OPEN PERFORMANCE ATTRIBUTION
For Windows

IDENTIFY
ANALYZE
QUANTIFY

NORTHFIELD
Open Performance Attribution

The NIS Open Performance Attribution System provides consultants, plan sponsors, and investment managers with a detailed explanation of a security portfolio’s performance. By characterizing a portfolio and its benchmark according to their exposure to factors and security groups, clients quickly identify the reasons for superior or sub-par performance. Through this examination, and a comparison with subsequent periods, investment "styles" are easily recognized, and can be evaluated against the stated investment philosophy.

Portfolio risk characteristics are identified for future change considerations. The system is especially useful for identifying the particular strengths of individual managers, and the way these affect a multi-manager investment process.

Key Features

- **Fast, Flexible Windows System**
  
  It is also available for Unix and Linux. A "command line" version can be run under DOS or Windows. The underlying performance attribution engine can also be called from programs written in Visual Basic, Java, C++, C#, and most other modern programming languages.

- **Ability to Use Any Multifactor Model**
  
  A crucial feature of the new system is the ability to operate with any multiple factor model of security behavior. All Northfield models can be used for attribution. Models from other vendors, and user-defined models, can be fully supported. This will allow investment firms to custom tailor their attribution process to specific investment products and themes. Users can define a variety of factor types (e.g. continuous variables such as beta or P/E, or binary variables such as sector membership).

- **Introduce User-defined Variables**
  
  It is also possible to combine an existing Northfield model with a user-defined variable such as an alpha forecast. For instance, this offers a convenient way of getting detailed analysis of the effectiveness of alpha estimates, net of the impact of a prescribed set of risk factors. Important distinctions between “bottom-up” and “top-down” strategies could be examined in this fashion.

- **Stratified Univariate and Multivariate Performance Attribution Reports**
  
  The system provides both stratified univariate attribution and multivariate factor-based analysis. Having both popular forms of attribution in the reports allows users to gain understanding of investment effects that would be hidden if either methodology were used alone. Users can also aggregate factors into groups of their own choosing, perhaps to distinguish between the effects of value and growth factors, or equity and fixed income aspects in a balanced portfolio. All reports can be viewed and printed through the Windows interface, or easily exported to Excel.

- **Detailed Reports for Risk Decomposition, both Predicted and Average Historic**
  
  All reporting procedures regarding risk estimation are entirely consistent with the methods and conventions used in Northfield’s Open Optimizer.
**Open Performance Attribution**

- **User Friendly, and Easily Handles Production Runs**

  Many of the underlying data files are shared with Northfield’s Open Optimizer. All files are in ASCII format and can be edited in the system’s Windows interface, or by using an external spreadsheet program. Data irregularities or conflicts with the problem set-up are extensively reported. In production mode, an activity log records all data sets processed, and reports problems encountered during a particular analysis run, while allowing the remainder of the batch process to proceed.

- **Flexible Observation Periods**

  The Open Performance System supports flexible observation periods such as daily or monthly. However, Northfield model data are delivered on a monthly observation basis because we believe that much conventional statistical analysis used for performance attribution is improper when daily data is involved. In a future release of the system, we anticipate incorporating a method to reconcile the results of using monthly “buy-and-hold” observations with daily observations. Alternatively, the Open system allows users to introduce their own data for their chosen observation frequencies.

- **Upcoming New Reports and Analysis**

  A “marginal contribution” is being designed to show whether securities positions that are contributing the most risk to a portfolio are also contributing the most return. From this information we can calculate a “realized information coefficient” to jointly estimate whether a manager is both good at forecasting security returns and weighting their positions so as to take best advantage of their forecasts. Other upcoming reporting items include a non-parametric measure of diversification (the J statistic), and a new analysis of portfolio turnover. Finally, we will be reporting a further delineation of factor return impacts to distinguish between the effects of consistent factor exposure bets (tilts) and the effects of factor exposure timing.
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**APPENDIX III: DESCRIPTION OF THE NORTHFIELD FUNDAMENTAL U.S. EQUITY MODEL** .......... 33
The Northfield Open Performance Attribution System is an analytical tool that uses various methods of factor attribution to determine the sources of active return of a portfolio versus a benchmark over a certain time period. These methods can be broken down into four main categories: stratification of individual factors, multivariate factor return impact, factor risk and return decomposition, and diagnostic reporting. By looking over suitable time periods, one can use inferential statistics to accept or reject the notion that a manager’s active contribution to return is demonstrative of skill versus random chance.

Whether you are a portfolio manager or a person evaluating a manager, understanding why a portfolio did well can be just as important as how well it did in an accounting sense. The portfolio manager might ask the question, “Can I demonstrate that my active investment decisions actually contributed to the portfolio’s return?”. A plan sponsor might ask, “Did the manager implement an investment strategy in line with my expectations, both in terms of methodology and at an acceptable level of risk?”. To consider these questions, we encapsulate within a context that both circumscribes the types of assets involved and the strategies most important to the investment process. A factor risk model is a useful tool for framing these types of discussions.

Risk models are used to define key characteristics, or factors, that contribute to the return on securities. Northfield has built a wide selection of models that are designed to evaluate the sensitivity of a collection of assets to risk on two levels: the systematic risk incurred by exposure to important factors, and the idiosyncratic risk that a company has by virtue of its individual properties that are independent of general market forces. One of our more popular models is the Northfield Fundamental Model. It is defined by the exposures of U.S. equities to a set of twelve style factors (e.g. Price/Earnings, Price/Book, Divided Yield, etc.) and fifty-five industry factors. Our other models include the Global Model that evaluates international securities exposures to factors such as industries, local markets, and currencies, and a series of models for single countries and regions. The US REIT Model looks at the characteristics of Real Estate Investment Trusts, and the Everything Everywhere model covers global equities, fixed income, derivatives and alternative investments for balanced portfolios, or enterprise-wide risk-adjusted performance analysis. Open Performance is designed to work not only with all of Northfield’s factor models but also ones that you have designed yourself.

