Comprehensive Risk and Performance Attribution

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Highlights

• A theory of attribution
• Returns-based risk decomposition
• Portfolio-based performance attribution
1. Theory
A new approach to attribution (Feldman, 2007)

• Start with a model of utility maximizing behavior
• Consider a random order model of the relative importance of the factors contributing to utility
• Look for a distribution over orders to describe the probability of being ordered by relative importance
• Assume
  – Random order consistency
  – Functional separability
  – Exclusion
• Proportional marginal attribution (PMA) is
  – The unique resulting expectation of factor contributions
  – A powerful and theoretically consistent attribution method
2. Returns-based risk attribution
Returns-based risk attribution / decomposition

• Let the utility function of an analyst be the $R^2$ of a factor model and use PMA
  – This PMVD
• Let the utility function of an analyst be based on the $|R|$ of a quantile regression and use PMA
  – This is PMQD
• *Linear covariance decomposition* or LCD is the standard method of variance decomposition
• The key advantage of PMA over LCD and statistical significance measures is that PMA is designed to take factor correlation into account
3. Variance decomposition: The two-factor case
Example: Portfolios of SPDRs and VIX futures

- PMVD SPDR attribution in blue, LCD SPDR attribution in red
- Horizontal axis shows SPDR portfolio percentage
- LCD gives negative attributions and those over 100%
- Correlation between SPDRs and VIX: -0.64

**LCD is not share monotonic**
Standard variance decomposition (LCD) in the general two-factor case

- Attributions diverge to plus and minus infinity as factor correlation approach -1.0
PMVD in the general two-factor case

- PMVD attributions are independent of correlation in the two-factor case
4. Negative factor relationships
Portable alpha example: A portfolio long the RAFI 1000 and short the Russell 1000

PMVD exposures consistent with the factor model

RAFI 1000 performance data courtesy of Research Affiliates.
Portable alpha example: PMVD and LCD compared

- Counterintuitive exposures
- Correlation induced bias
- Colinearity inflation

RAFI 1000 performance data courtesy of Research Affiliates.
5. PMVD put to a statistical test
Laudus Rosengberg Long/Short Fund
Is the LCG PMVD attribution reasonable?

Basic factor model

<table>
<thead>
<tr>
<th>Factor</th>
<th>Beta</th>
<th>T-Stat</th>
<th>p-value</th>
<th>PMVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCG</td>
<td>-18%</td>
<td>-1.23</td>
<td>0.221</td>
<td>-9.2%</td>
</tr>
<tr>
<td>LCV</td>
<td>-20%</td>
<td>-0.94</td>
<td>0.351</td>
<td>-6.4%</td>
</tr>
<tr>
<td>MCG</td>
<td>36%</td>
<td>2.50</td>
<td>0.015</td>
<td>5.8%</td>
</tr>
<tr>
<td>MCV</td>
<td>-20%</td>
<td>-0.88</td>
<td>0.382</td>
<td>-0.9%</td>
</tr>
<tr>
<td>SCG</td>
<td>-52%</td>
<td>-3.76</td>
<td>0.000</td>
<td>-26.7%</td>
</tr>
<tr>
<td>SCV</td>
<td>36%</td>
<td>1.78</td>
<td>0.079</td>
<td>3.6%</td>
</tr>
<tr>
<td>INTL</td>
<td>9%</td>
<td>0.67</td>
<td>0.503</td>
<td>0.7%</td>
</tr>
<tr>
<td>EMERG</td>
<td>2%</td>
<td>0.20</td>
<td>0.839</td>
<td>0.1%</td>
</tr>
<tr>
<td>HighYld</td>
<td>-8%</td>
<td>-0.47</td>
<td>0.643</td>
<td>-0.1%</td>
</tr>
<tr>
<td>LT</td>
<td>16%</td>
<td>1.15</td>
<td>0.255</td>
<td>1.1%</td>
</tr>
<tr>
<td>TBILL</td>
<td>-252%</td>
<td>-1.18</td>
<td>0.242</td>
<td>-1.0%</td>
</tr>
</tbody>
</table>

