantitative strategy performs.
  b) Demonstrating the sectors in which strategies based on fundamentals performs.
Cash-Flow or Discount Rate News

♦ The value of a share is the expected present value of its accruing cash flows

\[ P_t = \mathbb{E}_t \left[ \sum_{i=1}^{\infty} \frac{D_{t+i}}{\prod_{j=1}^{i}(1 + r_{t+j})} \right] \]

♦ Hence an unexpected change in the value, \( P_t - \mathbb{E}_{t-1}(P_t) \) must be due to new information (news) about either
   a) the cash flows (dividends) – Cash-flow news
   b) or the discount rates – Discount rate news.

♦ If discount rates are constant and known, then all price changes will be due to news about the future cash flows.
Cash Flow or Discount Rate News

♦ However discount rates may vary in cross-section (a time-varying conditional beta) because the
  
  – firm’s mix between safe and risky projects vary over time (Green, Berk and Naik, 1999) due to investment in new projects.
  
  – ‘value firms’ suffer from costly irreversible investment in bad times and therefore more risky (Zhang 2005).

♦ Or risk premium may vary over time due to investors’ changing attitudes towards risk (e.g. Campbell, Cochrane 1999).

BUT in these cases discount rate news will be correlated with cash flow news (but as Campbell and Cochrane note, the correlation can be quite low but not zero).

♦ Alternatively returns may be forecastable because of systematic biases in investors’ expectations
  
  – In Daniel, Hirshleifer and Subrahmanyan (2000) a fundamental to price ratio is a proxy for mispricing in the cross-section.
  
  – Barberis, Schleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyan (1998) where herding or overconfidence causes returns to predictable over time.
Data

We used two datasets for our empirical analysis:

♦ A Global Stock Universe
  – All constituents of the DJ World index.
  – Accounting data sourced from Worldscope, adjusted prices from Dow Jones.

♦ A US Stock Universe
  – All US-based constituents of the DJ World index.
  – Quarterly, accounting and price data from Compustat, macroeconomic series from the Federal Reserve and Bureau of Economic Analysis.
The Residual Income Model

♦ We illustrate our approach using a Residual Income Model

\[
P_t = \frac{B_t}{\text{Book value}} \cdot \frac{\sum_{i=1}^{\infty} \frac{E_{t+i} - rB_{t+i-1}}{(1+r)^i}}{\text{Residual income}} + \frac{\sum_{i=1}^{\infty} \rho^i (ROE_{t+i} - r)}{\text{Abnormal earnings}} \]

which implies

\[
r_t - r \approx \frac{B_t}{P_t} (1 + g) \Delta \mathbb{E}_t \sum_{i=1}^{\infty} \rho^i (ROE_{t+i} - r)
\]

where \( \rho = \frac{1 + g}{1 + r} \), \( g \) is the average growth rate and \( \Delta \mathbb{E}_t(.) = \mathbb{E}_t(.) - \mathbb{E}_{t-1}(.) \) is the change in expectations.

♦ Assume the discount rate is fixed, and Return on Equity \( E_t / B_{t-1} \) evolves

\[
ROE_t = \alpha + \phi ROE_{t-1} + \varepsilon_t
\]

then we can express the variance of cash-flow returns as

\[
\text{Var} \left[ \Delta \mathbb{E}_t \sum_{i=1}^{\infty} \rho^i (ROE_{t+i} - r) \right] = \frac{\sigma_{\varepsilon}^2 (\rho \phi)^2}{1 - (\rho \phi)^2}
\]
The importance of cash-flow news in explaining return variance: Estimates of $\phi$

Source: UBS estimates
The main model: Decomposing surprise stock returns

♦ By taking a log-linear approximation of the Residual Income Model, Vuolteenaho (2002) was able to drop the assumption of constant discount rates.

♦ Now the return surprise can be decomposed into cash flow or discount rate news

\[
\begin{align*}
    r_t - E_{t-1}(r_t) &= \Delta E_t \sum_{i=0}^{\infty} \phi^i e_{t+i} - \Delta E_t \sum_{i=1}^{\infty} \phi^i r_{t+i} + \kappa_t \\
    \text{Cash Flow News} & \quad \text{Expected Return News} \quad \text{approx. error}
\end{align*}
\]

where \( e_t = \log \left(1 + \frac{ROE_t - R^f_t}{1 + R^f_t}\right) \) and \( R^f_t \) is the risk free rate.

♦ Further instead of the earlier simple the model describing the evolution of ROE, we use a vector autoregression (VAR)

\[
    z_{i,t} = \Gamma z_{i,t-1} + u_{i,t}.
\]

where \( u_t \) are serially uncorrelated errors. The components of \( z_{i,t} \) will be **stock return** (the variable we are ultimately interested in), (log) **book to market ratio** and **ROE**, but also sector-level and macroeconomic factors.
The VAR model: example

♦ An example: The VAR below was estimated on data for all stocks in the Dow Jones Global Universe over our sample period

\[
\begin{bmatrix}
  r_t \\
  B_t / P_t \\
  e_t \\
  r_{t}^{\text{Sector}} \\
  (B / P)_{t}^{\text{Sector}} \\
  e_{t}^{\text{Sector}} \\
\end{bmatrix}
\begin{bmatrix}
  -0.00082 \\
  0.00055 \\
  -0.0003 \\
  0.00004 \\
  -0.00001 \\
  0.00002 \\
\end{bmatrix}
\begin{bmatrix}
  0.074 \\
  -0.059 \\
  0.041 \\
  0 \\
  0 \\
\end{bmatrix}
+ 
\begin{bmatrix}
  0.320 \\
  0.814 \\
  -0.037 \\
  0 \\
  0 \\
\end{bmatrix}
\begin{bmatrix}
  0.167 \\
  -0.125 \\
  0.514 \\
  0.198 \\
  0.198 \\
\end{bmatrix}
+ 
\begin{bmatrix}
  0.410 \\
  -0.084 \\
  -0.017 \\
  0.348 \\
  0.605 \\
\end{bmatrix}
\begin{bmatrix}
  0.516 \\
  0.142 \\
  0.031 \\
  0.799 \\
  0.165 \\
\end{bmatrix}
= 
\begin{bmatrix}
  r_{t-1} \\
  B_{t-1} / P_{t-1} \\
  e_{t-1} \\
  r_{t-1}^{\text{Sector}} \\
  (B / P)_{t-1}^{\text{Sector}} \\
  e_{t-1}^{\text{Sector}} \\
\end{bmatrix}
\begin{bmatrix}
  u_{1,t} \\
  u_{2,t} \\
  u_{3,t} \\
  u_{4,t} \\
  u_{5,t} \\
  u_{6,t} \\
\end{bmatrix}
\]

♦ Similar VARs were estimated for each global industry separately.
Results: Global model

Source: UBS

- Ratio of cash-flow news and discount rate news variance (LHS Axis)
- Correlation between return to relative value and return to not-growth (RHS Axis)
The ‘value premium correlation’ is the correlation between returns to relative-value portfolios and relative-not-growth portfolios.

Source: UBS
Deductions at the level of the Global universe

♦ **Utilities, Oil & Gas sectors and Basic Materials**: Even at the stock level, approximately 50% of the surprise return is due to news about discount rates. This suggests

– that if discount rate news is important, then returns are forecastable.
– If returns are forecastable, a quantitative strategy is more likely to deliver significant alpha.
– As corroboration, in these sectors returns to the value strategy are uncorrelated with the returns to a portfolio of low growth stocks.
– as returns are only partially driven by cash flow news, a strategy based on fundamental analysis will find it harder to out-perform.

♦ **Technology, Telecommunications and Consumer Services**: In these sectors, stocks surprise returns are largely driven by cash-flow news.

– Not just due to the TMT bubble.
– Suggests a strategy based on fundamental analysis is likely to deliver.
– Little predictability in returns, and hence simple quantitative strategies unlikely to outperform.
Results for the US market (1)

Ratio of cash-flow news and discount rate news

Source: UBS
Results for the US market (2)

♦ **Sector-level** cash-flow news (as a proportion of total cash-flow news) is more relevant in highly concentrated sectors.

♦ The columns display:

1. Proportion of cash-flow news originating at sector level
2. Average Herfindahl Hirschman index of concentration (calculated on market capitalization) over the sample period.

<table>
<thead>
<tr>
<th>Sector/Total</th>
<th>HH index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil &amp; Gas</td>
<td>0.09</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>0.03</td>
</tr>
<tr>
<td>Industrials</td>
<td>0.03</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>0.04</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.01</td>
</tr>
<tr>
<td>Consumer Services</td>
<td>0.03</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>0.15</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.11</td>
</tr>
<tr>
<td>Financials</td>
<td>0.11</td>
</tr>
<tr>
<td>Technology</td>
<td>0.05</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Source: UBS
Adding macroeconomic factors to the model (1)

- We augment the VAR model with macro factors. Our preferred specification is selected by maximising the Akaike information criterion. This model includes GDP growth, 3M Bill Rate and trade balance changes, and the three-month rate.