By comparing the manager’s exposure to the desired factors versus the exposures of a fair index portfolio one can focus attention on the active sources of return. Typically a user of Open Performance will go in with a predetermined question. For example, “Did the portfolio’s industry bets pay off, and was stock selection within the industries a significant contributor to return”. In what follows, we will explore the various flexible ways that Open Performance can help get the answers to such questions.
Open Performance Attribution

The Main Functions of Open Performance

Open Performance performs four main functions: stratification, factor return impact analysis, factor risk and return decomposition and diagnostic reporting. For each type of action the reports are based on how the factors are collected together in groups. First, we will talk briefly about the concept of factor groupings and then we will discuss each function one at a time.

The attribution process requires that all of the factors in a model be grouped together by factor type. There are two main types of factors: continuous factors and membership factors. Continuous factors represent a quantifiable attribute of all of the assets under consideration. For example, a variable that measures a company’s size, price/book ratio, or correlation of its return to the overall market return are captured by a continuous factor. In Open Performance they are referred to as a Continuous factor. Otherwise the factor is a membership factor. Each asset in the model is or is not a member of the factor. In other words, membership factors partition the asset universe into subsets. In Open Performance the three types of Membership factors are called: Discrete, Hybrid and Fractional factors. To explain the difference between the three it is helpful to first go into more detail about the importance of groups.

As a matter of necessity, membership factors are grouped together. For example, an asset is typically considered to be in one and only one industry. But is also meaningful and necessary to think about the overall impact of all of the industry bets together. Such a collection is considered to be a set of Discrete factors, or more succinctly a Discrete Group. On the other hand, one might consider a membership collection where the total membership of an asset is spread over more than one factor in the group. For example, in the case of REIT securities, any given REIT might be spread over one more different geographical region. In that case, we say that the Region factors in the REIT Model are a collection of Fractional factors, or a Fractional Group. Finally, there are Hybrid factors and Hybrid Groups that are functionally equivalent to Discrete types but have the additional property that a scalar value is associated with any particular asset that can be analyzed in a manner similar to a Continuous factor.

Continuous factors are grouped together by convenience. They can be in their own individual group or linked together conceptually. This will impact the way the information for each factor is displayed and the organization of the reports.

For example, in the Fundamental Model, there are 67 factors. 55 of these factors are industry factors and collected into the Industry Group of the Discrete type. The other twelve factors are in two Continuous groups: the Beta Group that comprises the Beta factor only and the Fundamental Group that consists of the remaining eleven “style” factors.
In principle, each of the main functions of attribution works the same way for all factor types. How the information is calculated is dictated by Group type. The Group also organizes how it is presented in the reports. In what follows, we will describe each of the reports generated by the attribution process first in terms of its primary purpose and second by specifically addressing how it might differ for a Continuous Group and a Discrete Group. A Fundamental Model based example will be used throughout the document.

Though we will not look at Hybrid or Fractional factors, everything that applies to Discrete factors and their Groups applies to Hybrid and Fractional as well. In future releases of Open Performance reports will be generated that cater to the unique properties of these two factor types.

The Open Performance system generates two types of output files: Single Period and Aggregate Output files. In general the period reports will be monthly reports if you use a Northfield model, but this can be adjusted to fit the periodicity of your own models. The Single Period reports provide discrete ‘snapshots’ of the portfolio’s performance while the aggregate provides a picture that incorporates the total performance over the lifetime of the problem.

The example reports in this document are primarily taken from a Single (monthly) Period report. Each report in the Single Period report has an analogous report in the Aggregate report. The versions in the Aggregate report are mainly an arithmetic average of the Single Period report, plus some extra information. Therefore, for the sake of brevity, definitions from both report types are listed together in what follows.

The Aggregate report provides a crucial way to measure a manager’s level of skill: the impact t-stat. The impact values in Open Performance are an analysis of the active return in terms of various strategies a manager may employ. For example, in a univariate factor stratification report, active return is broken down by the impact of being in a particular strata (Weight Impact) and by the stock selection within that strata (Selection Impact). The t-stat for the impact values over time provides a statistical test that measures how significant the difference is between an outcome and the hypothesized average value. Significantly large t-stat values will lead one to reject the null hypothesis that there is no skill.

Stratification reports allow you to break the portfolio and benchmark into sub-portfolios based upon either variable ranges or participation in a membership variable. We follow the well-established univariate Brinson-Fachler approach to stratification. While this does not account for the simultaneous influence of other factors the way the full cross-sectional regression model does, it can be useful. For example, suppose that during a certain time period, low PE stocks did well and high PE stocks did well, but the average PE stocks
did poorly. The PE factor return in the full regression will probably be low, because the relationship between PE and returns will not be linear. However, the nonlinear relationship will be apparent in the stratification reports.

