The Origin of Value

Ed Fishwick & Steve Satchell*

“If, among a nation of hunters, it usually costs twice the labour to kill a beaver which it does to kill a deer, one beaver should naturally exchange for - or be worth - two deer”

Adam Smith (The Wealth of Nations)

This presentation was prepared for Northfield’s 21st Annual Research Conference, Squaw Creek, Olympic Valley, California, March 2008. (*Steve Satchell is co-author of the paper on which this presentation is based. He did not participate in the preparation of this specific presentation and so is not responsible for its specific contents.)
“One of the great metaphysical ideas in economics is expressed by the word “value”. It does not mean market prices. Indeed, it is something which will explain how prices come to be what they are. What is it? Where shall we find it? Like all metaphysical concepts, when you try to pin it down it turns out to be just a word.

All the same, problems that have been turned upside down in pursuit of the causes of value are by no means empty of meaning...”

One period return is defined by the identity:

\[ R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \]  \hspace{1cm} (1.1)

(Where \( R, P, \) & \( D \) denote return, price, and \( t \) is an index of time)

Let \( E \) denote an expectation, and assume that expected returns are constant:

\[ E[R_{t+1}] = R \]  \hspace{1cm} (1.2)

Then, by taking expectations of the identity 1.1, assuming that long run growth is less than \( R \) (thus ruling out “rational bubbles”), repeated forward substitution and the Law of Iterated Expectations gives:

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1 + R)^n} \right]_t \]  \hspace{1cm} (1.3)
The DDM (\& variants) is ubiquitous. Because:

\[
P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n} \right]
\]  \hspace{1cm} (1.3)

We can go on to model the D, and thus estimate the price. Many variants exist, but the following is a common form:

\[
P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1+R)^n} \right] + E \left[ \sum_{n=j+1}^{j+K} \frac{D_{t+ft}}{(1+R)^n} \right] + E \left[ \left( \frac{D_{j+k}(1+g)}{R-g} \right) \left( \frac{1}{(1+R)^{j+k+1}} \right) \right]
\]  \hspace{1cm} (1.4)

1.4 is the well known 3 stage DDM

What makes this of interest of course is that the estimated price (the “fair value”, rarely equals the market price - making this (generically) a key tool (and concept) in active management
“Valuation” - DDMs etc

Because “value” and “price” can differ we need to write:

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1 + R)^n} \right]_t + e_t \]  

(1.5)

With respect to the R, this is typically assumed to be a linear function of a bunch of risk factors so:

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{\left(1 + \sum_{f=1}^{x} \beta_t R_{f,t} \right)^n} \right]_t + e_t \]  

(1.6)

This suggests a natural (and common) regression based approach:
“Valuation” - DDMs etc

For a universe of stocks $i = 1$ to $m$, solve for $R(i)$ - the Implied Rate of Return of $i$:

$$P_{i,t} = E \left[ \sum_{n=1}^{\infty} \frac{D_{i,t+n}}{(1 + R_{i,t})^n} \right] + e_{i,t}$$  \hspace{1cm} (1.7)

Since the $R(i,t)$ are a linear function of the betas, we can fit:

$$R_{i,t} = \sum_{f=1}^{x} \beta_{i,t} R_{f,t} + \rho_{i,t}$$  \hspace{1cm} (1.8)

Where the Implied of Return (IRR) is separated on the right into the Required Rate of Return (RRR) plus the Abnormal Rate of Return (ARR)

In practice the estimation of the regression maybe be simple or complex - but the separation of the Required from the Abnormal is the critical operation here
“Valuation” - DDMs etc

So:

\[ IRR_{i,t} = RRR_{i,t} + ARR_{i,t} \]  \hspace{1cm} (1.9)

Finally we can solve for “Fair Value”:

\[ FV_{i,t} = E \left[ \sum_{n=1}^{\infty} \frac{D_{i,t+n}}{(1 + RRR_{i,t})^n} \right] \]  \hspace{1cm} (1.10)

We might then compare FV to the market Price to derive some metric of mispricing, say:

\[ \%M_{i,t} = \frac{FV_{i,t}}{P_{i,t}} - 1 \]  \hspace{1cm} (1.11)
The key operation is here the separation of price and fair value.

