Return Forecasting by Quantile Regression

Northfield's 23rd Annual Research Conference

Sept 1, 2010

Larry Pohlman and Lingjie Ma

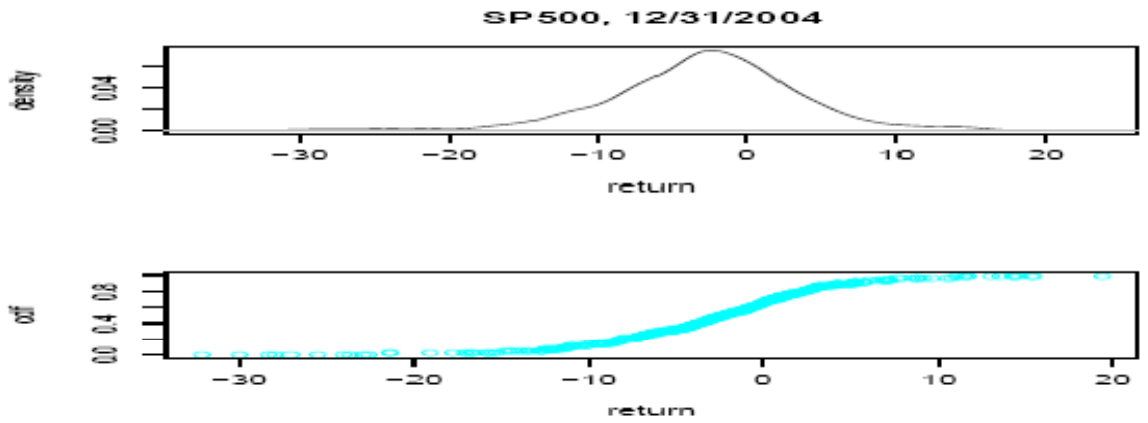
Outline

- The Math
- Examples
- Multivariate Model
- Results

The Math and Code

- Model $y = Xb + u$
- OLS Estimation $\hat{\beta}_{OLS} = \min_{b \in \mathrm{B}} \left[\sum (y_i - x_i^T b)^2 \right]$
- QR Estimation $\hat{\beta}_{QR}(\tau) = \min_b \left[\sum_{y_i \geq x_i^T \beta} \tau (y_i - x_i^T b) + \sum_{y_i < x_i^T b} (\tau - 1)(y_i - x_i^T b) \right]$
- R, S+, Stat, SAS

What does QR do?



- Sample Quantile:

$$F_y = \text{Prob}\left(\text{Y} \le \text{y}\right),$$
$$Q(\tau) = f\left\{y : F_y \ge \tau\right\}, \tau \in (0,1)$$

- Conditional Quantile:

$$y = x^T\beta + (x^T\delta)u$$
$$F_y^{-1}(\tau|x) = x^T\beta + x^T\delta F_u^{-1}(\tau)$$
$$Q_y(\tau|x) = x^T(\beta + \delta F_u^{-1}(\tau)) = x^T\beta(\tau)$$

Why QR?

A natural question is under what conditions
will QR be “better” than OLS?