Continuous Factor Stratification Report Example

<table>
<thead>
<tr>
<th>Range</th>
<th>Min</th>
<th>Max</th>
<th>Port Wt%</th>
<th>Bench Wt%</th>
<th>Actv Wt%</th>
<th>Port Ret%</th>
<th>Bench Ret%</th>
<th>Port Contrib</th>
<th>Bench Contrib</th>
<th>Weight Impact</th>
<th>Select Impact</th>
<th>Weight</th>
<th>Select</th>
<th>WI T</th>
<th>SI T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75</td>
<td>&lt; *</td>
<td>10.08</td>
<td>5.35</td>
<td>4.73</td>
<td>5.35</td>
<td>4.87</td>
<td>0.44</td>
<td>0.18</td>
<td>0.05</td>
<td>0.12</td>
<td>2.01</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>75</td>
<td>4.14</td>
<td>2.61</td>
<td>1.53</td>
<td>4.79</td>
<td>5.42</td>
<td>0.2</td>
<td>0.14</td>
<td>0.04</td>
<td>-0.03</td>
<td>2.19</td>
<td>-1.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>50</td>
<td>2.8</td>
<td>2.97</td>
<td>-0.17</td>
<td>3.62</td>
<td>5.85</td>
<td>0.14</td>
<td>0.18</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.3</td>
<td>-0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>40</td>
<td>2.8</td>
<td>6.26</td>
<td>-3.46</td>
<td>3.74</td>
<td>4.24</td>
<td>0.1</td>
<td>0.26</td>
<td>-0.04</td>
<td>-0.01</td>
<td>-0.68</td>
<td>-0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>35</td>
<td>3.67</td>
<td>5.18</td>
<td>-1.51</td>
<td>2.19</td>
<td>-0.29</td>
<td>0.08</td>
<td>-0.02</td>
<td>0.07</td>
<td>0.09</td>
<td>1.17</td>
<td>1.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>30</td>
<td>6.87</td>
<td>14.73</td>
<td>-7.86</td>
<td>3.42</td>
<td>1.89</td>
<td>0.23</td>
<td>0.28</td>
<td>0.07</td>
<td>0.10</td>
<td>0.59</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>25</td>
<td>12.54</td>
<td>14.05</td>
<td>-1.51</td>
<td>3.17</td>
<td>1.77</td>
<td>0.37</td>
<td>0.25</td>
<td>0.02</td>
<td>0.16</td>
<td>1.52</td>
<td>3.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>20</td>
<td>20.75</td>
<td>23.93</td>
<td>-3.18</td>
<td>3.3</td>
<td>1.98</td>
<td>0.68</td>
<td>0.46</td>
<td>0.03</td>
<td>0.27</td>
<td>1.05</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>15</td>
<td>19.55</td>
<td>15.38</td>
<td>4.16</td>
<td>3.87</td>
<td>2.93</td>
<td>0.79</td>
<td>0.46</td>
<td>0.01</td>
<td>0.19</td>
<td>0.47</td>
<td>0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>&lt; *</td>
<td>10</td>
<td>16.8</td>
<td>9.54</td>
<td>7.26</td>
<td>6.57</td>
<td>3.97</td>
<td>1.46</td>
<td>0.63</td>
<td>0.26</td>
<td>0.33</td>
<td>1</td>
<td>0.91</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: P/E Stratification Report

Column Headings:

- **Range Min and Max**: the minimum and maximum values for each range
- **Port Wt %**: the percentage of the portfolio in each factor range
- **Bench Wt %**: the percentage of the benchmark in each factor range
- **Actv Wt %**: the active weight in each factor range
- **Port Ret %**: the raw return for all assets in each range weighted as specified by the user
- **Bench Ret %**: the raw return of the sub-portfolio defined by the range
- **Port Contrib**: the return impact of over- or under-weighting the benchmark within a range, calculated as: Active Weight * (Return % of the benchmark range sub-portfolio - Benchmark Return)
- **Bench Contrib**: the return impact due to stock selection within a range, calculated as: Portfolio Weight * (Return % of the portfolio range sub-portfolio - Return % of the benchmark range sub-portfolio)
- **Weight Impact**: the return impact divided by the time-series standard deviation of the monthly impact and multiplied by the square root of the number of data points.
The information is the same as is contained in a Continuous Factor Stratification report, except that the lines of demarcation across sub-portfolios is along the members in the Group. For example, in the following example it is along industry classification.

For column definitions, see P/E Stratification Report Definitions listed above (Table 1)

<table>
<thead>
<tr>
<th>Factor</th>
<th>Port Wt%</th>
<th>Bench Wt%</th>
<th>Actv W%</th>
<th>Port Ret%</th>
<th>Bench Ret%</th>
<th>Port Contrib</th>
<th>Bench Contrib</th>
<th>Weight Impact</th>
<th>Select Impact</th>
<th>WI T</th>
<th>SI T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Major Banks</td>
<td>3.8</td>
<td>9.26</td>
<td>-5.45</td>
<td>4.33</td>
<td>4.33</td>
<td>0.16</td>
<td>0.4</td>
<td>-0.08</td>
<td>0</td>
<td>-2.5</td>
<td>0</td>
</tr>
<tr>
<td>2 Regional Banks</td>
<td>2</td>
<td>0.86</td>
<td>1.14</td>
<td>3.58</td>
<td>3.49</td>
<td>0.07</td>
<td>0.03</td>
<td>0.01</td>
<td>0</td>
<td>0.63</td>
<td>0.5</td>
</tr>
<tr>
<td>3 Savings &amp; Loans</td>
<td>0.6</td>
<td>0.65</td>
<td>-0.05</td>
<td>2.53</td>
<td>1.4</td>
<td>0.02</td>
<td>0.01</td>
<td>0</td>
<td>0.01</td>
<td>1.56</td>
<td>1.72</td>
</tr>
<tr>
<td>4 Financial Services</td>
<td>3.2</td>
<td>4.6</td>
<td>-1.4</td>
<td>2.91</td>
<td>2.34</td>
<td>0.09</td>
<td>0.11</td>
<td>0.01</td>
<td>0.02</td>
<td>0.31</td>
<td>0.46</td>
</tr>
<tr>
<td>5 Financial Misc.</td>
<td>1.8</td>
<td>0.81</td>
<td>0.99</td>
<td>5.56</td>
<td>4.99</td>
<td>0.1</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>1.1</td>
<td>2.16</td>
</tr>
<tr>
<td>50 Electric Utilities</td>
<td>5.94</td>
<td>2.73</td>
<td>3.2</td>
<td>2.15</td>
<td>1.92</td>
<td>0.13</td>
<td>0.05</td>
<td>-0.02</td>
<td>0.01</td>
<td>-0.19</td>
<td>0.45</td>
</tr>
<tr>
<td>51 Gas &amp; Water Utilities</td>
<td>1.4</td>
<td>0.29</td>
<td>1.11</td>
<td>0.38</td>
<td>0.84</td>
<td>0.01</td>
<td>0</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.45</td>
<td>-0.48</td>
</tr>
<tr>
<td>52 Oil Integrated Majors</td>
<td>1.2</td>
<td>4.35</td>
<td>-3.15</td>
<td>3.01</td>
<td>1.79</td>
<td>0.04</td>
<td>0.08</td>
<td>0.03</td>
<td>0.01</td>
<td>0.79</td>
<td>1.12</td>
</tr>
<tr>
<td>53 Oil Refining &amp; Sales</td>
<td>0.6</td>
<td>0.13</td>
<td>0.47</td>
<td>1.28</td>
<td>1.37</td>
<td>0.01</td>
<td>0</td>
<td>-0.01</td>
<td>0</td>
<td>-0.68</td>
<td>-0.22</td>
</tr>
<tr>
<td>54 Oil Extraction</td>
<td>1</td>
<td>0.56</td>
<td>0.44</td>
<td>1.2</td>
<td>0.99</td>
<td>0.01</td>
<td>0</td>
<td>-0.01</td>
<td>0</td>
<td>-0.32</td>
<td>1.62</td>
</tr>
<tr>
<td>55 Oil Services</td>
<td>1.8</td>
<td>0.85</td>
<td>0.96</td>
<td>4.5</td>
<td>2.13</td>
<td>0.08</td>
<td>0.02</td>
<td>0</td>
<td>0.04</td>
<td>-0.08</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 2: Industry Factor Stratification Report