- A triangular structure is imposed on the VAR:

\[
\begin{bmatrix}
\text{Stock Level Variables} \\
\text{Sector Level Variables} \\
\text{Macro Variables}
\end{bmatrix}_t = 
\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
0 & A_{22} & A_{23} \\
0 & 0 & A_{33}
\end{bmatrix}
\begin{bmatrix}
\text{Stock Level Variables} \\
\text{Sector Level Variables} \\
\text{Macro Variables}
\end{bmatrix}_{t-1} + u_t
\]

- For a sanity check;
  - We compare the VAR equilibrium values with historic means of the macro variables
  - We compare our model’s predictions for Q1 2007 with UBS forecasts

<table>
<thead>
<tr>
<th></th>
<th>Model Equilibrium</th>
<th>Sample Means</th>
<th>Model Forecast for Q1:2007</th>
<th>UBS forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>3.08%</td>
<td>2.93%</td>
<td>0.51% (Qtr)</td>
<td>0.45% (Qtr)</td>
</tr>
<tr>
<td>Trade balance</td>
<td>0.00%</td>
<td>-0.06%</td>
<td>-0.02%</td>
<td>0.10%</td>
</tr>
<tr>
<td>Short rate</td>
<td>3.55%</td>
<td>4.08%</td>
<td>4.62%</td>
<td>4.85%</td>
</tr>
</tbody>
</table>

Source: UBS. Source for UBS forecasts: “US Economic Perspectives”, Feb 16th. The UBS predicted 3-month rate is reduced by 15bp to ensure comparability with the historical figures from the Federal Reserve. UBS forecasts the change in the Trade Balance for good and services, whereas the VAR is for goods only. Mean levels are computed over the whole sample period.
Adding macroeconomic factors to the model (2)

- The significant of the macro variables on the respective sector return when measured relative to the market (robust F-test results):

<table>
<thead>
<tr>
<th>Sector</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sectors</td>
<td>0.020 **</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>0.298</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>0.020 **</td>
</tr>
<tr>
<td>Industrials</td>
<td>0.009 **</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>0.000 **</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.129</td>
</tr>
<tr>
<td>Consumer Services</td>
<td>0.005 **</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>0.481</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.897</td>
</tr>
<tr>
<td>Financials</td>
<td>0.030 **</td>
</tr>
<tr>
<td>Technology</td>
<td>0.657</td>
</tr>
</tbody>
</table>

** indicates 5% significance, * 10% significance in the robust F-test. Included macro variables are: 10Y rate, industrial production, GDP, trade balance, credit spread, 3M rate.

Source: UBS
Discussion of the findings for the US market

♦ In more concentrated sectors, the relevance of sector-level cash-flow news tends to increase.

♦ **Utilities, Oil & Gas sectors**: These sectors have the lowest ratio of cash flow to discount rate news. This implies
  – Even in the US returns are forecastable in these sectors.
  – As corroboration, in these sectors returns to the value strategy are uncorrelated with the returns to a portfolio of low growth stocks.

♦ **Technology, Telecommunications and Consumer Services**: In these sectors, stocks surprise returns are largely driven by cash-flow news.
  – Suggests a strategy based on fundamental analysis is likely to deliver.
  – However a considerable proportion of the news originates at the sector-level. A position in the sector alone would be sufficient to capture this alpha.
Using the US model to design a quantitative strategy

♦ We simulate a quantitative trading strategy based on our estimated model. This strategy is equivalent to weighted screens on momentum, ROE and Book/Price. The weights are the coefficients implicit within the VAR.

♦ For each quarter in the sample period, we rank all US stocks within a sector by predicted return one quarter ahead.

♦ Buy the top 30% stocks, equally weighted

♦ Short the bottom 30% stocks, equally weighted.

♦ All portfolios are self-financing

<table>
<thead>
<tr>
<th>Sector</th>
<th>Mean Excess Return</th>
<th>Annualised Volatility</th>
<th>Sharpe Ratio</th>
<th>Momentum weigh</th>
<th>Value weight</th>
<th>Quality weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilities</td>
<td>3.3%</td>
<td>9.0%</td>
<td>0.37</td>
<td>-0.0024</td>
<td>0.0056</td>
<td>0.0066</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>9.5%</td>
<td>16.4%</td>
<td>0.58</td>
<td>0.0266</td>
<td>0.0128</td>
<td>0.0046</td>
</tr>
<tr>
<td>Consumer Services</td>
<td>10.7%</td>
<td>12.8%</td>
<td>0.84</td>
<td>0.0030</td>
<td>0.0053</td>
<td>0.0151</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>5.6%</td>
<td>14.0%</td>
<td>0.40</td>
<td>-0.0049</td>
<td>-0.0030</td>
<td>0.0047</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.1%</td>
<td>12.4%</td>
<td>0.81</td>
<td>0.0036</td>
<td>0.0109</td>
<td>0.0099</td>
</tr>
<tr>
<td>Financials</td>
<td>9.0%</td>
<td>8.0%</td>
<td>1.12</td>
<td>-0.0007</td>
<td>0.0053</td>
<td>0.0095</td>
</tr>
<tr>
<td>Health Care</td>
<td>13.8%</td>
<td>23.6%</td>
<td>0.59</td>
<td>0.0022</td>
<td>0.0131</td>
<td>0.0105</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>9.9%</td>
<td>10.1%</td>
<td>0.99</td>
<td>-0.0005</td>
<td>0.0061</td>
<td>0.0124</td>
</tr>
<tr>
<td>Technology</td>
<td>13.4%</td>
<td>22.1%</td>
<td>0.61</td>
<td>0.0082</td>
<td>0.0195</td>
<td>0.0087</td>
</tr>
</tbody>
</table>

Source: UBS
Designing a Macro Strategy using the model

♦ The output of our forecasting model (VAR) can be combined with macroeconomic views and/or views about sector performances.

♦ Effectively this is a Bayesian or Black and Litterman approach;
  – The priors are the VAR forecasts
  – These are combined with the strategist views using the covariance matrix implicit in the VAR.
  – We are able to incorporate uncertainty in the strategist’s view into the model as well. In the case of the sector views we make the uncertainty proportional to the relevance of cash-flow news.

♦ As illustration of the methodology we incorporate the UBS US macroeconomic forecasts for 2007:Q1.
  − GDP +0.45%  Trade bal 0.1%  Short rate 4.85%

♦ And the following subset of sector views
  − Oil & Gas 0.00  Technology -2.5%  Industrials +2.5%

These returns are proportional to recommended sector weightings in the UBS Global Equity Strategy document of 9th December 2006. (For the other sectors, the recommended portfolio weights are in line with VAR predictions).
Designing a Macro Strategy using the model (2)

♦ Large adjustment for Oil & Gas where we attach more importance to the strategist’s view.

♦ Sectors like Health Care and Telecoms adjust because of the correlations among returns.

<table>
<thead>
<tr>
<th>Prediction of VAR</th>
<th>Prediction after Combination with Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil &amp; Gas</td>
<td>-0.137</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>0.007</td>
</tr>
<tr>
<td>Industrials</td>
<td>-0.006</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>0.069</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.034</td>
</tr>
<tr>
<td>Consumer Services</td>
<td>0.048</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>0.088</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.021</td>
</tr>
<tr>
<td>Financials</td>
<td>-0.048</td>
</tr>
<tr>
<td>Technology</td>
<td>-0.003</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>0.505</td>
</tr>
<tr>
<td>Change in Trade Balance</td>
<td>0.000</td>
</tr>
<tr>
<td>3M Bill Rate</td>
<td>4.618</td>
</tr>
<tr>
<td></td>
<td>4.834</td>
</tr>
</tbody>
</table>

Source: UBS
And of course, it predicts in which sectors a fundamental strategy is likely to be more successful…

Performance of a long-short strategy based on UBS analysts’ earnings upgrades forecasts
Conclusion

• We measure the relative importance of news about future cash-flows and news about future expected returns in explaining surprise returns.

♦ In the Utilities and Oil & Gas sectors, the majority of the return variance can be attributed to discount rate news. This suggests a quantitative strategy is more likely to deliver significant alpha.

♦ As corroboration, sectors where a significant proportion of return variance can be attributed to discount rate news are the very sectors where returns to a relative value strategy is uncorrelated with the returns to a portfolio of low growth stocks.

♦ For Industrials, Technology, Telecommunications and Consumer Services, return variance is largely driven by cash-flow news. The evidence is that strategies based on fundamental are more likely to outperform in these sectors.