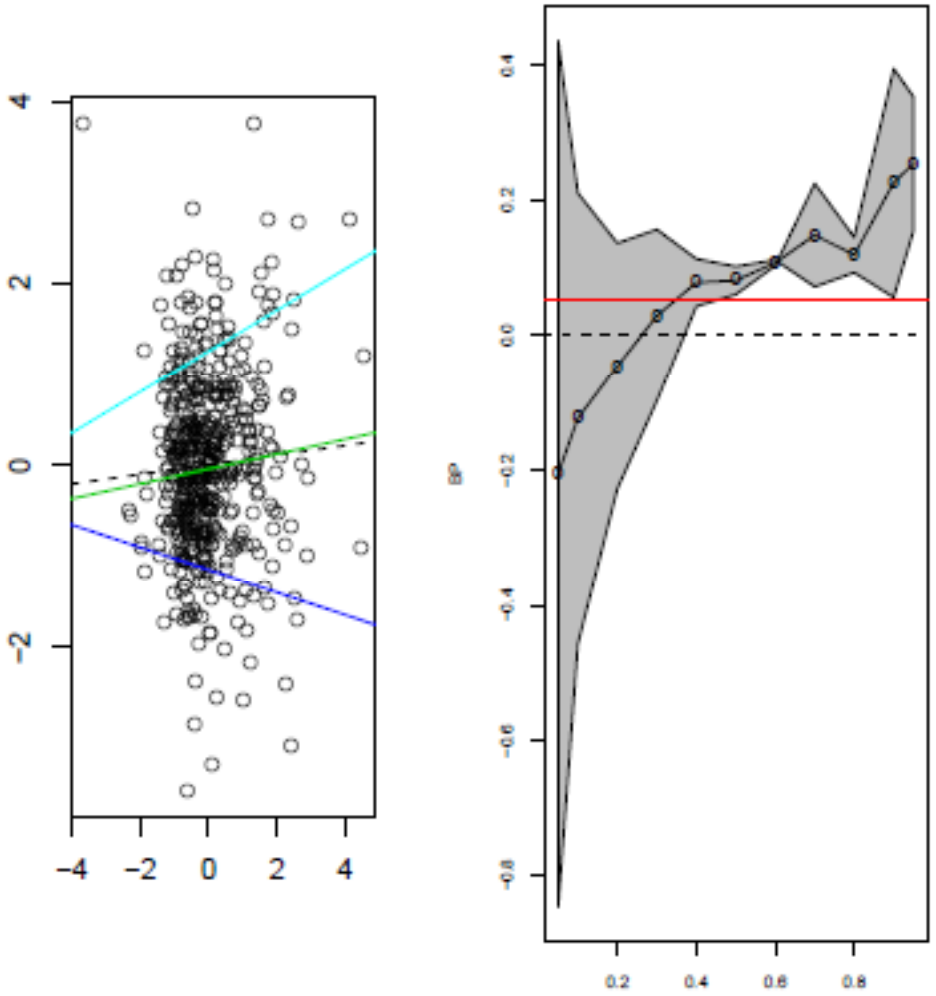
1. Full picture view: heterogeneity

- If there is heterogeneity, then QR will provide a more complete view of the relationship between variables through the effects of independent variables across quantiles of the response distribution.