This report is identical to the Stratification Report of a Discrete Group, except that the information on the membership factors has been aggregated into sectors as defined by the user. In the following example, the industry factors have been aggregated into ten industry sectors.

For column definitions, see Table 1 above.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Port Wt%</th>
<th>Bench Wt%</th>
<th>Actv W%</th>
<th>Port Ret%</th>
<th>Bench Ret%</th>
<th>Port Contrib</th>
<th>Bench Contrib</th>
<th>Weight Impact</th>
<th>Select Impact</th>
<th>WI T</th>
<th>SI T</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENERGY</td>
<td>5.88</td>
<td>5.83</td>
<td>0.05</td>
<td>14.34</td>
<td>8.67</td>
<td>0.84</td>
<td>0.51</td>
<td>-0.04</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FINANCIAL</td>
<td>11.76</td>
<td>20.98</td>
<td>-9.21</td>
<td>2.84</td>
<td>5.28</td>
<td>0.33</td>
<td>1.11</td>
<td>0.02</td>
<td>-0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TEL&amp;UTILITIES</td>
<td>23.53</td>
<td>7.14</td>
<td>16.39</td>
<td>11.83</td>
<td>8.4</td>
<td>2.78</td>
<td>0.6</td>
<td>0.37</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRANSPORTATION</td>
<td>0</td>
<td>1.24</td>
<td>-1.24</td>
<td>0</td>
<td>2.88</td>
<td>0</td>
<td>0.04</td>
<td>0.03</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TECHNOLOGY</td>
<td>11.76</td>
<td>18.05</td>
<td>-6.28</td>
<td>13.35</td>
<td>8.35</td>
<td>1.57</td>
<td>1.51</td>
<td>-0.06</td>
<td>0.46</td>
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<tr>
<td>HEALTH</td>
<td>0</td>
<td>14.94</td>
<td>-14.94</td>
<td>0</td>
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<td>0.36</td>
<td>0.46</td>
<td>0</td>
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</tr>
<tr>
<td>CONS_STAPLE</td>
<td>23.53</td>
<td>13.89</td>
<td>9.64</td>
<td>4.51</td>
<td>4.74</td>
<td>1.06</td>
<td>0.06</td>
<td>-0.26</td>
<td>0.15</td>
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<td></td>
</tr>
<tr>
<td>CONS_DISCRETE</td>
<td>0</td>
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<td>-5</td>
<td>0</td>
<td>6.77</td>
<td>0</td>
<td>0.34</td>
<td>-0.07</td>
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<td></td>
</tr>
<tr>
<td>BASIC INDUSTRY</td>
<td>17.65</td>
<td>10.65</td>
<td>6.99</td>
<td>7.62</td>
<td>2.12</td>
<td>1.35</td>
<td>0.23</td>
<td>1.05</td>
<td>-0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MISC</td>
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<td>2.28</td>
<td>3.6</td>
<td>7.53</td>
<td>5.06</td>
<td>0.44</td>
<td>0.12</td>
<td>0.1</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Industry Sector Stratification Report
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Stratification by Asset

This report shows for each asset in the portfolio and the benchmark: portfolio, benchmark, and active weights, the monthly return, the contribution the stock made to total return, and the impact of the holding on the relative return of the portfolio. It is available in the single period and aggregate report.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Port Wgt</th>
<th>Bench Wgt</th>
<th>Actv Wt</th>
<th>Return</th>
<th>Port Contrib</th>
<th>Bench Contrib</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.88</td>
<td>0.09</td>
<td>5.79</td>
<td>13.17</td>
<td>0.77</td>
<td>0.01</td>
<td>0.76</td>
</tr>
<tr>
<td>AA</td>
<td>0</td>
<td>0.23</td>
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<td>7.33</td>
<td>0</td>
<td>0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>AAPL</td>
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<td>0.06</td>
<td>-0.06</td>
<td>26.23</td>
<td>0</td>
<td>0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>5.88</td>
<td>0</td>
<td>5.88</td>
<td>-7.22</td>
<td>-0.42</td>
<td>0</td>
<td>-0.42</td>
</tr>
<tr>
<td>BA</td>
<td>0</td>
<td>0.26</td>
<td>-0.26</td>
<td>13.05</td>
<td>0</td>
<td>0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>BAC</td>
<td>0</td>
<td>1.33</td>
<td>-1.33</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>C</td>
<td>5.88</td>
<td>2.43</td>
<td>3.45</td>
<td>5.02</td>
<td>0.3</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>CA</td>
<td>0</td>
<td>0.11</td>
<td>-0.11</td>
<td>33.44</td>
<td>0</td>
<td>0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>ZMH</td>
<td>0</td>
<td>0.11</td>
<td>-0.11</td>
<td>-4.35</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Stratification by Asset Report

Factor Return Impact Reporting

The Factor Return Impact Report is a decomposition of the alpha contribution that was due to the manager’s active factor exposures. This multivariate approach affords a glimpse into the investor’s performance that is mindful of the interplay of the factors in question.

