

# Tax Adjusted Portfolio Optimization and Asset Location

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# Agenda

**Introduce Concept of Tax-Adjusted Portfolio Optimization (Asset allocation and location)**

**Outline Two Distinct Approaches**

Pretax Principal Values

After-Tax Principal Values

**Draw Extensions**

Multiple Dimension Taxing Scheme

Cost Basis

Traditional IRA as Distinct from Roth IRA

**Compare and Contrast Two Methods**



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- **Asset Allocation:** How should investors allocate their investment resources among various risky and risk-free assets?
- **Asset Location:** Given the availability of both taxable brokerage accounts (TBA) and tax-deferred retirement accounts (TDRA), how should investors locate the risky and risk-free assets in these two accounts?

# Tax Environment

## **TBA:**

Dividend and interest payments taxed at individual income tax rate

Capital Gains tax payable upon realization of capital gains

Tax-timing option

## **TDRA:**

No capital gains tax consequences

No tax-timing option

Tax at the time of withdrawal

Basis step-up provision



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# Asset allocation and location

## Empirical Works

Shoven and Sialm (2003, JPUBE)

Bergstresser and Poterba (2004, JPUBE),  
and Barber and Odean (2004, JPUBE )

## Dynamic Optimization Framework

Huang (2008, RFS), Dammon et al. (2004, JF),  
and Amromin (EFR, 2003)

## Static Optimization Framework

Wilcox et al. (2006, CFA Ins.),  
and Reichenstein (2001, JWM, and 2007, FSR)

## Implementing the model



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# What are the Implications?

- Taxes Affect Return
- Taxes Affect Risk
- Taxes Affect Asset Allocation
- Tax Structures/Locations
  - Each possible account-asset combination is a unique after-tax asset

# After-Tax Future Value Expressions for Various Tax Structures

Tax-Deferred Account

$$FVIF_{TDA} = (1 + r)^n (1 - t_n)$$

Annual Tax

$$FVIF_i = [1 + r(1 - t_i)]^n$$

Deferred Capital Gain

$$FVIF_{cg} = (1 + r)^n (1 - t_{cg}) + t_{cg}$$

Blended Tax Environment

$$FVIF_{Taxable} = (1 + r^*)^n (1 - T^*) + T^* - (1 - B)t_{cg}$$

$$r^* = (1 - p_i t_i - p_d t_d - p_{cg} t_{cg})$$

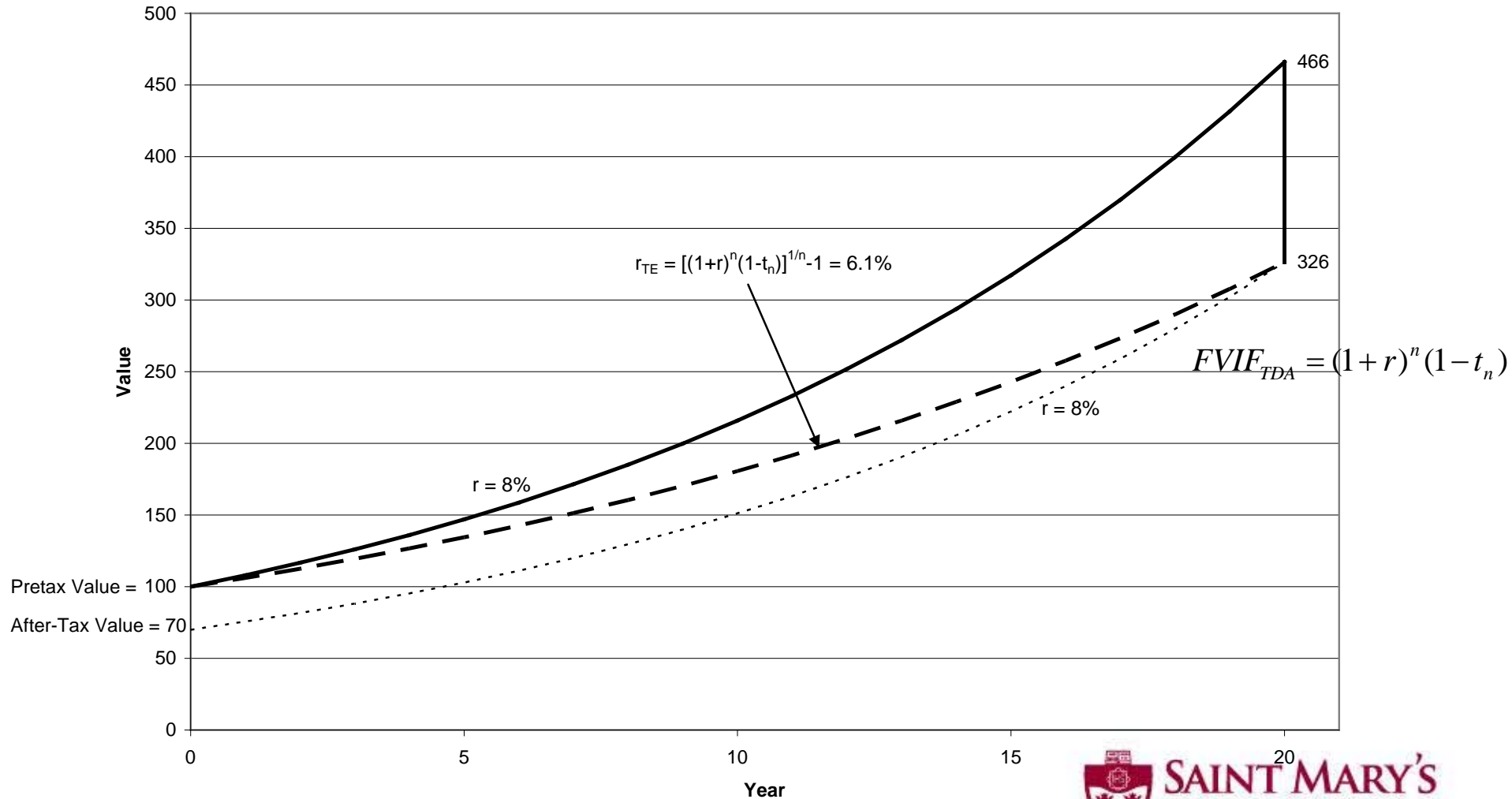
$$T^* = t_{cg} (1 - p_i - p_d - p_{cg}) / (1 - p_i t_i - p_d t_d - p_{cg} t_{cg})$$