This basic structure - is not limited to the DDM.

A common (and modern) approach is to substitute equity free cash flow for dividends.

But there is an endless variety of specifications of the type of model, with the key features as described above.

For instance we could write this as an EVA type model:
So if we define fair price as the sum of book value and EVA, we could write:

\[
P_{i,t} = B_{i,t} + E\left[\sum_{n=1}^{\infty} B_{i,t+n} \frac{r_{i,t+n} - k_{i,t+n}}{(1 + k_{i,t})^n}\right] + e_{i,t}
\]  

(1.12)

\[
P_{i,t} = B_{i,t} + EVA_{i,t} + e_{i,t}
\]  

(1.13)

Where B is book value, r is the return on capital, and k is the cost of capital.

We can operationalize this as (say) a conventional 3 stage model.

The e, as before, is mispricing. So:

\[
\%M_{i,t} = \frac{B_{i,t} + EVA_{i,t}}{P_{i,t}} - 1
\]  

(1.14)
Suppose there’s only 1 priced factor - say a CAPM type market - so:

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1 + rfr + \beta_{i,t} r_p m,t)^n} \right] + e_t \]

(1.15)

Where rfr is a risk free rate, rp is the factor risk premium. Also:

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1 + IRR_{i,t})^n} \right] \]

(1.16)

So we can solve for the IRR and fit:

\[ IRR_{i,t} = rfr + \beta_{i,t} r_p t + e_{i,t} \]

(1.17)

Which is the single factor case of 1.8. The advantage of which is it’s easy to draw:
IRR, Beta, Security Market Line

“Undervalued”
IRR, Beta, Security Market Line

"Undervalued"

"Overvalued"
Scientific methodology....
Scientific methodology...

Arbitrage Pricing Theory is not a Theory
Scientific methodology....

Arbitrage Pricing Theory is not a Theory

Discuss
Scientific methodology...

Theory: If it is raining the street will be wet
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is wet
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is wet

Conclusion: The theory *might* be correct
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is wet

Conclusion: The theory *might* be correct

But: We don’t know much more than before
Scientific methodology...

Theory: If it is raining the street will be wet
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is *not* wet
Scientific methodology...

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is not wet

Conclusion: The theory is disproved
Scientific methodology - Disproof as the cornerstone of science

Theory: If it is raining the street will be wet

Experiment: It is raining

Outcome: The street is not wet

Conclusion: The theory is disproved

And: We know much more than before
Scientific methodology - A well known confusion...

Theory: If the Earth orbits the sun Venus will exhibit phases
Scientific methodology - A well known confusion...

Theory: If the Earth orbits the sun Venus will exhibit phases

Experiment: Galileo looks through telescope
Scientific methodology - A well known confusion...

Theory: If the Earth orbits the sun Venus will exhibit phases

Experiment: Galileo looks through telescope

Outcome: Venus exhibits phases
Scientific methodology - A well known confusion...

Theory: If the Earth orbits the sun Venus will exhibit phases

Experiment: Galileo looks through telescope

Outcome: Venus exhibits phases

Galileo’s Conclusion: The Earth orbits the sun
Scientific methodology - A well known confusion...

Theory: If the Earth orbits the sun Venus will exhibit phases

Experiment: Galileo looks through telescope

Outcome: Venus exhibits phases

Galileo’s Conclusion: The Earth orbits the sun

But: Though the conclusion was ultimately correct, the science is bad
Scientific methodology - A different history / reality

Theory: If the Earth orbits the sun Venus will exhibit phases

Experiment: Look through telescope

Outcome: Venus does not exhibit phases

Conclusion: The Earth does not orbit the sun - the theory is disproved

So: This isn’t what happened - but the methodology is correct!
Scientific methodology....