A key element in this process is the factor return. The factor return is generated by an appropriate regression methodology over the factors in the whole model. In the Fundamental model, the 12 fundamental factors have priority over the industry group variables. A regression is first run on the 12 factors. An industry regression is then run on the residuals (or unexplained returns) from the factor regression. The sample size of some industries is so small that to assign those industries the same priority as the factors could lead to misleading results.
**Table 5: The Fundamentals Factor Return Impacts Report**

**Column Definitions:**

PORTFOLIO EXPOSURE = The (cap or equal) weighted average of each asset sensitivity to a factor. The weighted average is referred to as the portfolio factor exposure or portfolio factor beta.

BENCHMARK EXPOSURE = For all assets in the Benchmark, the (cap or equal) weighted average of each stock's sensitivity to a factor.

ACTIVE EXPOSURE = Portfolio Exposure - The Benchmark Exposure.

FACTOR RETURN = Return attributed to that factor based on a regression across the Northfield universe.

FACTOR IMPACT = (Active exposure) * (Factor return).

IMPACT T-STAT = (aggregate only) The mean of the monthly factor impact divided by the time-series standard deviation of the monthly impact and multiplied by the square root of the number of data points.
### The Industries Return Impacts Report

<table>
<thead>
<tr>
<th>Factor</th>
<th>Port Exposure</th>
<th>Bench Exposure</th>
<th>Active Exposure</th>
<th>Factor Ret</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Banks</td>
<td>5.56</td>
<td>9.19</td>
<td>-3.64</td>
<td>-1.07</td>
<td>0.0388</td>
</tr>
<tr>
<td>Regional Banks</td>
<td>0</td>
<td>0.86</td>
<td>-0.86</td>
<td>-0.72</td>
<td>0.0061</td>
</tr>
<tr>
<td>Savings &amp; Loans</td>
<td>0</td>
<td>0.66</td>
<td>-0.66</td>
<td>-1.76</td>
<td>0.0116</td>
</tr>
<tr>
<td>Financial Services</td>
<td>0</td>
<td>4.64</td>
<td>-4.64</td>
<td>-0.03</td>
<td>0.0014</td>
</tr>
<tr>
<td>Financial Misc.</td>
<td>5.56</td>
<td>0.81</td>
<td>4.74</td>
<td>-2.09</td>
<td>-0.0992</td>
</tr>
<tr>
<td>Insurance Life</td>
<td>0</td>
<td>0.67</td>
<td>-0.67</td>
<td>-5.66</td>
<td>0.0377</td>
</tr>
<tr>
<td>Insurance Other</td>
<td>0</td>
<td>4.15</td>
<td>-4.15</td>
<td>-0.21</td>
<td>0.0085</td>
</tr>
<tr>
<td>Building Construction</td>
<td>0</td>
<td>0.15</td>
<td>-0.15</td>
<td>4.87</td>
<td>-0.0074</td>
</tr>
<tr>
<td>Building Materials</td>
<td>0</td>
<td>0.37</td>
<td>-0.37</td>
<td>-1.33</td>
<td>0.005</td>
</tr>
<tr>
<td>Forest Products</td>
<td>0</td>
<td>0.25</td>
<td>-0.25</td>
<td>-1.59</td>
<td>0.004</td>
</tr>
<tr>
<td>Gas &amp; Water Utilities</td>
<td>0</td>
<td>0.28</td>
<td>-0.28</td>
<td>-0.54</td>
<td>0.0015</td>
</tr>
<tr>
<td>Oil Integrated Majors</td>
<td>0</td>
<td>4.36</td>
<td>-4.36</td>
<td>5.93</td>
<td>-0.2585</td>
</tr>
<tr>
<td>Oil Refining &amp; Sales</td>
<td>5.56</td>
<td>0.13</td>
<td>5.42</td>
<td>6.49</td>
<td>0.3518</td>
</tr>
<tr>
<td>Oil Extraction</td>
<td>0</td>
<td>0.53</td>
<td>-0.53</td>
<td>3.36</td>
<td>-0.0179</td>
</tr>
<tr>
<td>Oil Services</td>
<td>0</td>
<td>0.8</td>
<td>-0.8</td>
<td>7.4</td>
<td>-0.0594</td>
</tr>
</tbody>
</table>

Table 6: The Industries Return Impacts Report

### Column Definitions

- **PORTFOLIO EXPOSURE** = The percent of the portfolio in each factor (in this example, each industry).
- **BENCHMARK EXPOSURE** = The percent of the benchmark in each factor (here, each industry).
- **ACTIVE EXPOSURE** = Portfolio Exposure - Benchmark Exposure.
- **FACTOR RETURN** = The return attributed to that factor based on a regression across the Northfield universe.
- **FACTOR IMPACT** = (Active exposure) * (Factor return)
- **IMPACT T-STAT** = (aggregate only) The mean of the monthly factor impact divided by the time-series standard deviation of the monthly impact and multiplied by the square root of the number of data points.
This report is identical to the Factor Return Impact Report of a Discrete Group, except that the information on the membership factors has been aggregated into sectors as defined by the user. In the following example, the industry factors have been aggregated into ten industry sectors.