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# After-Tax Return – Tax Deferred Account

Value of a TDA Over Time



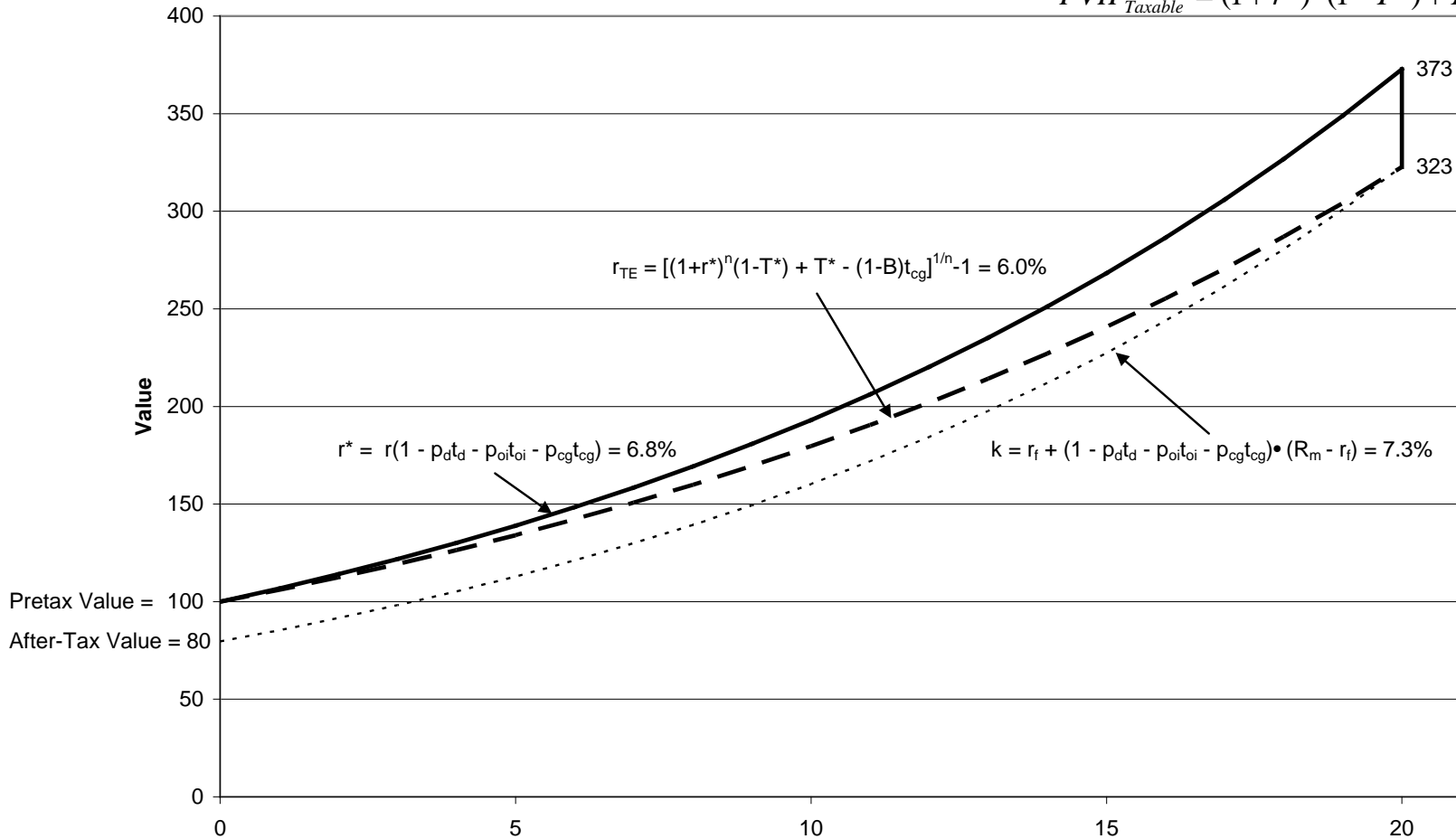
Value of a tax-deferred account over time assuming an 8% pretax return and a 30% terminal tax rate.