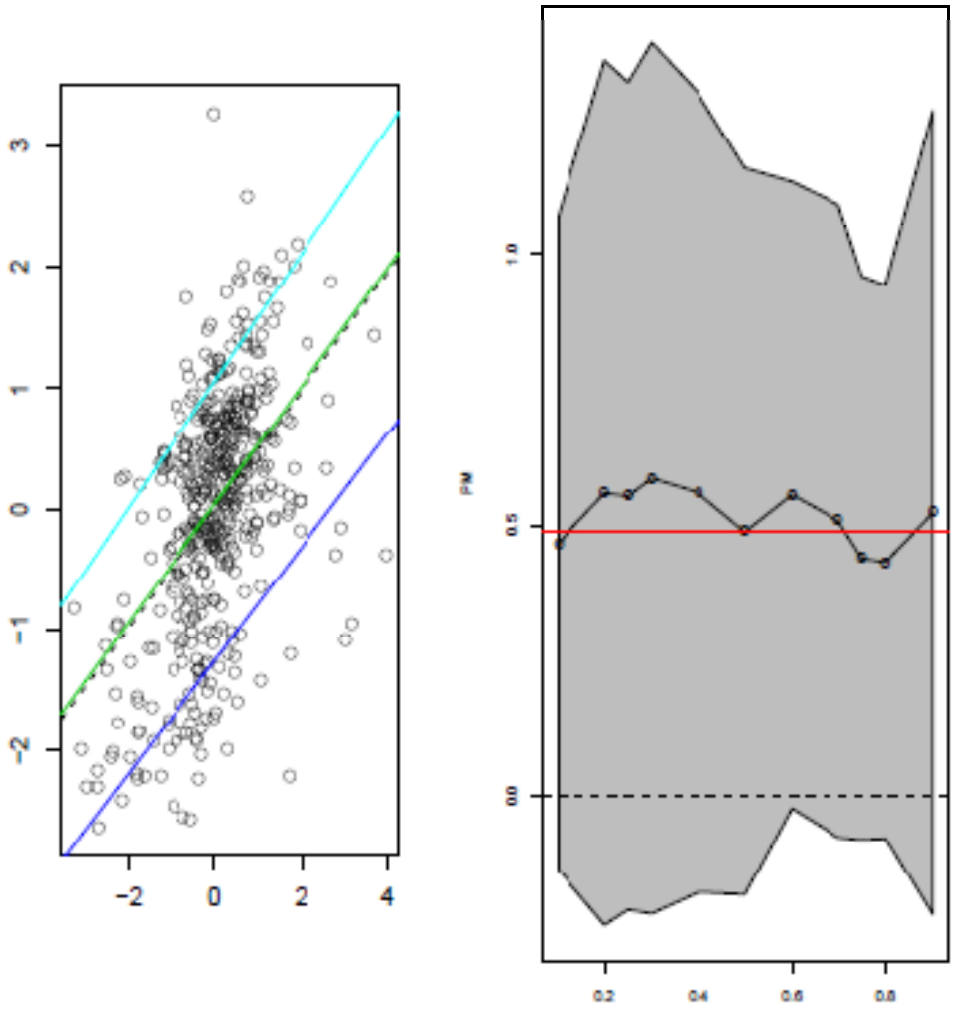
2. Robustness: fat-tail distribution

- If the conditional return distribution is not Gaussian but fat-tailed, the QR estimates will be more robust and efficient than the conditional mean estimates