### Sector Return Impact Report Example

<table>
<thead>
<tr>
<th>Factor</th>
<th>Port Exposure</th>
<th>Bench Exposure</th>
<th>Active Exposure</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENERGY</td>
<td>5.56</td>
<td>5.83</td>
<td>-0.27</td>
<td>0.016</td>
</tr>
<tr>
<td>FINANCIAL</td>
<td>11.11</td>
<td>20.98</td>
<td>-9.87</td>
<td>0.0049</td>
</tr>
<tr>
<td>TEL&amp;UTILITIES</td>
<td>22.22</td>
<td>7.14</td>
<td>15.08</td>
<td>0.0444</td>
</tr>
<tr>
<td>TRANSPORTATION</td>
<td>0</td>
<td>1.24</td>
<td>-1.24</td>
<td>0.0001</td>
</tr>
<tr>
<td>TECHNOLOGY</td>
<td>11.11</td>
<td>18.05</td>
<td>-6.94</td>
<td>0.0095</td>
</tr>
<tr>
<td>HEALTH</td>
<td>0</td>
<td>14.94</td>
<td>-14.94</td>
<td>-0.5767</td>
</tr>
<tr>
<td>CONS_STAPLE</td>
<td>22.22</td>
<td>13.89</td>
<td>8.33</td>
<td>-0.1621</td>
</tr>
<tr>
<td>CONS_DISCRET</td>
<td>0</td>
<td>5</td>
<td>-5</td>
<td>0.0913</td>
</tr>
<tr>
<td>BASIC INDUSTRY</td>
<td>16.67</td>
<td>10.65</td>
<td>6.01</td>
<td>-0.0225</td>
</tr>
<tr>
<td>MISC</td>
<td>5.56</td>
<td>2.28</td>
<td>3.28</td>
<td>-0.1149</td>
</tr>
</tbody>
</table>

Table 7: The Industries Sector Return Impacts Report

For column definitions, see Table 6 above.

### Risk and Return Decomposition Reporting

This report analyzes the sources of risk defined as the variance in the difference of returns of the portfolio vs. the benchmark. The first portion gives an assessment of risk, as measured by contributed variance for all of the factors in the model. The risk predicted is based on factor exposures as of the end of the period, and on the historical volatilities associated with the factors. The second portion gives a list of summary statistics about the overall risk characteristics of the portfolio and benchmark.
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Risk Decomposition Report Example

<table>
<thead>
<tr>
<th>Factor</th>
<th>Active Exposure</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Beta</td>
<td>0.12</td>
<td>6.57</td>
</tr>
<tr>
<td>2 Price/Earnings</td>
<td>0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td>3 Price/Books</td>
<td>-0.88</td>
<td>3.94</td>
</tr>
<tr>
<td>4 Dividend Yield</td>
<td>0.38</td>
<td>3.14</td>
</tr>
<tr>
<td>5 Trading Activity</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>6 Relative Strength</td>
<td>-0.19</td>
<td>8.52</td>
</tr>
<tr>
<td>7 Market Cap</td>
<td>-1.18</td>
<td>22.29</td>
</tr>
<tr>
<td>8 Earnings Variability</td>
<td>0.52</td>
<td>1.6</td>
</tr>
<tr>
<td>9 EPS Growth Rate</td>
<td>-0.1</td>
<td>0.62</td>
</tr>
<tr>
<td>10 Price/Revenue</td>
<td>-0.57</td>
<td>18.89</td>
</tr>
<tr>
<td>63 Gas &amp; Water Utilities</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>64 Oil Integrated Majors</td>
<td>-0.04</td>
<td>0.94</td>
</tr>
<tr>
<td>65 Oil Refining &amp; Sales</td>
<td>0.05</td>
<td>-0.42</td>
</tr>
<tr>
<td>66 Oil Extraction</td>
<td>-0.01</td>
<td>0.15</td>
</tr>
<tr>
<td>67 Oil Services</td>
<td>-0.01</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Factor Tracking Variance = 108.51
Stock Specific Tracking Variance: 138.31
Total Tracking Variance = 246.83
Tracking Error = 15.71
Total Risk of Port = 26.99
Total Risk of Bench = 23.36

Table 8: The Risk Decomposition Report

Risk Decomposition Definitions:

ACTIVE EXPOSURE = Portfolio Exposure - Benchmark Exposure

RISK = Contributed Variance = The Active Exposure $^2$ * (The Factor Variance) + Covariance effects with other Factors

Factor Tracking Variance = The sum of the factor variances

Stock Specific Tracking Variance = The portion of total variance that arises because the specific stocks of the portfolio and the specific stocks of the benchmark are different. Equal to the active weight of each stock squared times the variance of its stock specific returns

Total Tracking Variance = Factor Tracking Variance + Stock Specific Tracking Variance

Tracking Error = The square root of the Total Tracking Variance

Total Risk of Port = The square root of the estimated portfolio variance

Total Risk of Bench = The square root of the estimated benchmark variance
The return decomposition report provides a grand summary of all sources of excess return. The raw return on the portfolio and benchmark is displayed along with the active return. The active return is broken down into two portions: Factor Model Specific and Stock Specific. The Factor Model Specific portion is the sum of factor return impacts; it encapsulates the active exposures the manager had to the various factors. The Stock Specific portion is a reflection of the manager’s stock selection ability.

The Factor Model Specific portion is further decomposed into parts based on subgroups of the factors in the model, as seen in the Factor Return by Group section of the report. The return value for each group is the sum of the factor return impact values for that subgroup.

<table>
<thead>
<tr>
<th>Return Decomposition</th>
<th>Return</th>
<th>Model SD</th>
<th>Tstat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Free Rate</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Port</td>
<td>7.92</td>
<td>7.79</td>
<td>1.02</td>
</tr>
<tr>
<td>Bench</td>
<td>5.35</td>
<td>6.74</td>
<td>0.79</td>
</tr>
<tr>
<td>Total Active</td>
<td>2.57</td>
<td>4.54</td>
<td>0.57</td>
</tr>
<tr>
<td>Benchmark Excess</td>
<td>5.26</td>
<td>6.74</td>
<td>0.78</td>
</tr>
<tr>
<td>Stock Specific</td>
<td>-0.67</td>
<td>3.39</td>
<td>-0.2</td>
</tr>
<tr>
<td>Factor Model Specific</td>
<td>3.24</td>
<td>3.01</td>
<td>1.08</td>
</tr>
<tr>
<td>Factor Return by Group</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Industries</td>
<td>-0.71</td>
<td>1.15</td>
<td>-0.62</td>
</tr>
<tr>
<td>The Fundamentals</td>
<td>2.91</td>
<td>2.68</td>
<td>1.09</td>
</tr>
<tr>
<td>Beta</td>
<td>1.04</td>
<td>0.74</td>
<td>1.4</td>
</tr>
<tr>
<td>Asset Portion (Port)</td>
<td>7.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Portion (Port)</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reported Port Return</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unidentified (Port)</td>
<td>5.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Port Return (without cash)</td>
<td>8.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Portion (Bench)</td>
<td>5.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Portion (Bench)</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reported Bench Return</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unidentified (Bench)</td>
<td>5.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bench Return (without cash)</td>
<td>5.35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: The Return Decomposition Report