# After-Tax Return – Taxable Account

Value of a Taxable Account Over Time

$$FVIF_{Taxable} = (1 + r^*)^n (1 - T^*) + T^* - (1 - B)t_{cg}$$



Value of a taxable account over time assuming an 8% return. The pretax return is 8%. The proportion of return taxed each year as ordinary income, dividends, and capital gains is 25%, 25%, and 0%, respectively. The tax rates on each of these forms of return are assumed to be 30%, 30%, and 25%, respectively. The original cost basis is assumed to be 60% of the initial pretax market value. The risk-free rate is 3%, the beta is one, and the market risk premium is 5%.



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# Pretax Value vs. After-Tax Value (ATV)

- Taxable Portfolios
  - A portfolio in a *tax-free* environment must worth more than the same portfolio in a *taxable* environment
- After-Tax Portfolio Value (ATV)
  - The present value of after-tax cash flows the portfolio is likely to generate
  - Incorporates expected tax liability associated with dividends, taxable income, realized gains, and unrealized gains
  - An investor's economic welfare is determined by ATV



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# Simple Example of After-Tax Asset Allocation

Account Type	Asset Class	Pretax Market Value	Pretax Weights	After-Tax Market Value	After-Tax Weights
TDA	Stock	\$1,500,000	75%	\$900,000	64.3%
Tax-Exempt	Bonds	500,000	25%	500,000	35.7%
Total		\$2,000,000	100%	\$1,400,000	100%

Note: Withdrawals at the end of the investment horizon are assumed to be taxed at a rate of 40 percent.



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# Taxes and Investment Risk

Consider a \$100,000 investment with the following potential outcomes

Outcome	Prob.	Pretax Accumulation	Pretax Return	After-Tax Market Value	After-Tax Returns
Good	1/3	\$125,000	25%	\$115,000	15%
Average	1/3	110,000	10%	106,000	6%
Bad	1/3	95,000	-5%	97,000	-3%
Exp. Value		\$110,000	10%	\$106,000	6%
Std. Dev. ( $\sigma$ )			12.25%		7.35%

Note: Investment returns are assumed to be taxed at a rate of 40 percent in the year they are earned.