♦ We illustrate how our model can be used in practice to guide asset allocation, possibly by incorporating macroeconomic views and sector outlooks and/or earnings forecasts at the firm level.
References


Appendix
Derivation of Vuolteenaho’s Decomposition (1)

♦ We can write the return as

\[
(1 + R_t + r_t^{rf}) = \left(\frac{P_t + D_t}{P_{t-1}}\right) = \left(\frac{P_t + D_t}{D_t}\right) \left(\frac{D_t}{D_{t-1}}\right) \left(\frac{D_{t-1}}{P_{t-1}}\right)
\]

♦ Campbell and Shiller (1988) take a log-linear approximation, to show

\[
r_t + r_t^{rf} = \log \left(1 + e^{-\delta_t}\right) + \Delta d_t + \delta_{t-1}
\]

\[
\approx \left(\kappa - \rho \delta_t\right) + \Delta d_t + \delta_{t-1}
\]

\[
\text{log linear approx} + \text{log of dividend growth} + \text{log of Dividend Yield}
\]

♦ If returns in any period are higher than expected then either

1) Dividend growth is higher than expected or
2) Yields have fallen.
Derivation of the Decomposition Formula (2)

♦ Similarly note that the clean surplus accounting relation can be written

\[
(1 + \frac{E_t}{B_t}) = \left( \frac{B_t + D_t}{B_{t-1}} \right) = \left( \frac{B_t + D_t}{D_t} \right) \left( \frac{D_t}{D_{t-1}} \right) \left( \frac{D_{t-1}}{B_{t-1}} \right)
\]

and taking logs implies

\[
e_t + r_t^{\text{rf}} \approx \left( \kappa - \rho \gamma_t \right) \quad + \quad \Delta d_t \quad + \quad \gamma_{t-1}
\]

log linear approx log of dividend growth log(Dividend/Book)

♦ Subtract this relation from Campbell and Shiller relation

\[
e_t - r_t \approx \rho \left( \delta_t - \gamma_t \right) - \left( \delta_{t-1} - \gamma_{t-1} \right)
\]

log(Book/Market) lagged log(Book/Market)

♦ If ROE is greater than returns then the Book to Market Ratio increases (by an amount related to payout ratios).
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<table>
<thead>
<tr>
<th>UBS rating</th>
<th>Definition</th>
<th>UBS rating</th>
<th>Definition</th>
<th>Rating category</th>
<th>Coverage¹</th>
<th>IB services²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buy 1</strong></td>
<td>FSR is &gt; 6% above the MRA, higher degree of predictability</td>
<td><strong>Buy 2</strong></td>
<td>FSR is &gt; 6% above the MRA, lower degree of predictability</td>
<td>Buy</td>
<td>44%</td>
<td>36%</td>
</tr>
<tr>
<td><strong>Neutral 1</strong></td>
<td>FSR is between -6% and 6% of the MRA, higher degree of predictability</td>
<td><strong>Neutral 2</strong></td>
<td>FSR is between -6% and 6% of the MRA, lower degree of predictability</td>
<td>Hold/Neutral</td>
<td>43%</td>
<td>36%</td>
</tr>
<tr>
<td><strong>Reduce 1</strong></td>
<td>FSR is &gt; 6% below the MRA, higher degree of predictability</td>
<td><strong>Reduce 2</strong></td>
<td>FSR is &gt; 6% below the MRA, lower degree of predictability</td>
<td>Sell</td>
<td>13%</td>
<td>26%</td>
</tr>
</tbody>
</table>

1: Percentage of companies under coverage globally within this rating category.
2: Percentage of companies within this rating category for which investment banking (IB) services were provided within the past 12 months.

Source: UBS; Rating allocations as of 31 December 2006.

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Alpha Scaling Revisited

June 21, 2007

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Motivation

- Portfolio construction
  = transferring investment skill efficiently into positions
  central to the success of an asset management firm

- Traditional portfolio construction incorporates qualitative information

- Quant, particularly optimization, uses information in the form of risk and return

- investment views → return forecasts → positions
Examples of Views

- Tech Analyst – “IBM is a strong buy”
- Strategist – “Financials will mildly outperform over the next year”
- Model – “On a scale from 1-10, Siemens is an 8”
Alpha Scaling/Adjustment

I. Extract all the information contained in the view to formulate a best return forecast

II. Given a set of best forecasts, condition them, so they are suitable for use in an optimizer
I: Extracting Information

- Seek the best prediction of the future given the information
- Suppose the analyst overreacts and is at times wrong
  - The best forecast of the future tempers the analyst’s opinion
  - On the other hand, if the analyst is exceedingly cautious, the best forecast should amplify the opinion
- Convert information (e.g. ratings) to returns
II: Conditioning for Optimization

- Optimizers seek extremes (by mandate!)
- Inputs are estimated with error
- Optimized selection introduces bias

- Conditioning deals with optimization under uncertain inputs, a large and separate topic
  - Northfield is building a set of tools to address this
Overview of Alpha Scaling Presentation

- Standard methods of constructing good forecasts spelled out
- Standard method of combining sets of forecasts
- Northfield’s upcoming alpha scaling tool
How to make signals (views) into forecasts?

- One approach - fit a linear model
  
  \[
  \hat{y}(\hat{g}) = A \hat{g} + b
  \]

- Minimize expected squared error
  
  \[
  \hat{y}(\hat{g}) = E(y) + \text{cov}(y, \ g) \text{cov}(g, \ g)^{-1} [\hat{g} - E(g)]
  \]
Linear Model (cont)

- e.g. $g_i$ = stock i’s analyst rating (1-5)
  stock i’s earnings surprise
  stock i’s percentile rank
  change in 90 day T-bill yield

- e.g. $y_k$ = stock k’s return
  stock k’s return net of market $\beta$ and industry

- Important observation: each $y_k$ is built separately
  $\hat{y}_k(\hat{g}) = E(y_k) + \text{cov}(y_k, g) \text{cov}(g, g)^{-1} [\hat{g} - E(g)]$
One Signal Per Stock – Grinold

- Forecast $y_k$ using only signal $g_k$
  - e.g. forecast IBM’s return from only IBM’s rating

- $\hat{y}_k(\hat{g}_k) = a \hat{g}_k + b$

  choose $a$ and $b$ to minimize expected squared error:
  
  $\hat{y}_k(\hat{g}) = E(y_k) + \text{cov}(y_k, g_k) / \text{var}(g_k) [\hat{g}_k - E(g_k)]$

  
  $\text{IC} = \frac{E(y_k) + \rho(y_k, g_k) \times \text{std}(y_k) \times \text{std}(g_k)}{\text{volatility}} \times \frac{[\hat{g}_k - E(g_k)] / \text{std}(g_k)}{\text{score}}$
IC = correlation (signal, return being forecast)

Volatility is the volatility of the return being forecast

Score is the z-score of that instance of the signal

IC can be estimated over a group of securities (e.g. same cap/industry/volatility) if the model works equally well on them

Expect lower IC’s for volatile securities (harder to predict) than for less volatile ones (easier to predict)

Using a single IC exaggerates volatile securities’ alphas
Grinold Example

- The upcoming period is
  - good for DELL (z-score of 1)
  - better for MSFT (z-score of 2)
  - great for PEP (z-score of 3)

- **Stock-specific volatility**: $\sigma_{ss}^{DELL} = 27\%$, $\sigma_{ss}^{MSFT} = 25\%$, $\sigma_{ss}^{PEP} = 9\%$

- **Skill, corr(signal,return)**: $IC_{tech} = .10$, $IC_{consumer} = .15$

- **Assume** $E[y] = 0$, stock-specific return averages 0 over time

  \[
  \hat{y}_{DELL} = 0 + .10 \times 27\% \times 1 = 2.7\%
  \]
  \[
  \hat{y}_{MSFT} = 0 + .10 \times 25\% \times 2 = 5.0\%
  \]
  \[
  \hat{y}_{PEP} = 0 + .15 \times 9\% \times 3 = 4.0\%
  \]
Grinold Cross-Sectionally

- \( \hat{y}_k \) = stock k’s return over a benchmark
  \( \hat{g}_k \) = the relative attractiveness of stock k

  e.g. forecast IBM’s return over the market using IBM’s %ile in a stock screen

- \( \hat{y}_k(\hat{g}_k) = E(y_k) + IC(y_k, g_k) \times std(y_k) \times score(\hat{g}_k) \)

Assume 1) The volatility of what you are predicting is the same across all stocks
2) All stocks are equally likely to have a given level of relative attractiveness
   e.g. utility co is as likely to be a strong buy as tech co
Grinold Cross-Sectionally (cont)

- \( \hat{y}_k(\hat{g}_k) = E(y_k) + IC(y_k, g_k) \times \text{std}(y) \times \text{score}(\hat{g}_k) \)

  - \( \text{std}(y) \) can be estimated by cross-sectional return vol
  - \( \text{score}(\hat{g}_k) \) can be estimated by \( \hat{g}_k \)'s cross-sectional score

If skill is the same across all securities,

- \( \text{IC} \) can be estimated by correlation between cross-sectional score and relative return

- \( \hat{y}_k(\hat{g}_k) = IC \times \text{xc volatility} \times \text{xc score} \)
Cross-Sectional Grinold Example

- Relative to other stocks,
  - DELL will outperform (z-score of 2)
  - MSFT will strongly outperform (z-score of 3)
  - PEP will slightly outperform (z-score of 1)

- Cross-sectional volatility of 1 year returns is 15%

- Skill, corr(xc signal, xc return): IC_{tech stocks} = .08, IC_{consumer stocks} = .12

  \[
  \hat{y}_{MSFT} = .08 \times 15\% \times 3 = 3.6\%
  \hat{y}_{DELL} = .08 \times 15\% \times 2 = 2.4\%
  \hat{y}_{PEP} = .12 \times 15\% \times 1 = 1.8\%
  \]
Combining Sets of Good Forecasts: Black Litterman

- Asset managers have different sets of information
  - IBM will return 5%
  - SP500 will beat R2000 by 4%

- Once cleaned up (see previous slides), how can they be fused into 1 forecast per stock?