Return Decomposition Definitions:

Risk Free Rate = Return on cash. In the Fund Model, the return on a three month T-bill
Port Return = Return of Portfolio based on all assets identified
Bench Return = Return of Benchmark based on all stocks identified
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Total Active = Port Return – Bench Return
Benchmark Excess = Bench Return – Risk Free Rate
Stock Specific Return = idiosyncratic portion of active return
Factor Model Specific = systematic portion of active return
Factor Return by Group = breakdown of the systematic return by factor group
Asset Portion (Port or Bench) = portion of return from non-cash assets
Cash Portion (Port or Bench) = portion of return from cash
Reported (Port or Bench) Return = a return value supplied externally by the user
Unidentified (Port or Bench) = Return – Reported Return
(Port or Bench) Return (without cash) = return on the holdings with cash ignored

Model SD = the standard deviation that the factor model predicts for the return type. The standard deviation for a particular return is:

\[
\text{SQRT(} \text{Est. var. } \text{)} / \text{SQRT (Periodicity term)}
\]

where:
Est. var. = an estimated variance. The estimated variance is a function of the Risk Decomposition process and the context of the particular return. For further elaboration on how variance is estimated, please read Appendix II: Understanding Model Standard Deviation.

Periodicity term = the number of per-period data points in a year used by the factor model (for the purposes of annualization). For example, the Fundamental Model uses monthly data points; therefore the Periodicity term is always 12. If you were using a daily model, then it would equal the number of business days in the year.

T-stat (single period) = (Return) / (Model SD).

Under certain circumstances, the estimated variance by Group may be negative. Therefore the Model SD and T-stat values are undefined. In this case the Model SD and T-stat value are both reported as “N/A”.

Aggregate Only Definitions

The following columns (not shown in the example) appear in the Aggregate file only:

- CUM RET: the geometric mean of the returns over time
- REAL SD: the standard deviation of the returns over time

Interaction effect = (Portfolio CUM RET) - (Benchmark CUM RET) - (Active CUM RET)
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Diagnostic Reports
The various reports not covered above. Some are Group type specific reports, views of information over time, and other tools to judge the efficacy of the attribution process.

Factor Summary Report (Continuous Groups only)
This report displays the weighted average of the raw factor loadings for the portfolio and the benchmark.

All fundamental factor loadings for the Northfield Fundamental Factor Model (FFM) are recalculated monthly. Each of the eleven factors is presented in standardized form, using the mean and cross-sectional standard deviation of each factor value from a reference universe. The reference universe consists of all common stocks with a minimum capitalization of $250 million. During the regression analysis, used to calculate returns to these factors, the influence of each stock in the reference universe is weighted by the square root of the market capitalization.

For factors Price/Earnings, Price/Book and Price/Revenue, the actual calculations are done with the reciprocals of the values in order to avoid dividing by zero (e.g. P/E is measured as E/P). Once the appropriate weighted average values are calculated across a portfolio, the values are again inverted to their original form (e.g. standardized E/P back to standardized P/E).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Port</th>
<th>Bench</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price/Earnings</td>
<td>20.8</td>
<td>20.69</td>
</tr>
<tr>
<td>Price/Books</td>
<td>1.26</td>
<td>2.8</td>
</tr>
<tr>
<td>Dividend Yield</td>
<td>2.82</td>
<td>1.8</td>
</tr>
<tr>
<td>Trading Activity</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>Relative Strength</td>
<td>0.9</td>
<td>1.03</td>
</tr>
<tr>
<td>Market Cap</td>
<td>25333.66</td>
<td>84976.79</td>
</tr>
<tr>
<td>Earnings Variability</td>
<td>0.71</td>
<td>0.5</td>
</tr>
<tr>
<td>EPS Growth Rate</td>
<td>9.46</td>
<td>11.24</td>
</tr>
<tr>
<td>Price/Revenue</td>
<td>0.47</td>
<td>1.34</td>
</tr>
<tr>
<td>Debt/Equity</td>
<td>1.15</td>
<td>0.76</td>
</tr>
<tr>
<td>Price Volatility</td>
<td>0.33</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 10: The Fundamentals Factor Summary Report

What remains after accounting for factor specific returns is the stock-specific returns. This report is useful in evaluating analyst performance because it measures performance relative to the benchmark's returns by each member in the group. For example, in the Fundamental Model Industry Group, the average of the residual returns for all stocks in an industry is the industry return. For each industry, the universe stock-specific return is zero.

The Net value in the report provides information on an analyst's stock picking abilities. Together with the standard deviation of Net this can, over time, help assess analyst performance.
### Table 11: The Industries Specific Return By Factor Report

#### Specific Return by Factor Report Definitions:

Portfolio Stock-specific Return = The equal-weighted average specific return of portfolio stocks.

Benchmark Specific Return = The equal weighted average specific return of benchmark stocks. Note that unlike for the universe, this number need not be zero for the benchmark.