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# Tax-Adjusted Discount Rate for ATV

- Tax- Adjusted Volatility

$$\sigma_{pretax,Taxable} \approx (1 - t_{a,i})\sigma_i$$

- Tax-Adjusted, Risk-Adjusted Discount Rate

$$k_{AfterTax,i} \approx r_f + (1 - t_{a,i})\beta_i[E(r_m) - r_f]$$

- It Does NOT Equal

$$k_{AfterTax,i} = (1 - t_{a,i})\{r_f + \beta_i[E(r_m) - r_f]\} = (1 - t_{a,i})k_{BeforeTax,i}$$

# Sample After-Tax Correlation Matrix

- Simple Example
  - Two uncorrelated asset classes
  - Two types of accounts

		Taxable		TDA	
		Equity	Bonds	Equity	Bonds
Taxable	Equity	<b>1</b>	<b>0.0</b>	<b>1</b>	<b>0.0</b>
	Bonds	<b>0.0</b>	<b>1</b>	<b>0.0</b>	<b>1</b>
TDA	Equity	<b>1</b>	<b>0.0</b>	<b>1</b>	<b>0.0</b>
	Bonds	<b>0.0</b>	<b>1</b>	<b>0.0</b>	<b>1</b>

# Summary of Expressions

**Table 1.** Expressions for calculating tax-adjusted returns, volatilities, and covariances when an asset's principal is assumed to be denominated in pretax units or after-tax units.

	<i>Pretax Principal Value</i>	<i>After-Tax Principal Value</i>
<i>Panel A: Tax Deferred Account</i>		
Return = $r_{i,TDA}$	$(1 + r_i)(1 - t_n)^{1/n} - 1$	$r_i$
Volatility = $\sigma_{i,TDA}$	$(1 - t_n)^{1/n} \sigma_i$	$\sigma_i$
Covariance = $\sigma_{ij,TDA}$	$(1 - t_n)^{2/n} \rho_{ij} \sigma_i \sigma_j$	$\rho_{ij} \sigma_i \sigma_j$
<i>Panel B: Taxable Account</i>		
Return = $r_{i,Taxable}$	$[(1 + r^*)^n(1 - T^*) + T^* - (1 - B)t_{cg}]^{1/n} - 1$	$r_f + (1 - t_{a,i})\beta_i[E(r_m) - r_f]$
Volatility = $\sigma_{i,Taxable}$	$(1 - t_{a,i})\sigma_i$	$(1 - t_{a,i})\beta_i\sigma_m$
Covariance = $\sigma_{ij,Taxable}$	$(1 - t_{a,i})(1 - t_{a,j})\rho_{ij}\sigma_i\sigma_j$	$(1 - t_{a,i})(1 - t_{a,j})\beta_i\beta_j\sigma_m^2$
<i>Panel C: Roth-type Accounts</i>		
Return = $r_{i,Roth}$	$r_i$	$r_i$
Volatility = $\sigma_{i,Roth}$	$\sigma_i$	$\sigma_i$
Covariance = $\sigma_{ij,Roth}$	$\rho_{ij}\sigma_i\sigma_j$	$\rho_{ij}\sigma_i\sigma_j$
<i>Panel D: Covariances Across Account Types</i>		
$Cov(r_{i,TDA}, r_{j,Taxable})$	$(1 - t_n)^{1/n}(1 - t_{a,j}) \rho_{ij}\sigma_i\sigma_j$	$(1 - t_{a,j})\beta_i\beta_j\sigma_m^2$
$Cov(r_{i,TDA}, r_{j,Roth})$	$(1 - t_n)^{1/n} \rho_{ij}\sigma_i\sigma_j$	$\rho_{ij}\sigma_i\sigma_j$
$Cov(r_{i,Roth}, r_{j,Taxable})$	$(1 - t_{a,j})\rho_{ij}\sigma_i\sigma_j$	$(1 - t_{a,j}) \beta_i\beta_j\sigma_m^2$

# Sample Inputs

	<i>Taxable</i>		<i>TDA</i>		<i>Roth</i>	
	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>
Pretax Return ( $r$ )	8.0%	4.0%	8.0%	4.0%	8.0%	4.0%
Pretax Std. Dev. ( $\sigma$ )	15.0%	6.0%	15.0%	6.0%	15.0%	6.0%
Tax Rate – Ordinary Income ( $t_{oi}$ )	30%	30%				
Tax Rate – Dividends ( $t_d$ )	15%	15%				
Tax Rate – Capital Gains ( $t_{cg}$ )	25%	25%				
Tax Rate – TDA Withdrawals ( $t_n$ )			30%	30%		
Return – Ordinary Income ( $p_{oi}$ )	0.05	1				
Return – Dividends ( $p_d$ )	0.25	0				
Return – Capital Gains ( $p_{cg}$ )	0.25	0				
Cost Basis ( $B$ )	0.9	1				
Beta	1.0	0.2	1.0	0.2	1.0	0.2
Correlation		0.1		0.1		0.1
Pretax Account Balances		40%		40%		20%