Arbitrage Pricing Theory is not a Theory

Discuss
Scientific methodology - Valuation...

Theory: If prices are set according as the discounted value of dividends (or EFCF, etc) price setting will obey 1.3

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n} \right]_{t} \]  

(1.3)
Theory: If prices are set according as the discounted value of dividends (or EFCF, etc) price setting will obey 1.3

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n} \right]_t \]  

(1.3)

Experiment: Estimate 1.3
IRR, Beta, Security Market Line

![Graph showing the relationship between IRR and Beta with a scattered plot and a red line indicating the Security Market Line.](image)
Scientific methodology - Valuation...

Theory: If prices are set according as the discounted value of dividends (or EFCF, etc) price setting will obey 1.3

\[ P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n} \right]_t \]  

(1.3)

Experiment: Estimate 1.3

Outcome: Price setting does not obey 1.3
Scientific methodology - Valuation...

Theory: If prices are set according as the discounted value of dividends (or EFCF, etc) price setting will obey 1.3

\[
P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n} \right]_t
\]  

(1.3)

Experiment: Estimate 1.3

Outcome: Price setting does not obey 1.3

Conclusion: The theory is disproved (The authors suggest that no other theory exists that has been so comprehensively disproved)
Scientific methodology - Valuation...

Theory: If prices are set according as the discounted value of dividends (or EFCF, etc) price setting will obey 1.3

\[ P_t = E\left[\sum_{n=1}^{\infty} \frac{D_{t+n}}{(1+R)^n}\right] \tag{1.3} \]

Experiment: Estimate 1.3

Outcome: Price setting does not obey 1.3

Conclusion: The theory is disproved (The authors suggest that no other theory exists that has been so comprehensively disproved)

But: No buts
Valuation

Proposition: *All that the 1000s of variations of DDMs, EFCFM, EVAMs, etc have done is to disprove beyond question the theory that prices are set with respect to these variables*
Valuation

Proposition: *All that the 1000s of variations of DDMs, EFCFM s, EVAM s, etc have done is to disprove beyond question the theory that prices are set with respect to these variables*

\[ P_{i,t} = FV_{i,t} + e_{i,t} \]  \hspace{1cm} (1.18)

Is a mis-representation of the original theory

We can propose any arbitrary model of FV and 1.18 will always be true
Proposition: All that the 1000s of variations of DDMs, EFCFMs, EVAMs, etc have done is to disprove beyond question the theory that prices are set with respect to these variables

\[ P_{i,t} = FV_{i,t} + e_{i,t} \quad (1.18) \]

Is a mis-representation of the original theory

We can propose any arbitrary model of FV and 1.18 will always be true

Say FV equals the company zip code raised to the power of yesterday's median temperature in degrees Fahrenheit at company HQ - equation 1.18 will still hold

We begin with an identity (1.1) make assumptions to derive (1.3) but find it never holds. The introduction of the error term renders the analysis meaningless
“If, among a nation of hunters, it usually costs twice the labour to kill a beaver which it does to kill a deer, one beaver should naturally exchange for - or be worth - two deer”

Adam Smith (The Wealth of Nations)
“If, among a nation of hunters, it usually costs twice the labour to kill a beaver which it does to kill a deer, one beaver should naturally exchange for - or be worth - two deer”

Adam Smith (The Wealth of Nations)

“What status should be given to this proposition? It is not metaphysical. It could serve as a hypothesis to be tested. But it is a hypothesis derived neither from observation nor analysis. It belongs rather to the realm of myth - a hypothesis of the same kind as that God ordered the sun to go round the earth so as to divide day from night”

Joan Robinson (Economic Philosophy, 1962)
“What status should be given to this proposition? It is not metaphysical. It could serve as a hypothesis to be tested. But it is a hypothesis derived neither from observation nor analysis. It belongs rather to the realm of myth - a hypothesis of the same kind as that God ordered the sun to go round the earth so as to divide day from night”

Joan Robinson (Economic Philosophy, 1962)
Adam Smith began the search. But the story of beavers and deer has no real analytical content. It is derived largely from *moral preconceptions*.