- Ans: Black-Litterman
Black Litterman

- Motivated by need to stabilize asset allocation optimization
- Bayesian Approach
- Assume a prior distribution on the vector of mean returns
  - Centered at implied alpha that makes market portfolio optimal (stability)
  - Covariance is proportional to covariance of returns
New information given as portfolio forecasts with error

- IBM’s will return 5% ± 2%
  i.e. return of portfolio holding 100% IBM is 5% ± 2%

- MSFT will outperform IBM by 3% ± 4%
  i.e. return of portfolio long MSFT short IBM is 3% ± 4%

- S&P500 will outperform R2000 by 4% ± 2%

Combined forecast is expected value given prior and information
Black-Litterman (cont)

- prior on mean returns:
  - \( \mathbf{m} \sim \mathcal{N} (\mathbf{m}_0, \Sigma_0) \)

- forecasts impart new information:
  - \( \hat{\mathbf{g}} = \mathbf{P} \mathbb{E}[\mathbf{m} | \text{info}] + \mathbf{\varepsilon} \)
  - \( \mathbf{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \Omega) \)

- \( \hat{\mathbf{y}} = [\Sigma_0^{-1} + \mathbf{P}^\top \Omega^{-1} \mathbf{P}]^{-1} [\Sigma_0^{-1} \mathbf{m}_0 + \mathbf{P}^\top \Omega^{-1} \hat{\mathbf{g}}] \)
  - \( = \mathbf{m}_0 + [\Sigma_0^{-1} + \mathbf{P}^\top \Omega^{-1} \mathbf{P}]^{-1} \mathbf{P}^\top \Omega^{-1} (\hat{\mathbf{g}} - \mathbf{Pm}_0) \)
  - \( = \mathbf{m}_0 + [\Sigma_0 - (\mathbf{P} \Sigma_0)\mathbf{P}^\top (\Omega + \mathbf{P} \Sigma_0 \mathbf{P}^\top)^{-1} \mathbf{P} \Sigma_0] \mathbf{P}^\top \Omega^{-1} (\hat{\mathbf{g}} - \mathbf{Pm}_0) \)

- Because of the prior’s covariance, one security tells us about another. e.g. if IBM and DELL are correlated, information about IBM says something about DELL
Black-Litterman Example

- Prior on IBM and DELL of $(2\%, 5\%)$, with respective variances $4\%^2$, $9\%^2$ and correlation $.5$

- Predict that IBM will return $5\% \pm 3\%$

- $m_0 = (2\% \ 5\%)^T$, $\Sigma_0 = (4 \ 3; \ 3 \ 9) \ %^2$
  $P = (1 \ 0)$, $\hat{g} = 5\%$, $\Omega = 9\%^2$

- Updated forecasts: $\hat{y}_{IBM} = 2.9\%$, $\hat{y}_{DELL} = 5.7\%$
Extending Black Litterman

- Consider as underlying securities all the stock specific returns and all the returns to factors, e.g. \( m = (m_{ss}^{IBM}, m_{ss}^{DELL}, \ldots, m_{E/P}, m_{GROWTH}, \text{ etc.}) \)

- Make forecasts at different levels
  - Net of style and industry, IBM will return 5% ± 4%
  - The dividend yield factor will return 2% ± 3%
  - Inclusive of all effects, DELL will return 9% ± 6%
  - S&P500 will outperform R2000 by 4% ± 2%

- Information gets projected onto all securities. e.g. forecast about S&P500 over R2000 → return on market cap → return on large and small cap stocks which aren’t in S&P500 or R2000

- Easy to implement
Suppose the best forecast is that IBM beats the benchmark by 5% over the next 6 months, and you have no opinion beyond.

What is the forecast alpha if you plan to hold IBM for 6 mos? A year?

Combined forecast ≈ time-weighted average over reference holding period of each interval’s best forecasts

e.g. 2 yr, 8% annualized over 1st 6 mo, 1% over remaining 18 → ¼ × 8% + ¾ × 1% = 2.75%
Seek a theoretically sound, information preserving, robust way of refining investment views.

Have client’s forecast alpha. Don’t know alpha generating process.

Sophisticated methods leverage information. Better to be simple than falsely precise.

Beginning from alpha forecasts (not individual stock scores) necessitates a cross-sectional framework: Cross-sectional Grinold.
Preprocess for Robustness: Rank Rescaling into Scores

- Map raw signals by rank onto standard normal
  e.g. 25\(^{th}\) percentile $\rightarrow$ $F^{-1}(0.25)$
Estimate Cross-Sectional Volatility

- Expected market weighted cross sectional variance
  \[ E[\sum_s w_s (r_s - r_m)^2] \]
  where \( r_m = \sum_s w_s r_s \)
  \[ = E[\sum_s w_s (r_s - \mu_s + \mu_s - \mu_m + \mu_m - r_m)^2] \]
  \[ = \sum_s w_s \sigma_s^2 - \sigma_m^2 + \sum_s w_s (\mu_s - \mu_m)^2 \]
  \[ \approx \sum_s w_s \sigma_s^2 - \sigma_m^2 \]
  \[ = \text{avg stock variance} - \text{variance of the market} \]

- Numbers come straight from risk model
- If forecasting return net of \( \beta \), industry, etc., easy to calculate risk net of these effects
Put The Pieces Together

- IC – user parameter
  cross sectional volatility – from risk model
  score – signal after rank mapping to std N

- Forecast of return above market
  \[ \text{Forecast of return above market} = \text{IC} \times \text{volatility} \times \text{score} \]
Standard practice alpha scaling methods can be arrived at by following your nose. No hidden magic or sophistication.

Being clear about the inputs and what’s being forecast is this first step in scaling alphas well.

Adjustments for horizon and signal decay are important, particularly in low-turnover portfolios.

Northfield’s upcoming alpha scaling functionality can make your life easier.
References


Portfolio Construction in a Regression Framework

James Sefton

June 2007
Motivation

♦ The approach can be used to answer the following questions:

– Are the weights in my portfolio the result of one or two abnormal events?
– How well are my portfolio weights estimated? Are some positions critical to the portfolio performance?
– Are my holdings sub-optimal? Is the evidence strong enough to warrant a trade?
– How different are two portfolios? Is one more efficient than the other? Or are they both efficient but just offer different risk-return trade-offs?
– Do I have significant tilts in my portfolio? Are these a result of stock picks or style tilts?
– What are the costs in terms of portfolio efficiency of my investment constraints?
– Is the new information sufficient to warrant a rebalancing?
What is so different about this approach?

♦ *Modified Mean-Variance Problem*: Given a time series of returns to a set of assets, estimate portfolio weights that maximise portfolio return for a given level of risk.

  – Step 1. Assume normality, and estimate from the time series data, the mean and covariance matrix of asset returns.
  – Step 2. Using an optimiser, estimate the optimal portfolio weights.

♦ Many authors (Jobson and Korkie, 1981; Jorian, 1992; Broadie, 1993; Michaud, 1989, Best and Grauer, 1991) have noted how sensitive the portfolio weights in Step 2 are to sampling errors in Step 1. Michaud coined the term that optimisers are ‘*error maximisers*’.

♦ The regression approach is 1 step procedure from data to portfolio. We can use all the developed regression diagnostics to analyse our portfolio.
Resampled Efficient Portfolios

Michaud proposed an *ad hoc* procedure to limit the sampling error problem that has received considerable attention. It depends crucially on imposing a no-shorting constraint on all estimated portfolios.