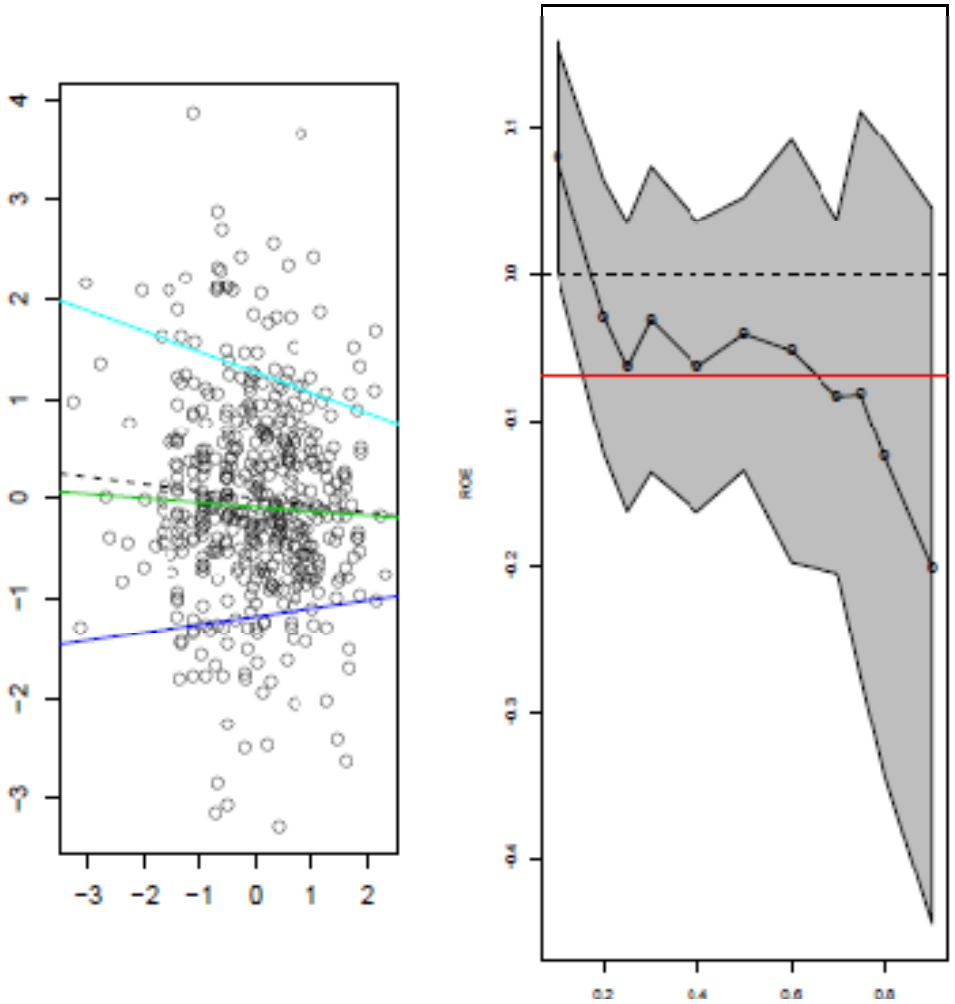
Example: Book to Price



Example Price Momentum



Example Return on Equity



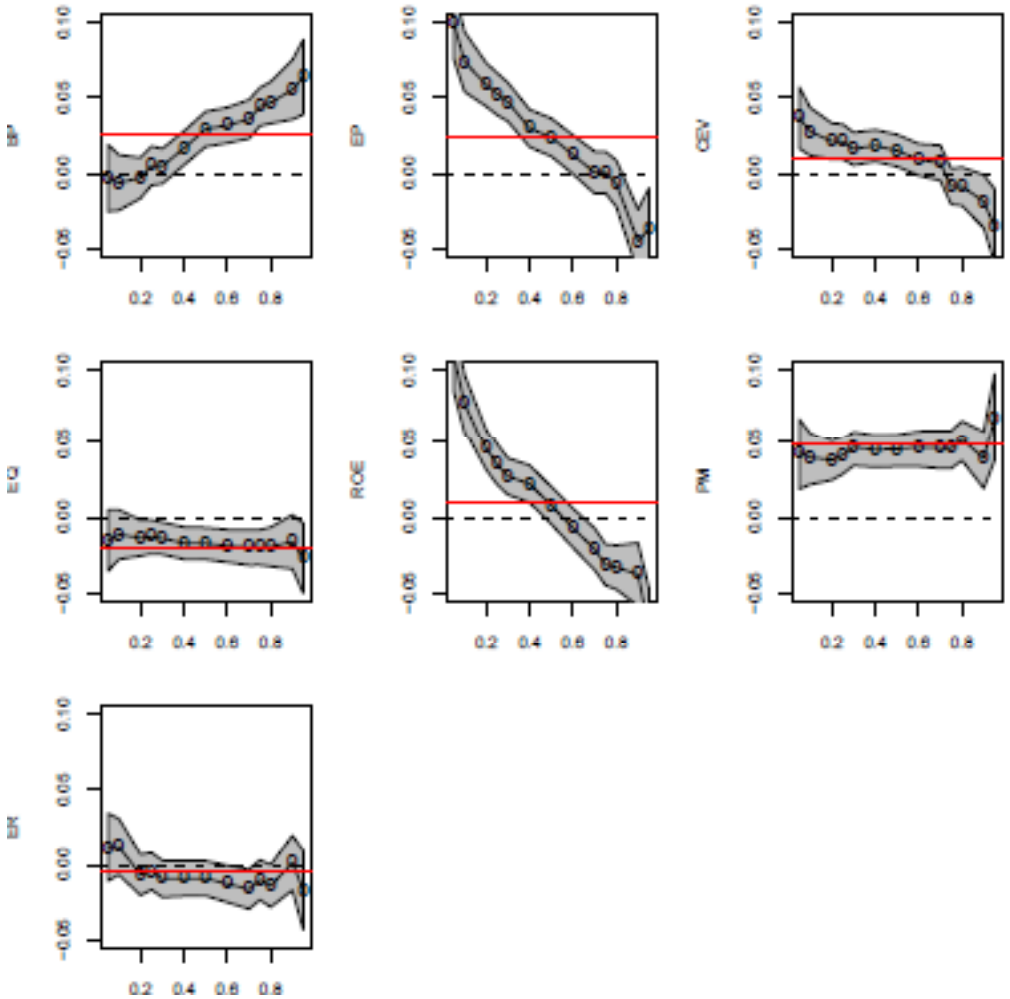
Which Tau?