David Ricardo also tried “labour-units” as both a metric for, and a cause of, value. He realised that other stuff mattered (capital), but never figured out what to think about it.

Alfred Marshall tried to rescue Ricardo. But for Marshall labour alone determines relative prices - it determines value - and if labour created it surely it should have it! Which sounds like:

Marx..
1 quarter corn = x cwt iron

“tells us that in two different things there exists something common to both”

“the two things must be equal to a third, which is neither one nor the other”

“each of them, so far as it is use value, must be reducible to this third”

“they have one common property, that of being products of labour”
1 quarter corn = x cwt iron

“tells us that in two different things there exists something common to both”

“the two things must be equal to a third, which is neither one nor the other”

“each of them, so far as it is use value, must be reducible to this third”

“they have one common property, that of being products of labour”

“Human labour power has been expended in their production. Human labour is embodied in them. When looked at as crystals of this social substance, common to them all, they are - Values”
“Human labour power has been expended in their production. Human labour is embodied in them. When looked at as crystals of this social substance, common to them all, they are - Values”

“This theory of prices is not a myth, like Smith’s tale of beavers and deer. Nor is it an original contribution to science. It is simply dogma.”

Joan Robinson

“The conflation of the idea of labour as the measure of value and labour as the cause of value was taken over from Ricardo, and is a fatal misunderstanding”

John Hicks

“Is it value that determines prices, or prices that determine values?”
3 Stage DDM

\[ P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1+R)^n} \right] + t E \left[ \sum_{n=j+1}^{j+K} \frac{D_{j+ft}}{(1+R)^n} \right] + E \left[ \left( \frac{D_{j+k} (1+g)}{R - g} \right) \left( \frac{1}{(1+R)^{j+k+1}} \right) \right] \] (1.4)

What I don’t know well:
- Short term D/EFCF/etc
- Approximate factor pricing model
- Factor betas

What I really don’t know:
- Medium/long term D/EFCF/etc
- Factor risk premiums
- Exact factor pricing model

Total parameter uncertainty

Why would rational market agents do this?
3 Stage DDM

\[ P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1+R)^n} \right] + t \cdot E \left[ \sum_{n=j+1}^{j+K} \frac{D_{j+ft}}{(1+R)^n} \right] + E \left[ \frac{D_{j+k}(1+g)}{R-g} \right] \left( \frac{1}{(1+R)^{j+k+1}} \right) \] (1.4)

Why would rational market agents do this?
3 Stage DDM

\[
P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1+R)^n} \right] + t \cdot E \left[ \sum_{n=j+1}^{j+K} \frac{D_{j+ft}}{(1+R)^n} \right] + E \left[ \left( \frac{D_{j+k}(1+g)}{R-g} \right) \left( \frac{1}{(1+R)^{j+k+1}} \right) \right] \quad (1.4)
\]

Why would rational market agents do this?

Answer - they don’t!
3 Stage DDM

\[ P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1+R)^n} \right] + t \cdot E \left[ \sum_{n=j+1}^{j+K} \frac{D_{j+ft}}{(1+R)^n} \right] + E \left[ \frac{D_{j+k} (1+g)}{R - g} \right] \left( \frac{1}{(1+R)^{j+k+1}} \right) \]  

(1.4)

Why would rational market agents do this?

Answer - they don’t !

The Strange Tale of “Excess volatility”....
Excess volatility

“Fact”: We can derive a rational bound on expected dividend volatility $\text{RSIGMA}(D)$ from time-series.

Likewise, we can derive a rational bound on expected risk premium volatility $\text{RSIGMA}(R)$ from time-series.