- **Step 1.** From the observed asset returns series, estimate a mean vector, $\mu_0$, and covariance matrix $V_0$ of the asset returns.
- **Step 2.** Using the estimated distribution, generate another set of asset returns and re-estimate a new mean vector, $\mu_i$, and covariance matrix $V_i$ of the generated asset return series.
- **Step 3.** Estimate a minimum variance and maximum return portfolio from the distribution $N(\mu_i, V_i)$. Also calculate the other 8 ranked decile portfolios with volatility equally spaced between the minimum and maximum.
- **Step 4.** Go to step 2 and repeat $n$ times.
- **Step 5.** The 10 resampled efficient ranked portfolios are then the simply the average of the $n$ estimated decile portfolios.
The resampled frontiers all lie below the estimated frontier estimated form $\mu_0$ and $V_0$. These portfolios are averaged to get the 5th decile portfolio and the maximum return portfolio.
Resampling is a shrinkage estimator – Scherer (2004)

- The no-short constraint means that
  - average weights of low return/high volatility assets are biased up
  - average weights of high return/low volatility assets are biased down

![Diagram showing portfolio weights for different assets with and without no-short constraints.](image-url)
The Regression Approach – Britten-Jones (1999)

- The issue is not the magnitude of the estimation error *per se* but rather the way it impacts upon the portfolio construction process – Broadie (1993).

- The advantages of a regression based approach is that
  1. It effectively maps returns directly into portfolio weights. A one step procedure!
  2. It is statistically rigorous and has a long pedigree!
  3. There is a substantial toolkit that can be brought to bear on regression problems.
  4. As it is simple, it can be easily extended to look at many practical problems e.g. incorporating priors, forecasts, sensitivity analysis.
The Regression Framework

♦ Define the matrices of asset returns and indicators as

\[
R = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1N} \\
    r_{21} & r_{22} & \cdots & r_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{T1} & r_{T2} & \cdots & r_{TN}
\end{bmatrix} \in \mathbb{R}^{T \times N} \quad e_T = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{T} \quad e_N = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{N}
\]

♦ And the portfolio weights were the least squares estimate in the regressions

\[\gamma e_T = Rw + \varepsilon\]

For some constant \(\gamma\) subject to the restriction that \(e_N'w = 1\)
The Regression Framework

- As $\gamma$ varies the regression weights trace out the portfolios on the efficient frontier.

$$\varepsilon'\varepsilon = \sum_t (\gamma - \mu_E)^2 + w' (R - \mu)' \left( \frac{1}{\sigma_E^2} (R - \mu) \right) w$$
An Asset Allocation Example

To illustrate the technique we build a portfolio of six equity funds of six markets. The returns to the funds are the returns to the relevant Dow Jones Global Index between Oct-96 and Sep-06.

A summary of the fund returns is

<table>
<thead>
<tr>
<th></th>
<th>Annualised Mean Return</th>
<th>Annualised Volatilities</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>US</td>
</tr>
<tr>
<td>US</td>
<td>9.6</td>
<td>17.4</td>
<td>1.00</td>
</tr>
<tr>
<td>Canada</td>
<td>14.3</td>
<td>18.5</td>
<td>1.00</td>
</tr>
<tr>
<td>UK</td>
<td>10.6</td>
<td>16.4</td>
<td>1.00</td>
</tr>
<tr>
<td>Euro Area</td>
<td>12.2</td>
<td>20.4</td>
<td>1.00</td>
</tr>
<tr>
<td>Japan</td>
<td>3.9</td>
<td>22.6</td>
<td></td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>6.0</td>
<td>20.1</td>
<td></td>
</tr>
</tbody>
</table>
Estimating the efficient frontier

As we vary $\gamma$ so we estimate the portfolio along the frontier.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Minimum Variance $= \frac{\gamma}{\beta} = 0.16$</th>
<th>Tangency Portfolio $= \frac{(1+\alpha)}{\gamma} = 23.32$</th>
<th>46.65</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portfolio Weights</strong></td>
<td><strong>Std. Err</strong></td>
<td><strong>Std. Err</strong></td>
<td><strong>Std. Err</strong></td>
</tr>
<tr>
<td>US</td>
<td>0.269 (0.06)</td>
<td>-0.263 (0.68)</td>
<td>-0.800 (1.37)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.127 (0.05)</td>
<td>1.137 (0.65)</td>
<td>2.155 (1.31)</td>
</tr>
<tr>
<td>UK</td>
<td>0.645 (0.06)</td>
<td>0.656 (0.75)</td>
<td>0.667 (1.50)</td>
</tr>
<tr>
<td>Euro Area</td>
<td>-0.341 (0.06)</td>
<td>-0.010 (0.70)</td>
<td>0.324 (1.40)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.180 (0.03)</td>
<td>-0.150 (0.36)</td>
<td>-0.482 (0.72)</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>0.121 (0.04)</td>
<td>-0.370 (0.50)</td>
<td>-0.864 (0.99)</td>
</tr>
<tr>
<td><strong>Average Return, $\mu_N$</strong></td>
<td>8.49</td>
<td>17.73</td>
<td>27.03</td>
</tr>
<tr>
<td><strong>Volatility, $\sigma_N$</strong></td>
<td>13.97</td>
<td>20.18</td>
<td>32.41</td>
</tr>
<tr>
<td><strong>Zero-Beta Return, $r_Z$</strong></td>
<td>$-\infty$</td>
<td>0.00</td>
<td>4.26</td>
</tr>
<tr>
<td><strong>Portfolio Sharpe Ratio</strong></td>
<td>N/A</td>
<td>0.88</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Assume a normal distribution for 5 assets. Generate 24 months of return data.

- True Efficient Frontier – the assumed frontier
- Estimated Frontier – the estimated frontier
- Actual Frontier – The achieved performance from the estimated portfolios

Source: reproduced with permission from Mark Broadie (1993), Annals of Operations Research
Leverage Statistics enables the detection of outliers

- Leverage and Influence statistic suggest September 1998 has undue impact. Dropping this **one** observation increases the underweight position on Asia by 20% and reduce the UK by 10%.
Testing Portfolio Efficiency

♦ For a given $\gamma$, we can test the efficiency of a portfolio $P$ by an F-test of whether the estimated portfolio weights $w=w_E$ are equal to the weights of portfolio $P$.

$$Z(\gamma) = \left( \frac{T - N - 1}{N - 1} \right) \left( \frac{SSR(w = w_P) - SSR(w_E)}{SSR(w_E)} \right) \sim F_{N-1,T-N-1}$$

$$Z(\gamma) \approx -\frac{\left( \frac{\mu_E - r_Z}{\sigma_E} \right)^2}{1 + \left( \frac{\mu_P - r_Z}{\sigma_P} \right)^2}$$

♦ However $\gamma$ is unknown. We therefore propose as a statistic the minimum over all $\gamma$

$$Z = \text{Min}_{\gamma} Z(\gamma)$$
Shanken’s Test of Portfolio Efficiency

♦ If a portfolio $P$ is on the efficient frontier then there exists a zero-beta return, $r_Z$ such that for any asset $i$

$$E(r_i) - r_Z = \beta_i \left( E(r_P) - r_Z \right)$$
Shanken’s Test of Portfolio Efficiency

♦ Hence if a portfolio $P$ is efficient frontier, there exists a zero-beta return, $r_Z$ such that for all assets the constant $\alpha_i$ in the regression

$$r_{it} - r_Z = \alpha_i + \beta_i (r_{pt} - r_Z) + \varepsilon_{it}$$

is equal to zero.

♦ Hence Shanken proposes that the efficiency of portfolio $P$ can be tested by an F-test of the null hypothesis that the constants $\alpha_i$ are all zero. This test statistic is

$$W(r_z) = \left( \frac{T - N}{N - 1} \right) \left( \frac{\alpha' \Sigma^{-1} \alpha}{1 + S_P^2} \right) \sim F_{N-1,T-N}$$

where

$$S_P = \left( \frac{\mu_P - r_Z}{\sigma_P} \right)$$

is the Sharpe ratio of $P$.

where $\Sigma$ is the covariance matrix of the residuals $\varepsilon$, $T$ is the number of periods and $N$ the number of assets.
Shanken’s Test of Portfolio Efficiency

♦ This supposes that the zero beta return, $r_z$, is known.

♦ Shanken therefore proposes taking the minimum of this statistic overall returns $r_z$,

$$W = \text{Min}_{r_z} W(r_z)$$

He shows that this minimum exists and gives a strict test of efficiency – in the sense that if portfolio $P$ fails this test, then we can reject the hypothesis that $P$ is efficient.

♦ Note: Finding the minimum amounts to solving a quadratic equation. We shall return to this later.
Theorem

- After a great deal of algebra, it is possible to show that

\[ W(r_z) = Z(\gamma) \]

when

\[ r_z = \frac{(1 + \alpha - \chi \gamma)}{(\chi - \beta \gamma)} \]

where

\[ \alpha = \mu'V^{-1}\mu \quad \beta = e_N'V^{-1}e_N \quad \chi = \mu'V^{-1}e_N \]

- Hence \( W = Z \). Further we can give this minimum a geometric interpretation in the next slide.
Geometric Interpretation

♦ The Shanken’s quadratic condition for the minimum can be rephrased as requiring that the line $r_z - P$ be perpendicular to the line $\gamma - P$.