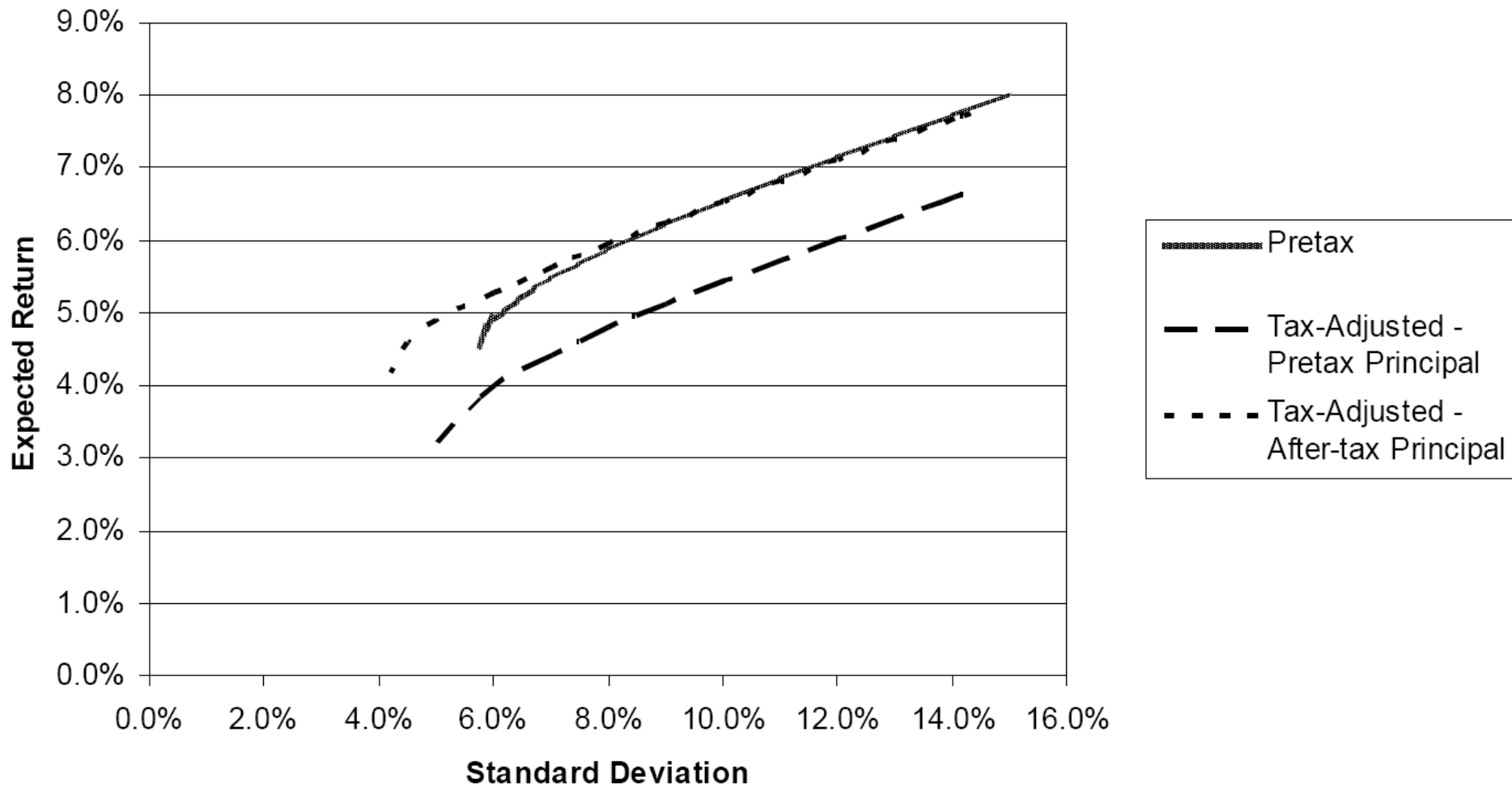


# Pretax Efficient Frontier

**Table 3.** Standard pretax efficient frontier equity allocation.

<i>Expected Return</i>	<i>Standard Deviation</i>	<i>Equity Allocation</i>
8.0%	15.0%	100.0%
7.7%	14.0%	93.0%
7.4%	13.0%	85.9%
7.1%	12.0%	78.7%
6.9%	11.0%	71.3%
6.5%	10.0%	63.6%
6.2%	9.0%	55.6%
5.9%	8.0%	46.8%
5.5%	7.0%	36.8%
4.9%	6.0%	22.2%
4.5%	5.7%	12.7%

