THE SECOND HALF OF MARKOWITZ
GOALS FOR THIS DISCUSSION

- Describe the most prevalent aspect of portfolio theory that most investors get wrong, the determination of appropriate risk tolerance
- Introduce the “Discretionary Wealth Hypothesis” (DWH) from Wilcox (2003)
- Show how to derive the Northfield risk acceptance parameter (RAP) from discretionary wealth theory
- Resolve the conflict between use of the DWH and the single period assumption in Markowitz

This theory says that an investor can form an efficient frontier of differently composed portfolios:
- Each portfolio has the maximum return for a given level of risk
- Each portfolio has the minimum risk for a given level of return
- Time is defined as a single long period

But what did Markowitz say about how an investor should choose which efficient portfolio to hold?
- Nothing
TWENTY SEVEN YEARS LATER

- Assumes investors want to maximize the expectation of the log of their wealth
- The mean-variance formulation is derived from a Taylor series approximation to the log of wealth. It's just the first two terms

\[ U = \alpha - \frac{\sigma^2}{\text{RAP}} \quad \text{or} \quad U = \alpha - \lambda \sigma^2 \quad \lambda = \frac{1}{\text{RAP}} \]

\( \lambda \) is just the slope of the tangent line to the frontier
Have you ever asked a passer-by on the street “What is your risk tolerance parameter in mean-variance space?”

- My grandmother would have taken great offense at an impertinent question and slapped me.

Investors constantly use terms of art such as “conservative” or “aggressive” to describe their posture without actually understanding what that means.

Portfolio compositions change for a muddle of two reasons:

- Expectations have changed about the risk or return of various assets.
- The investor’s risk aversion has changed, probably without being recognized or consciously done.
THE DISCRETIONARY WEALTH HYPOTHESIS

- Simply put it says that investors should not put more money at risk than they can afford to lose
- *Think of your life as a balance sheet*, including the present value of future savings and the present value of the liability for expected expenditures
  - Low discount rates for important liabilities, higher discount rates for non-essential expenditures
  - What is the debt/equity ratio for your life?

Northfield INFORMATION SERVICES, INC.
Wilcox derives that the optimal risk aversion for an investor:

- Let L = total assets / net worth
- Optimal risk aversion $\lambda = L / 2$
- Allowing time variation in $\lambda$ maximizes the expected median of future wealth, rather than the mean

This implies that optimal risk aversion varies in both unpredictable and predictable ways

- Market volatility will change our net worth in unpredictable ways
- Getting closer to retirement age, or having a child graduate university changes our balance sheet in predictable ways

This implies that we have some ability to forecast our optimal asset allocation for times in the future, requiring a multi-period framework
EVEN MORE ON DISCRETIONARY WEALTH

- Following the Discretionary Wealth Hypothesis is similar to portfolio insurance for individual investors
  - You are increasing aggressiveness when you can afford to do so
  - You are taking a more conservative posture when you must
- These changes only impact your risk tolerances
  - Changes in portfolio composition must also reflect changes in expectations
- The DWH approach is now included in CFA Institute curriculum
- DWH approach can also incorporate uncertainty in the balance sheet formation
  - We don’t know how long we’ll live
  - Will our children require financial support for education or not
Use of the DWH requires a change to the traditional Markowitz assumption of a time being a single long period.

If we know that our portfolio will be changing over time, and those changes require transaction costs, we need to weigh the benefits of improvements in expected utility against the trading costs in the right way.

Traditional optimization procedures that assume a single period can be substantially improved by incorporating a simple approximation.
Imagine I have a portfolio, \( P_1 \) with return \( \alpha \) (net of fees and expenses) and standard deviation \( \sigma \). Our usual utility function would say:

\[
U_1 = \alpha - \frac{\sigma^2}{RAP}
\]

Where RAP is equal to our risk acceptance parameter.

Now let’s imagine there is another portfolio, \( P_2 \) that has a higher utility, because either the return is higher or the standard deviation is lower.

This portfolio has completely different positions than the initial portfolio. Let’s assume that this portfolio has a higher return by increment \( \Delta \), so

\[
U_2 = (\alpha + \Delta) - \frac{\sigma^2}{RAP}
\]
Since $U_2$ is greater than $U$, we should be willing to pay some transaction costs to switch from $P_1$ to $P_2$.

Now let’s consider a different way to improve our returns:

- We go back to the manager of Portfolio 1 and ask them to reduce their fees by $\Delta$, so now our revised utility on $P_1$ is $U_{1,L}$ for “lowered fees”.
- Notice that $U_{1,L}$ and $U_2$ are equal. So if we invest our money in either $P_2$ or $P_1$ (after lowering the fees), the expected value of wealth at the end of time is the same.
- This suggests that we should be willing to pay the manager an upfront fee to lower his management fees that is equal to the trading costs we would be willing to pay to switch from the initial portfolio. **As long as conditions never change, this is valid**
THE KEY CONCEPT

- Since $P_2$ and $P_{1L}$ have different securities, the performance will be different from month to month
  - Even if the long term average return and volatility are identical
  - So over any finite time horizon, we cannot be sure which of the portfolios will perform better

- $P_{1L}$ will always perform better than $P_1$, over all time horizons, as it is just the same portfolio with lower fees
  - For $P_{1L}$ the probability of outperforming $P_1$ is always 1
  - $P_2$ is guaranteed to be better than $P_1$ in the long run if conditions don’t change, but the probability that $P_2$ will actually outperform $P_1$ over any finite horizon is between .5 and 1
  - Amortizing transaction costs over a single period is equivalent to assuming that this value is always one
IMPLEMENTATION

- Let’s assume a single period optimization for a strategy with expected turnover of M% per annum
  - In this case we want to amortize the cost of each transaction over a four year expected holding period, or M% per annum
  - In our multi-period world, we want to amortize by M divided by the probability that the revised portfolio will actually realize a better risk adjusted return over the finite holding period
- We use the tracking error between the original portfolio P₁ and the modified portfolio P₂, as the standard error on the increase in expected utility between U₂ and U₁
  - The tracking error between P₁ and P₁L is zero
  - Just pick your distribution assumption, convert to a T-stat and calculate the probability value by which to adjust amortization
- Northfield optimizer does this already
CONCLUSIONS

- Traditional usage of Modern Portfolio Theory provides little guidance to investors on how to formulate their risk tolerance.
- The Discretionary Wealthy Hypothesis can be utilized to rationally quantify risk tolerance, and even more importantly required changes in risk tolerance over the investor life cycle.
- Implementing the DWH requires converting to a multi-period framework in which the classical treatment of transaction costs must be modified.
A QUICK RULE OF THUMB

- Let’s assume we don’t know the life balance sheet for the investor, but they are willing to deal with a 3% annual active risk
  - If we assume a “worst case” scenario of a three standard deviation event, this equates to a 9% drawdown
  - We must believe that 91% of our wealth is not to be put at risk
- Doing out the algebra and converting from decimal to percents, this equates to a Northfield RAP of 18
- A reasonable value for risk tolerance is in the neighborhood of six times acceptable annual volatility