♦ As we know that angles subtended on the circumference of a circle are $90^0$, the circle with diameter $\gamma - r_z$ connects all ‘close’ portfolios.
Testing the efficiency of investing only in the US

- We can test whether a portfolio invested entirely in the US is efficient, i.e. a null of $w=[1 \ 0 \ 0 \ 0 \ 0 \ 0]$, at different point on the frontier.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>US Portfolio</th>
<th>Japanese Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$=\frac{\chi}{\beta}$</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>$=\frac{(1+\alpha)}{\chi}$</td>
<td>23.32</td>
<td></td>
</tr>
<tr>
<td>Statistic $Z(\gamma)$</td>
<td>0.55</td>
<td>1.63</td>
</tr>
<tr>
<td>No. of Restrictions, $P$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$(T-N-1)/P Z(\gamma)$</td>
<td>57</td>
<td>167.9</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Zero-Beta Return, $r_Z$</td>
<td>$-\infty$</td>
<td>-3589</td>
</tr>
<tr>
<td>Sharpe Ratio of $w_P$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Sharpe Ratio of $w_E$</td>
<td>0.55</td>
<td>0.19</td>
</tr>
<tr>
<td>Sharpe Ratio of $w_E$</td>
<td>0.88</td>
<td>0.65</td>
</tr>
</tbody>
</table>
The $Z$-distance induces a geometry

- We map the risk-return space into a return-efficiency space.
- Portfolios are different because they are more or less efficient or offer a different return.
Bayesian Extensions

♦ The approach can be extended very easily by extending the regression

\[ \gamma e_{ext} = R_{ext} w + \varepsilon \]

where

\[ e_{ext} = \begin{bmatrix} e_T \\ 1 \\ 0 \end{bmatrix} \quad R_{ext} = \begin{bmatrix} R \\ r_{\text{forecast}}^{\text{forecast}} \\ V_{\text{Prior}}^{1/2} \end{bmatrix} \quad \text{Var}(\varepsilon) = \sigma^2 \begin{bmatrix} I & 0 & 0 \\ 0 & t_0^{-1} & 0 \\ 0 & 0 & t_0^{-1} \end{bmatrix} \]

♦ The regressors are extended by the forecast returns and the prior covariance matrix. The parameter \( t_0 \) is a measure of confidence in the priors in data units.
Bayesian Extensions 2

♦ The risk matrix is now a weighted average of the prior and the sample risk matrix

\[ V_{ext} \approx \frac{T}{T + t_0} V + \frac{t_0}{T + t_0} t_0 V_{Prior} \]

and the posterior estimates of the returns is

\[ \mu_{ext} = \frac{T}{T + t_0} \mu + \frac{t_0}{T + t_0} \mu_{forecast} \]

♦ Further we could find the best portfolio \( w \) subject to a set of constraints on the holdings, e.g. no-short or industry neutral.
Inclusion of priors shrink the portfolios back

- We assume a single factor model and the implied equilibrium returns as our prior. As we increase $t_0$ so the shrinkage increases and std errors improve.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$t_0 = 520$</th>
<th>$t_0 = 2080$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum Variance = $\chi/\beta = 0.17$</td>
<td>Tangency Portfolio = $(1+\alpha)/\chi = 27.29$</td>
</tr>
<tr>
<td>Portfolio Weights</td>
<td>Std. Err</td>
<td>Std. Err</td>
</tr>
<tr>
<td>US</td>
<td>0.21 (0.03)</td>
<td>0.34 (0.43)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.15 (0.03)</td>
<td>0.41 (0.42)</td>
</tr>
<tr>
<td>UK</td>
<td>0.26 (0.03)</td>
<td>0.24 (0.43)</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.05 (0.03)</td>
<td>0.20 (0.42)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.18 (0.03)</td>
<td>-0.07 (0.33)</td>
</tr>
<tr>
<td>Asia Pacific</td>
<td>0.15 (0.03)</td>
<td>-0.12 (0.39)</td>
</tr>
<tr>
<td>Average Return, $\mu_W$</td>
<td>9.10</td>
<td>11.21</td>
</tr>
<tr>
<td>Volatility, $\sigma_W$</td>
<td>15.84</td>
<td>17.58</td>
</tr>
<tr>
<td>Zero-Beta Return, $r_Z$</td>
<td>$-\infty$</td>
<td>0.00</td>
</tr>
<tr>
<td>Portfolio Sharpe Ratio</td>
<td>N/A</td>
<td>0.64</td>
</tr>
</tbody>
</table>
We can estimate the cost of an investment constraint

- The constrained frontier lies within the efficient frontier.
- The Z-distance is now between a point of this frontier and its nearest neighbour on the efficient frontier.
Testing for the cost of investment constraints

- We look at two constraints. Investing only in EAFE markets and the imposing a no-short constraint.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>EAFE Portfolio</th>
<th>No Short Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = \frac{\chi}{\beta} = \frac{(1+\alpha)}{\chi}$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>$\times 100$</td>
<td>23.32</td>
<td>23.32</td>
</tr>
<tr>
<td>Statistic Z($\gamma$)</td>
<td>0.103</td>
<td>0.066</td>
</tr>
<tr>
<td>No. of Restrictions, $P$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$(T-N-1)/P \times Z(\gamma)$</td>
<td>0.0066</td>
<td>0.0024</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Zero-Beta Return, $r_Z$</td>
<td>$-\infty$</td>
<td>$-\infty$</td>
</tr>
<tr>
<td>N/A</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sharpe of Constrained Portfolio</td>
<td>8.47</td>
<td>-8.30</td>
</tr>
<tr>
<td>N/A</td>
<td>0.66</td>
<td>0.80</td>
</tr>
<tr>
<td>Sharpe Ratio of Efficient Portfolio</td>
<td>N/A</td>
<td>1.34</td>
</tr>
<tr>
<td>N/A</td>
<td>0.39</td>
<td>1.31</td>
</tr>
</tbody>
</table>
Conclusions

♦ We have rephrased our original questions into the regression framework.

♦ Is there sufficient new information to justify a rebalancing of my portfolio?
  – This can be rephrased as is my current portfolio, statistically different from an optimal portfolio designed using all my current information?

♦ Are the tilts or positions in my portfolio significantly different from the consensus or efficient set of portfolios?
  – This could be rephrased as simply whether my portfolio is significantly different from the efficient set.
  – Or more informatively, what is the confidence I need to have in my forecasts to justify my current positions?
References

Distinguishing Between Being Unlucky and Unskillful

Dan diBartolomeo
Summer 2007 Investment Seminar – London
June 2007
What We’re After

• There are numerous performance metrics used as proxies for manager skill such as alpha, and information ratio
  – Most of these rarely have statistically significant values because you need a long time series of data, over which time conditions are presumed but not guaranteed to be stable
  – We would like a measure that uses more information so we can get statistically meaningful results over a shorter window

• Manager’s occasionally experience very bad return outcomes for a period of time
  – We need a means to discriminate the manager being bad from a truly random event
What We Probably Don’t Care About

• There is an enormous literature in finance regarding whether asset managers collectively exhibit skill
  – Obvious implications for concepts of market efficiency
  – Most of this work is based on the concept of “performance persistence”: those that perform consistently well must be skillful

• But we want to evaluate only one manager
Usual Methods

• There is lots of literature on using traditional return performance metrics such as alpha and information ratio as proxies for manager skill:

• You need very long time series of return observations to have enough data to get anything statistically significant by which time conditions may change

• Just going to daily data doesn’t help
More on Skill Detection

• Some research has been done on CUSUM methods

• Tries to isolate what portion of a manager’s history is likely to be relevant to current activities
  – Throw away data from before the most likely date of a regime shift
The Breakdown Problem

• Consider a real manager who maintains a below 3% ex-ante tracking error and has a cumulative return of -6.3% over a one year period
  – Is this a 2.4 standard deviation event? If so the manager was very unlucky
  – Was the risk model wrong? Maybe the risk model was underestimating the risk so it’s not such a rare event
  – Expected tracking error averaged 2.74%, realized was 2.80%

• Ex-ante tracking error estimate is the expectation of the standard deviation of the active return, which is measured around the mean
  – Mean active return was -.54% per month

IR as Skill


- \( IR = IC \times \text{Breadth} \)
  - \( IR = \frac{\text{alpha}}{\text{tracking error}} \)
    - \( IC = \text{correlation of your return forecasts and outcomes} \)
    - \( \text{Breadth} = \text{number of independent “bets” taken per unit time} \)

- If we know how good we are at forecasting and how many bets we act on, we know how good our performance should be for any given risk level
The Fundamental Law Makes Big Assumptions

• There are no constraints at all on portfolio construction
  – Positions can be long or short and of any size
• We measure only “independent” bets
  – Buying 20 different stocks for 20 different reasons is 20 different bets
  – Buying 20 stocks because they all have a low PE is one bet, not 20!
• Transaction costs are zero, so bets in one time period are independent of bets in other periods
  – This is the property that casinos depend on. Once we have the odds in our favor, we want to make lots of bets
• Research resources are limitless so our forecasting effectiveness (IC) is constant as we increase the number of eligible assets
Enter the Transfer Coefficient


- \( IR = IC \times TC \times Breadth^{.5} \)
  - \( IR = \text{alpha} / \text{tracking error} \)
    - \( IC = \text{correlation of your return forecasts and outcomes} \)
    - \( TC = \text{the efficiency of your portfolio construction (TC < 1)} \)
    - \( Breadth = \text{number of independent “bets” taken per unit time} \)
What Drives the Transfer Coefficient?