## Efficient Frontiers for Different Optimization Frameworks



# Tax-Adjusted Efficient Frontier: Pretax Principal Values

	<i>Taxable</i>		<i>TDA</i>		<i>Roth</i>	
	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>
Tax-Adjusted Return	6.5%	2.8%	6.1%	2.2%	8.0%	4.0%
Tax-Adjusted Std. Dev.	13.2%	4.2%	14.7%	5.9%	15.0%	6.0%

*Panel B: Tax-Adjusted Asset Allocation and Location*

<i>Expected Return</i>	<i>Standard Deviation</i>	<i>Equity Allocation</i>	<i>Taxable</i>		<i>TDA</i>		<i>Roth</i>	
			<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>
6.6%	14.2%	100.0%	40.0%	0.0%	40.0%	0.0%	20.0%	0.0%
6.6%	14.0%	98.8%	40.0%	0.0%	40.0%	0.0%	18.8%	1.2%
6.3%	13.0%	91.8%	40.0%	0.0%	40.0%	0.0%	11.8%	8.2%
6.0%	12.0%	84.6%	40.0%	0.0%	40.0%	0.0%	4.6%	15.4%
5.7%	11.0%	77.3%	40.0%	0.0%	37.3%	2.7%	0.0%	20.0%
5.4%	10.0%	69.7%	40.0%	0.0%	29.7%	10.3%	0.0%	20.0%
5.1%	9.0%	61.7%	40.0%	0.0%	21.7%	18.3%	0.0%	20.0%
4.8%	8.0%	53.2%	40.0%	0.0%	13.2%	26.8%	0.0%	20.0%
4.4%	7.0%	41.4%	0.0%	40.0%	21.4%	18.6%	20.0%	0.0%
4.0%	6.0%	31.4%	0.0%	40.0%	31.4%	8.6%	0.0%	20.0%
3.2%	5.0%	11.2%	0.0%	40.0%	0.0%	40.0%	11.2%	8.8%

# Tax-Adjusted Efficient Frontier: After-Tax Principal Values

	<i>Taxable</i>		<i>TDA</i>		<i>Roth</i>	
	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>
Tax-Adjusted Return	7.4%	3.7%	8.0%	4.0%	8.0%	4.0%
Tax-Adjusted Std. Dev.	13.3%	2.1%	15.0%	6.0%	15.0%	6.0%

## *Panel B: Tax-Adjusted Asset Allocation and Location*

<i>Expected Return</i>	<i>Standard Deviation</i>	<i>Equity Allocation</i>	<i>Taxable</i>		<i>TDA</i>		<i>Roth</i>	
			<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>	<i>Equity</i>	<i>Bonds</i>
7.8%	14.3%	100.0%	40.0%	0.0%	40.0%	0.0%	20.0%	0.0%
7.7%	14.0%	97.7%	40.0%	0.0%	37.7%	2.3%	20.0%	0.0%
7.4%	13.0%	89.5%	40.0%	0.0%	29.5%	10.5%	20.0%	0.0%
7.1%	12.0%	81.1%	40.0%	0.0%	21.1%	18.9%	20.0%	0.0%
6.8%	11.0%	73.7%	31.0%	9.0%	22.7%	17.3%	20.0%	0.0%
6.5%	10.0%	66.9%	16.8%	23.2%	30.1%	9.9%	20.0%	0.0%
6.2%	9.0%	60.3%	1.2%	38.8%	39.0%	1.0%	20.0%	0.0%
5.9%	8.0%	51.8%	0.0%	40.0%	31.8%	8.2%	20.0%	0.0%
5.6%	7.0%	42.8%	0.0%	40.0%	22.8%	17.2%	20.0%	0.0%
5.3%	6.0%	32.9%	0.0%	40.0%	12.9%	27.1%	20.0%	0.0%
4.9%	5.0%	29.7%	0.0%	40.0%	29.7%	10.3%	0.0%	20.0%
4.2%	4.2%	6.6%	0.0%	40.0%	0.0%	40.0%	6.6%	13.4%

# Conclusions

Derivation and implementation of models

Two Approaches to After-Tax Portfolio Optimization

- Pretax Principal Values
- After-Tax Principal Values

## Advantages

## Disadvantages

**Pretax Principal**

Familiar

Negative returns  
for short time  
horizons

**After-Tax Principal**

Better represents  
capital at risk

Low volatility of  
non-equity  
assets



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