$R^2$: 55.52%

Combined LC factor

<table>
<thead>
<tr>
<th>Factor</th>
<th>Beta</th>
<th>T-Stat</th>
<th>p-value</th>
<th>PMVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>-35%</td>
<td>-2.11</td>
<td>0.038</td>
<td>-16.6%</td>
</tr>
<tr>
<td>MCG</td>
<td>36%</td>
<td>2.59</td>
<td>0.011</td>
<td>6.2%</td>
</tr>
<tr>
<td>MCV</td>
<td>-22%</td>
<td>-1.12</td>
<td>0.265</td>
<td>-1.2%</td>
</tr>
<tr>
<td>SCG</td>
<td>-52%</td>
<td>-3.83</td>
<td>0.000</td>
<td>-25.1%</td>
</tr>
<tr>
<td>SCV</td>
<td>37%</td>
<td>1.87</td>
<td>0.066</td>
<td>2.8%</td>
</tr>
<tr>
<td>INTL</td>
<td>9%</td>
<td>0.64</td>
<td>0.522</td>
<td>0.8%</td>
</tr>
<tr>
<td>EMERG</td>
<td>2%</td>
<td>0.22</td>
<td>0.829</td>
<td>0.1%</td>
</tr>
<tr>
<td>HighYld</td>
<td>-9%</td>
<td>-0.52</td>
<td>0.608</td>
<td>-0.2%</td>
</tr>
<tr>
<td>LT</td>
<td>16%</td>
<td>1.24</td>
<td>0.219</td>
<td>1.2%</td>
</tr>
<tr>
<td>TBILL</td>
<td>-254%</td>
<td>-1.21</td>
<td>0.232</td>
<td>-1.0%</td>
</tr>
</tbody>
</table>

$R^2$: 55.19%

- F-test constraint p-value = .47
- Correlations degrade LCG stat. sig. levels
6. Extreme positive colinearity
Example from Feldman (2006): LB US Long Term Credit

Data supplied by Ibbotson Associates.

Data supplied by Ibbotson Associates.
PMVD: December 1992 to August 2007

Data courtesy of Morningstar.
PMVD analysis based on daily data

Data courtesy of Morningstar.
LCD with daily data

Data courtesy of Morningstar.
Long-term credit example conclusions

- PMVD appears to capture information in credit markets, sometimes before it is reflected in spreads
- Many historical events clearly identified
- Other factors also reflected in PMVD results
- Noise likely also present (Gromping, 2006)
- LCD appears to capture little information
7. Proportional marginal quantile risk attribution
Quantile regression

- Koenker and Bassett (1978) and Bassett and Chen (2001)
- Exact analogy to least squares
- Estimate conditional quantile (e.g. median) instead of conditional mean
- Minimize the sum of absolute quantile deviations
- Attribution approach is exactly analogous to OLS
- Operational difference with OLS: choice of quantile
PMQD example: Analysis of CTA performance

- Based on Premia Capital Management daily P/L for 2005

<table>
<thead>
<tr>
<th>Sector</th>
<th>PMVD</th>
<th>Quantile 1%</th>
<th>50%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>20%</td>
<td>66%</td>
<td>16%</td>
<td>10%</td>
</tr>
<tr>
<td>Energy</td>
<td>13%</td>
<td>31%</td>
<td>10%</td>
<td>58%</td>
</tr>
<tr>
<td>Metals</td>
<td>19%</td>
<td>1%</td>
<td>26%</td>
<td>7%</td>
</tr>
<tr>
<td>Livestock</td>
<td>37%</td>
<td>2%</td>
<td>34%</td>
<td>5%</td>
</tr>
<tr>
<td>Precious</td>
<td>1%</td>
<td>0%</td>
<td>8%</td>
<td>19%</td>
</tr>
<tr>
<td>UST5TR</td>
<td>10%</td>
<td>1%</td>
<td>7%</td>
<td>0%</td>
</tr>
<tr>
<td>R1 or R2</td>
<td>1.9%</td>
<td>24.5%</td>
<td>1.5%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

- PMVD and median quantile PMQD components similar
- Agriculture and energy dominate 1% quantile downside risk
- Energy dominates 99% quantile upside potential
- Metals and livestock show median quantile risk but little tail risk

P/L data courtesy of Premia Capital Management.
8. Portfolio performance attribution
Attribution of portfolio performance

- Use a utility function to quantify the benefit of historical or forecast investment performance of an optimized portfolio of assets
  - Determine the utility share of each asset
  - Divide the utility share of an asset by its portfolio weight

- This is proportional marginal performance attribution (PMPA)
## Standard portfolio optimization problem