• Imagine a manager with a diverse team of analysts that are great at forecasting monthly stock returns on a large universe of stocks, but whose portfolio is allowed to have only 1% per year turnover
  – Good monthly forecasts, diverse reasons and a large universe imply high IC and high breadth
  – But if we can never act on the forecasts because of the turnover constraint TC can be zero or even negative

• If we can’t short a stock that we correctly believe is going down, or take a big position in a stock that we correctly believe is going up, TC declines
  – The more binding constraints we have on our portfolio construction, the more return we fail to capture when our forecasts are good
  – For bad forecasters, a low TC is good. You hurt yourself less when you constrain your level of activity
Limitations of IR

• Managers often talk about IR, but it really doesn’t correspond to investor utility except in extreme cases
  – Consider a manager with an alpha of 1 basis point and a tracking error of zero
  – IR is infinite but value added for the investor is very, very small

• The statistical significance of a ratio is hard to calculate
Our Solution is to Incorporate Cross-Sectional Information

• Successful active management involves forecasting what returns different assets will earn in the future, and forming portfolios that will efficiently use the valid information contained in the forecast
  – We usually have a large universe of assets to work with, so we get statistical significance quickly
  – In mathematical terms, this means that the position sizes within our portfolios balance the marginal returns, risks and costs
  – If we know how good we are at forecasting future asset returns, we can forecast how well our portfolios should perform if they are efficiently constructed. If we do less well than we should, our portfolio construction is at fault
A Quant Way to Think About It

• Every portfolio manager must believe that the portfolio they hold is optimal for their investors
  – If they didn’t they would hold a different portfolio
• If we describe investor goals as maximizing risk adjusted returns, we know that the marginal risks associated with every active position must be exactly offset by the expected active returns
  – Guaranteed by the Kuhn Tucker conditions for finding the maximum of a polynomial function
  – For every portfolio, there exists a set of alpha (active return) expectations that would make the portfolio optimal. We call these the implied alphas
The Effective Information Coefficient

- We define the EIC as the skill measure.
- EIC is the pooled average rank correlation of the implied alphas and the realized returns at the security level.
  - If our forecasting skill is good (high IC) and our portfolio construction skill is good (high TC) then EIC will be high.
  - If either IC or TC is low, EIC will be low.
- As this measurement involves every active position during each time period, the sample is large and statistical significance is obtained quickly.
Using EIC to Dissect Performance

- If we have EIC values for a given period (e.g. month), we can estimate the expected magnitude of alpha.
- The expected alpha is just the EIC times the cross-sectional dispersion of the asset returns.
- So we can look at returns as:

\[ P_t - B_t = EIC_t \times D_t + \varepsilon_t \]

- \( P_t \) = portfolio return during period t
- \( B_t \) = benchmark return during period t
- \( EIC_t \) = skill for period t
- \( D_t \) = cross-sectional dispersion of asset returns
- \( \varepsilon_t \) = residual returns due to luck

- You can now look at the time series standard deviation of the \( \varepsilon_t \) to see if the risk model is predicting risk accurately.
An Alternative View

• Active managers can add value in two ways:
  – Being right more often than they are wrong about which securities will outperform the market. Sort of like a batting average in baseball
  – Getting bigger magnitude returns on gainers than on losers. You can have a batting average below 50% and still make money if you hit a decent number of “home runs”

• Peter Lynch used to refer to “ten baggers”
  – Stocks that go up ten fold in value while you hold them
  – Just a couple can have a huge effect on portfolio returns

• Batting average concept first formalized in:
Attribution to Batting Average and Active Return Skew

• A formalization was proposed by hedge fund manager Andrei Pokrovsky (formerly Northfield staff) in 2006

• We can easily measure batting average
  – It is the percentage of cases in which active returns and active weights are of the same sign
  – High numbers are good

• Take the vector product of active weights and active returns. Measure the skew statistic of the distribution
  – Positive skew in active returns is a measure of portfolio construction efficiency
Style Dependency of Batting Average and Active Skew

- Value managers will tend to have high batting average and low skew
- Growth/momentum managers will tend to have lower batting average but higher skew
- Trend following behavior creates the skew
Conclusions

• Its often difficult to assess whether a period of extraordinary performance (good or bad) is the result of luck or skill
• We propose the Effective Information Coefficient as a measure of skill
  – It is estimated both over time and across assets so sample sizes get large quickly
  – It incorporates both key aspects of investment skill, forecasting returns and forming efficient portfolios
• We present an alternative representation of skill as “batting average” and “payoff skew”
A Market Impact Model that Works

Dan diBartolomeo
Summer 2007 Investment Seminar – London
June 2007
Main Points for Today

- Of the inputs needed to optimally rebalance a portfolio, market impact of large trades is the least researched.
- Rational boundary conditions need to be incorporated into the empirical estimation of market impact models.
- We have developed a market impact model that has excellent in-sample explanatory power in several major equity markets, over a database of more than 1.5 million trades.
Motivation

- Northfield client comment in 1997:
  - “Your optimizer just told me to buy five million shares of Ford. From who?”
- Clearly, we want to incorporate market impact into portfolio optimization processes
- We would also like to incorporate good market impact estimates into “trade scheduling” that balances reducing market impact against opportunity costs and risk
Total Trading Costs

- Most people see trading costs as having several components
  - Agency costs
  - Bid/Asked Spread
  - Market Impact (my trade moves the price)
  - Other people’s trades move the price, maybe in my favor. Traders call this “trend cost”. We call it risk

- Often overlooked ingredients
  - My large concurrent trades (my trade in Ford impacts the price of GM)
  - The implicit opportunity cost of waiting. Unless we’re passive, we want to buy stocks before they go up, not after. If we’re selling we want to sell before they go down, not after
Let’s See What We Can Reasonably Estimate

- Agency Costs are essentially known in advance
- Bid/Asked Spreads: Some time variation but reasonably stable
- Market Impact: Lots of models exist. Underlying factors are highly significant, but explanatory power is typically quite low
- Trend Costs: They can move the price for or against us. Ex-post is often the largest part of the costs. Pretty darn random. Or so it seems.
- Market impact and trend costs are hard to disentangle so maybe the market impact models work better than we think
Lots of market impact models look like this. Market impact increases with trade size either linearly or at a decreasing rate.

\[ M = A_i + (B_i \times X) \]

OR

\[ M = A_i + (C_i \times |X^{0.5}|) \]

M is the expected cost to trade one share
X is the number of shares to be traded
A_i is the fixed costs per share
B_i, C_i are coefficients expressing the liquidity of the stock as a function of fundamental data.
The Need for Boundary Conditions

- Our optimizer allows for a market impact formula that combines the linear and square root processes
  \[ M = A_i + [(B_i \times X_t) + (C_i \times |X_t^{0.5}|)] + \ldots \]

- When clients started to put in their own values for B and C, we often saw bizarre results such as forecast selling costs over 100%!
  - This arose because their coefficients were based on empirical estimations from data sets that did not contain very large trades that traders never try to do because they would be too costly

- Coefficients for B and C must provide rational results in the entire range of potential trade size from zero to all the shares of a firm
Let’s consider a hostile takeover as the “worst case” scenario for market impact

- We’re going to buy up all the shares of a company and tell the entire world we’re doing it. The takeover premium can be viewed as an extreme case of market impact.

If we believe only in the linear market impact process, we can set our coefficient to the expected takeover premium for a stock divided by shares outstanding.