- Classic study of food expenditures vs income. Subjects tend to stay in the same income level. So use the tau for their percentile.
- Stocks frequently move between percentiles.
- Since we are interested in ranking stocks choose the tau with the steepest significant slope.

Multivariate Model

- Book to Price
- Earnings to Price
- Cashflow to Enterprise Value
- Balance sheet Accruals
- Return on Equity
- Price Momentum (9 months)
- Earnings Momentum (9 months)

Model Variable Plots



Results: Equal Weight Quintiles

	Q1	Q2	Q3	Q4	Q5	Q1-Q5
Return						
OLS	2.51(4.10)	1.96(2.36)	0.50(2.41)	-1.11(2.64)	-3.89(4.10)	6.40(7.45)
QR(0.10)	3.26(3.72)	1.59(2.71)	-0.04(2.29)	-1.54(2.38)	-3.27(4.61)	6.54(7.45)
QR(0.50)	1.85(3.96)	1.65(2.48)	0.95(2.57)	-1.06(2.73)	-3.41(4.67)	5.26(7.52)
QR(0.90)	-1.29(3.87)	-0.50(2.47)	1.21(2.27)	0.09(2.66)	0.45(3.49)	-1.74(6.41)

Results: Cap Weighted Qunitiles

	Q1	Q2	Q3	Q4	Q5	Q1-Q5
Return						
OLS	3.20(4.83)	1.65(3.66)	0.65(3.64)	-1.93(3.79)	-4.22(5.10)	7.42(8.31)
QR(0.10)	3.91(4.78)	1.42(3.46)	0.50(3.88)	-2.68(3.70)	-3.59(5.50)	7.50(8.28)
QR(0.50)	2.54(4.57)	0.57(3.73)	1.72(3.83)	-1.92(3.87)	-3.78(5.31)	6.32(8.05)
QR(0.90)	-1.49(5.21)	-1.38(3.64)	1.45(3.48)	1.19(4.21)	-0.53(5.25)	-0.97(8.24)

Optimized Portfolios TE=3%

	OLS	QR(0.1)	QR(0.5)	QR(0.9)
Long Only				
Total Return	8.33(14.19)	9.12(14.35)	9.49(15.26)	7.83(15.79)
Excess Return	1.11(2.34)	1.90(2.38)	1.22(2.84)	-0.44(2.61)
IR	0.47	0.80	0.43	-0.17
Market Neutral				
Total Return	0.80(2.68)	2.27(3.27)	0.55(2.55)	0.28(3.06)
Excess Return	0.80(2.68)	2.27(3.27)	0.55(2.55)	0.28(3.06)
SR	0.30	0.69	0.22	0.09

Optimized Portfolios TE=6%

	OLS	QR(0.1)	QR(0.5)	QR(0.9)
Long Only				
Total Return	8.98(14.76)	9.65(14.49)	8.91(15.57)	7.33(15.78)
Excess Return	1.76(3.54)	2.43(3.71)	1.69(4.13)	0.11(4.31)
IR	0.50	0.65	0.41	0.03
Market Neutral				
Total Return	2.63(4.64)	2.92(5.34)	2.26(4.36)	-0.13(5.30)
Excess Return	2.63(4.64)	2.92(5.34)	2.26(4.36)	-0.13(5.30)
SR	0.57	0.55	0.52	-0.02

Conclusion

- Conditional mean method is still attractive
- QR provides a full-picture distributional view
- Link between distribution estimates and point portfolio.