<table>
<thead>
<tr>
<th>MANAGER</th>
<th>Historical Return</th>
<th>Return Adjustment</th>
<th>Adjusted Return</th>
<th>Std. Dev.</th>
<th>Annualized Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFA U.S. Large Company Institutional</td>
<td>0.90%</td>
<td>0.40%</td>
<td>1.30%</td>
<td>3.34%</td>
<td>1.19</td>
</tr>
<tr>
<td>Robeco WPG 130/30 Large Cap Core Intl</td>
<td>0.75%</td>
<td>0.60%</td>
<td>1.35%</td>
<td>3.93%</td>
<td>1.06</td>
</tr>
<tr>
<td>Bridgeway Ultra-Small Company</td>
<td>1.96%</td>
<td>-0.20%</td>
<td>1.76%</td>
<td>5.44%</td>
<td>1.02</td>
</tr>
<tr>
<td>Profunds Real Estate UltraSector Svc</td>
<td>1.67%</td>
<td>-0.30%</td>
<td>1.37%</td>
<td>6.78%</td>
<td>0.63</td>
</tr>
<tr>
<td>AllianceBernstein Intl Growth C</td>
<td>1.70%</td>
<td>-0.10%</td>
<td>1.60%</td>
<td>3.93%</td>
<td>1.28</td>
</tr>
<tr>
<td>Oppenheimer Emerging Growth A</td>
<td>1.28%</td>
<td>0.30%</td>
<td>1.58%</td>
<td>6.23%</td>
<td>0.80</td>
</tr>
<tr>
<td>Matthews China</td>
<td>2.15%</td>
<td>-0.50%</td>
<td>1.65%</td>
<td>5.02%</td>
<td>1.04</td>
</tr>
<tr>
<td>Putnam High Yield M</td>
<td>0.87%</td>
<td>-0.10%</td>
<td>0.77%</td>
<td>1.43%</td>
<td>1.51</td>
</tr>
<tr>
<td>Vanguard Long-Term Bond Index</td>
<td>0.55%</td>
<td>0.25%</td>
<td>0.80%</td>
<td>2.57%</td>
<td>0.88</td>
</tr>
<tr>
<td>Mellon Short Term U.S. Govt Secs M</td>
<td>0.20%</td>
<td>0.20%</td>
<td>0.40%</td>
<td>0.39%</td>
<td>2.24</td>
</tr>
</tbody>
</table>

- 10 Managers
- Historical returns, standard deviations and correlations based on 60 months of date to June 2007
- Return adjustments constitute one relatively optimistic assessment of the coming year
- MVO: Quadratic utility with \( \lambda = 10 \)

Data courtesy of MPI
Performance attribution example:
Assets and performance characteristics

<table>
<thead>
<tr>
<th>MANAGER</th>
<th>MVO Portfolio Allocation</th>
<th>PMA</th>
<th>PMPA</th>
<th>Annualized Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFA U.S. Large Company Institutional</td>
<td>5.2%</td>
<td>6.3%</td>
<td>1.20</td>
<td>1.19</td>
</tr>
<tr>
<td>Robeco WPG 130/30 Large Cap Core Intl</td>
<td>11.8%</td>
<td>13.1%</td>
<td>1.11</td>
<td>1.06</td>
</tr>
<tr>
<td>Bridgeway Ultra-Small Company</td>
<td>8.5%</td>
<td>7.2%</td>
<td>0.85</td>
<td>1.02</td>
</tr>
<tr>
<td>ProFunds Real Estate UltraSector Svc</td>
<td>1.9%</td>
<td>0.3%</td>
<td>0.17</td>
<td>0.63</td>
</tr>
<tr>
<td>AllianceBernstein Intl Growth C</td>
<td>15.7%</td>
<td>16.3%</td>
<td>1.03</td>
<td>1.28</td>
</tr>
<tr>
<td>Oppenheimer Emerging Growth A</td>
<td>0.0%</td>
<td>0.0%</td>
<td>--</td>
<td>0.80</td>
</tr>
<tr>
<td>Matthews China</td>
<td>21.7%</td>
<td>25.0%</td>
<td>1.15</td>
<td>1.04</td>
</tr>
<tr>
<td>Putnam High Yield M</td>
<td>0.0%</td>
<td>0.0%</td>
<td>--</td>
<td>1.51</td>
</tr>
<tr>
<td>Vanguard Long-Term Bond Index</td>
<td>32.6%</td>
<td>31.3%</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td>Mellon Short Term U.S. Govt Secs M</td>
<td>2.7%</td>
<td>0.6%</td>
<td>0.21</td>
<td>2.24</td>
</tr>
</tbody>
</table>

- PMPA: Proportional marginal portfolio attribution
- PMPA takes manager correlations to the portfolio into account
- PMPA results surprisingly similar to to Sharpe ratio for some managers
9. Limitations
PMA Limitations

• Computing time
  – Increases sharply with number of factors
  – Solution: approximation methods

• Attribution precision
  – Examined in Feldman (2005) and Gromping (2007)
  – PMA is estimated consistently
  – Bootstrapped confidence intervals
10. Your turn
11. Wrap up
Get more information from your data

- PMA methods have a strong theoretical foundation
- PMA is share monotonic
- PMA methods work
- LCD and statistical significance measures have serious limitations as risk attribution methods
  - LCD and statistical significance measures are vulnerable to correlation-driven risk attribution distortions
  - LCD risk exposures can be completely suppressed
References