If we believe only in the square root process for market impact, we can set our coefficient to the expected takeover premium for a stock divided by the square root of shares outstanding.
More Impact for Dummies

- If we don’t know which process to believe in, we can just do both with a weighting summing to one.

\[
B_i = W \times \left( \frac{E[P_i]}{S_i} \right) \\
C_i = (1-W) \times \frac{E[P_i]}{(S_i^{0.5})}
\]

\begin{itemize}
  \item $B_i$ = the coefficient on the linear process
  \item $C_i$ = the coefficient on the square root process
  \item $P_i$ = the takeover premium in percent
  \item $S_i$ = the number of shares outstanding
  \item $W$ = a weight
\end{itemize}
Accounting for Different Liquidity

- If we assume takeover premiums are lognormal, we can easily express the expected takeover premium as a function of a liquidity measure

\[ \mathbb{E}[P_i] = QP / ((1 + K/100)^{Z_i}) \]

- \( P \) = % average price premium in a hostile takeover
- \( K \) = the log percentage standard error around \( P \)
- \( Z_i \) = the \( Z \) score of a liquidity measure for stock \( i \)
- \( Q \) = a scalar between zero and one (the takeover scenario is “worst case” for information leakage, so a smaller value could fit better)
An Empirical Example

- Various academic studies using M & A databases have reported average takeover premiums from 37% to 50% with a standard deviation around 30%
- Let's take a hypothetical company with $5 Billion market cap, $50 share price and 100 million shares outstanding
  - Assume $P = 37$, $K = 40$, $W = .25$, $Z_i = 0$
- 1,000,000 share trade impact = 2.88%
- 10,000 share trade impact = 0.27%
The Empirical Data

- Our dataset was provided through our strategic relationship with Instinet, with whom we have created a trade scheduling algorithm
  - Over 1.5 million orders over an 18 month period with fine detail such as time stamps, arrival price, execution price, order type (buy/sell, limit/market), tracking of cancelled orders, etc.
  - Totally anonymous. We have no information on what firms traded or which orders belong to whom
  - Most of the data is from the US, with good representation of major markets such as Japan, UK, Canada, etc.

- Very large dataset on which to estimate a model with only two free parameters, Q and W

- Security level liquidity measures are derived from various aspects of the Northfield risk model for a given market, such as typical trading volumes, stock volatility, etc.
Measuring Market Impact?

- One way to measure market impact would be to compare the price we got on our trade versus the price on the previous trade as a measure of how much our trade “moved the price”
  - This may not be relevant to us. We care about how much the price moved from the price it was at when we decided to trade the stock
- We use an “arrival price” measure. It’s the percentage in price between the execution price and the price that existed in the market when we got the order to transact the stock
  - Implementation Shortfall described in Perold (1988)
  - Very noisy relationships since a limit order can sit for hours between arrival and execution. Prices can move around a lot during that time from other people’s trades, not ours
Criteria To Judge A Model

- **Unbiasedness**
  - On average the forecasts of market impact should match observed costs

- **Low Error**
  - The absolute difference between the forecasts and the observations as a percentage of the forecast

- **High Explanatory Power**
  - The model should accurately predict when the market impact a trade will be high or low

- **All three of the above criteria are calculated on a trade dollar weighted basis**
  - It’s a lot more important to get things right on a million share trade than a hundred share trade
An Estimation Subtlety

- Using “arrival price cost” as the measure, it’s possible that the realized market impact of a trade could be negative or zero
  - We’re buying a stock and the price went down before our order was executed

- However, the forecast market impact is always positive, so we calculate percentage of error with the forecast in the denominator

- This means that a low percentage of error and a high R-squared are not exactly congruent measures
  - There is more room to be wrong on one side than the other
Example Results for A Given Q,W

<table>
<thead>
<tr>
<th></th>
<th>Average Impact Cost BP</th>
<th>Avg % Absolute (Error)</th>
<th>% R-squared</th>
<th>Time Length of Order (HR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>13.9</td>
<td>205</td>
<td>74.2</td>
<td>1:32</td>
</tr>
<tr>
<td>Swiss</td>
<td>24.8</td>
<td>158</td>
<td>64.2</td>
<td>6:32</td>
</tr>
<tr>
<td>Canada</td>
<td>28.5</td>
<td>217</td>
<td>33.8</td>
<td>2:39</td>
</tr>
</tbody>
</table>
Empirical Results Discussion

- We are able to isolate values of Q,W that are economically reasonable and fit the data extremely well in several countries (US, Canada, Switzerland) based on our current choice of liquidity measures.
- The same liquidity measures fit poorly in other countries such as the UK and Japan. More research is needed.
- We will continue this line of research until we cover all stocks in our Global Equity Risk Model.
In the Portfolio Optimization Context

- Most people who use an optimizer for rebalancing portfolios use some form of transaction cost estimate.
- Many people use some form of non-linear transaction cost function that includes a market impact component.
- If interdependencies between market impacts are not accounted for, large trades will be incorrectly specified.
  - Consider two sets of orders:
    - Buy 5 Million shares of Ford, Buy 5 Million shares of GM
    - Buy 5 Million shares of Ford, Sell 5 Million shares of GM
    - Are expected market impacts the same?
So Where Are We Now?

- Our optimizer now has a flexible functional form built into the objective function that takes cross-market impact and liquidity limits into account. It looks like:

\[ M = A_i + B_i X_t + C_i |X_t|^{0.5} + D_i (\max[X_t - L_t, 0])^2 + Y_i \]

where:

\[ Y_i = \sum_{j=1}^{m} (B_i X_j + C_i X_j^{0.5} + D_i \max[X_j - L_j, 0]^2) * P_{ij} * Q_{ij} * \left(\frac{2}{(m-1)}\right) \text{ \forall } i \neq j \]

\[ P_{ij} = \text{correlation between } i \text{ and } j \text{ from our risk model} \]

\[ Q_{ij} = \text{sign}[\Delta \text{shares}_i * \Delta \text{shares}_j] \]
Amortization Functions

- In a portfolio optimization process, total estimated costs must be traded off against expected alpha and risk
  - Amortize trading costs over the expected holding period
  - Adjust the amortization rate, $\Gamma$, to reflect “the probability of realization” which is less than one for finite holding periods
- For small transaction costs, arithmetic amortization is sufficient, but if costs are large we need to consider compounding
- Assume a trade with 20% trading cost and an expected holding period of one year.
  - We can get an expected alpha improvement of 20%. But if we give up 20% of our money now, and invest at 20% for one period, we only end up with 96% of the money we have now.
Optimal Trade Scheduling

- If we know the urgency of trades, and the likely impact, we can create optimal trade “schedules” to break up large trades into a series of smaller trades
  - We still need an assumption about the extent that market impact in one period is a permanent move in the price and how much is transient.

- Once we have that, the problem becomes a dynamic optimization. Normally solved using Bellman equation methods
  - Our formulation uses a nearly traditional optimization with time made endogenous. Think of many stocks all called IBM that each can only be traded in one period: IBM (to trade Monday), IBM (to trade Tuesday), etc.

- Our trade scheduling algorithm with Instinet went live a few months ago
Setting the Objective for Scheduling

- Consider a set of undone orders as a long short-portfolio that you are liquidating
  - You are long shares you do have and don’t want
  - You are short shares you do want and don’t have
- The normal mean-variance objective function

\[ U = \alpha - \sigma^2/T - (C\Gamma) \]

- Works just fine except the sign on alpha is reversed from the norm
  - You are currently short stocks that you do want. The reason you want them is that they have positive alpha
  - We can’t get all our trades done in one shot, so we need a multi-period representation
Permanent and Temporary Market Impact

- If our trades in any given stock are far apart in time, price movements caused by our trades will be independent of one another.

- If our trades follow each other with little time in between, market impact effects will have a cascading effect as each trade moves the price from where the previous trade left it.

- We call this persistent portion of market impact “stickiness”, and account for it in the solution when the length of our discrete time blocks is short.
Why Mean Variance?

- Some trading algorithms try minimizing the standard deviation of trading costs
  - From a process control perspective minimizing the uncertainty (standard deviation) rather than variance seems intuitive

- However, the variance in trading costs impact ending portfolio values is half the variance, and is not linearly related to the standard deviation
  - Consider trading a 100 Yen portfolio with a trading cost of 10% twice. Ending wealth is 81 Yen
  - Consider two trades with an average of 10% cost, 0% cost and 20% cost. Ending wealth is 80 Yen
  - The decimal variance of the second case is 0.01 so the expected loss is 0.005 per observation. This checks since 0.005 * 100 * 2 = 1
The Trade Schedule

- Lets assume we want to finish all our open trades over a two trading day period
- We can break the two days into discrete time blocks, either by clock time or by “expected share of day’s volume” (e.g. each block is the length of clock time that usually trades 5% of the day’s volume)
- Think of a spreadsheet where each order is a row and each time block is a column.
  - We want the matrix of orders such that all orders are completed by the end of the schedule
  - That maximizes our objectives: capturing short term alpha, minimizing risk and market impact
- After each period is experienced, we can check that the expected orders were executed; if not, we can re-run the schedule based on the remaining shares and time periods
Conclusions

- Our search for a market impact model that both fits the data well (at least in some markets) and is rationality bounded has been fruitful.
- We can incorporate reasonable estimates of market impact for any size trade into both optimization and trade scheduling processes.
- We expect to incorporate this market impact model into the Instinet trade scheduling system during 2007.
References


References