Theory: Prices are set as the discounted value of dividends so price setting will obey $1.3$.

$$P_t = E \left[ \sum_{n=1}^{\infty} \frac{D_{t+n}}{(1 + R)^n} \right]_t$$

Because market agents are rational, price volatility will not exceed a rational bound $\text{RSIGMA}(P)$ implied by $(1.3)$ and $\text{RSIGMA}(D)$ and $\text{RSIGMA}(R)$. 
Excess volatility

Experiment: Estimate SIGMA(P) and RSIGMA(P)
Excess volatility

Experiment: Estimate SIGMA(P) and RSIGMA(P)

Outcome: SIGMA(P) = 13 * IGMA(P)
Excess volatility

Experiment: Estimate SIGMA(P) and RSIGMA(P)

Outcome: SIGMA(P) = 13 RSIGMA(P)

Conclusion: Prices exhibit “excess volatility” - Market agents are irrational
Excess volatility

Experiment: Estimate SIGMA(P) and RSIGMA(P)

Outcome: SIGMA(P) = 13 RSIGMA(P)

Conclusion: Prices exhibit “excess volatility” - Market agents are irrational

But: Didn’t we just disprove the theory?

(ie this is not how stocks are priced)
The “order of magnitude” argument...

The excess volatility findings have been subject to wide critique

Mainly around technical debate on the expectations formation mechanisms

So Shiller invokes an “order of magnitude” argument:

The scale of the excess volatility is so great that any technical debate is really a side-issue - prices are simply “too volatile” to be rational
The “order of magnitude” argument...

The excess volatility findings have been subject to wide critique

Mainly around technical debate on the expectations formation mechanisms

So Shiller invokes an “order of magnitude” argument:

The scale of the excess volatility is so great that any technical debate is really a side-issue - prices are simply “too volatile” to be rational

We could put it differently - “the evidence completely refutes the theory, so we will ignore this completely, and simply assert that price setting is irrational”
The “order of magnitude” argument...

The excess volatility findings have been subject to wide critique

Mainly around technical debate on the expectations formation mechanisms

So Shiller invokes an “order of magnitude” argument:

The scale of the excess volatility is so great that any technical debate is really a side-issue - prices are simply “too volatile” to be rational

We could put it differently - “the evidence completely refutes the theory, so we will ignore this completely, and simply assert that price setting is irrational”

“This ... is not a myth, like Smith’s tale of beavers and deer. Nor is it an original contribution to science. It is simply dogma.”
3 Stage DDM

\[ P_t = E \left[ \sum_{n=1}^{j} \frac{D_{t+n}}{(1 + R)^n} \right] + t E \left[ \sum_{n=j+1}^{j+K} \frac{D_{j+ft}}{(1 + R)^n} \right] + E \left[ \left( \frac{D_{j+k}(1 + g)}{R - g} \right) \left( \frac{1}{(1 + R)^{j+k+1}} \right) \right] \] (1.4)

Why would rational market agents do this?

Answer - they don’t!
“Rational” forecasting

Krugman makes the following point:

In 1973 exchange rates float

A rational investor has no idea how to forecast exchange rates

She forms 2 models - a PPP (Value) model, and a momentum model

She is a rational bayesian, and puts 50% weight on each model at the start, and updates the weights as the sample evolves

By the mid 80s the weight on the value model is close to zero

This is rational
Stock Selection - One example - Sporting Bet..

UK based internet gaming company

Explosive growth, but

Complete uncertainty around D/E/FCF/etc at even short horizon

Factor pricing??

US Federal Law

Only useful information is around delta of short term information flow

This is rational
The origins of value - some observations

Separation of price and value is problematic

Default free bonds clearly obey something like 1.3 - but price equals value

Equities clearly don’t obey anything like 1.3 - so why pretend that they do?

Risk aversion / liquidity / crises

Sub-prime

Security selection / GTAA / etc

“Economic rationale” for models!?
“Value will not help. It has no operational content. It is just a word.”