Net = Portfolio specific return  - Benchmark specific return
Issue Detail Report
This report shows, for all assets in the portfolio, the exposures to all of the factors asset by asset.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Beta</th>
<th>Price/Earnings</th>
<th>Price/Book</th>
<th>Dividend Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.23</td>
<td>-1.11</td>
<td>-0.07</td>
<td>-0.72</td>
</tr>
<tr>
<td>B</td>
<td>0.48</td>
<td>0.56</td>
<td>-0.24</td>
<td>0.76</td>
</tr>
<tr>
<td>C</td>
<td>1.16</td>
<td>0.64</td>
<td>-0.38</td>
<td>0.06</td>
</tr>
<tr>
<td>D</td>
<td>0.64</td>
<td>0.92</td>
<td>-0.1</td>
<td>0.99</td>
</tr>
<tr>
<td>E</td>
<td>0.26</td>
<td>1.62</td>
<td>-0.36</td>
<td>1.46</td>
</tr>
<tr>
<td>F</td>
<td>1.1</td>
<td>-0.42</td>
<td>-0.65</td>
<td>0.8</td>
</tr>
<tr>
<td>G</td>
<td>0.43</td>
<td>-0.18</td>
<td>-1.12</td>
<td>0.1</td>
</tr>
<tr>
<td>K</td>
<td>0.04</td>
<td>0.21</td>
<td>-1.13</td>
<td>0.49</td>
</tr>
<tr>
<td>L</td>
<td>1.71</td>
<td>-1.11</td>
<td>0.47</td>
<td>-0.72</td>
</tr>
<tr>
<td>N</td>
<td>1.41</td>
<td>-1.11</td>
<td>1</td>
<td>-0.72</td>
</tr>
<tr>
<td>O</td>
<td>0.36</td>
<td>0.21</td>
<td>-0.28</td>
<td>1.69</td>
</tr>
<tr>
<td>Q</td>
<td>2.54</td>
<td>-1.11</td>
<td>4</td>
<td>-0.72</td>
</tr>
<tr>
<td>R</td>
<td>0.58</td>
<td>0.72</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>S</td>
<td>1.32</td>
<td>2.82</td>
<td>0.3</td>
<td>0.53</td>
</tr>
<tr>
<td>T</td>
<td>2.24</td>
<td>-1.11</td>
<td>0.65</td>
<td>0.99</td>
</tr>
<tr>
<td>V</td>
<td>1.95</td>
<td>-1.11</td>
<td>2.6</td>
<td>1.38</td>
</tr>
<tr>
<td>X</td>
<td>1.37</td>
<td>0.72</td>
<td>1.63</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

Table 12: The Issue Detail Report

Asset Weights Summary Report
An accounting of how the weights of assets in the portfolio and benchmark are assigned with cash and without cash.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Cash excluded</th>
<th>Cash included</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>$$</em></td>
<td>5.56</td>
<td>5.56</td>
</tr>
<tr>
<td>A</td>
<td>5.88</td>
<td>5.56</td>
</tr>
<tr>
<td>B</td>
<td>5.88</td>
<td>5.56</td>
</tr>
<tr>
<td>C</td>
<td>5.88</td>
<td>5.56</td>
</tr>
<tr>
<td>D</td>
<td>5.88</td>
<td>5.56</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 13: Portfolio Weights Summary Report
This report displays any security identifiers not recognized by the system. The usual reasons for identifiers not being recognized are a recent ticker change, a difference in handling identifier formats, or inclusion of a security not covered in the model.

<table>
<thead>
<tr>
<th>Portfolio Exceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>H not found in database file d:\NorthInfo\data\f20030430.csv</td>
</tr>
<tr>
<td>H not found in return file d:\NorthInfo\data\tret20030531.csv</td>
</tr>
</tbody>
</table>

Benchmark Exceptions
no exceptions

Table 14: Exception Report

For multiple period aggregate reports, the following return values are reported for each time period.

1. Risk Free
2. Portfolio Return
3. Benchmark Return
4. Active Return
5. Benchmark Excess
6. Stock Specific
7. Factor Model Specific
8. Return By Factor Group

For multiple period aggregate reports, the following risk forecasts are reported for each time period.

1. Factor Tracking Variance
2. Stock Specific Tracking Variance
3. Total Tracking Variance
4. Tracking Error
5. Total Risk of Portfolio
6. Total Risk of Benchmark
7. Tracking Variance by Factor Group

For each of the factors in the risk model, the active portfolio exposure is given for each time period. This exposure is in the normalized factor values used by the risk model, not converted to measured values as it is in the aggregate factor exposure reports.
The purpose of this document is to explain how to construct a problem for use in the NIS Open Performance Attribution System. It goes through a sample .perf file line-by-line, explains what each parameter is, suggests acceptable choices for the parameter and describes any file formats associated with the parameter. Further details are available in the help file.

The Sample Problem using the Northfield Fundamental US Equity Model provided with the analytical system and discussed here.

The sample problem (“Nisperf32_fund.perf”) is a user-defined portfolio (portfolio code: “port”) compared to a user defined benchmark portfolio (portfolio code: “bench”). Both are equal-weighted in this simple example. Monthly attribution reports will be generated for the period spanning July 2001 through September 2001. The attribution will be considered in the context of the NIS Fundamental Factor Model. An aggregate of the three monthly reports will also be created.

The various factors of the model are grouped together by type. There are four types of Factors: Continuous, Discrete, Hybrid, and Fractional.

These collections of factors are called Groups. It is essential that every Factor in your model belong to one (and only one) Group. You can specify a Group that contains only one Factor. Any non-Continuous-Factor Group (i.e. a Group of Discrete Factors) can be further collected into Sectors.

In the Fundamental Model there are two Groups: the Industry Group (a collection of Discrete Factors) and the Fundamental Group (a collection of Continuous Factors).

Each boldface heading in this section represents a line item in our ‘Nisperf32_fund.perf’ file. The ordering of the items does not matter. Blank lines and lines starting with “#” are ignored. Unless indicated, each line must be represented in the .perf file, and no specified parameter’s value should be empty.

Number of Factors=67

The total number of distinct Factors in the model. The Fundamental Model has 67 Factors. This number must equal the total number of rows in the Factor Description File. (In fact, the Model Variance File and Correlation File have the same row count as well. The Distributional Parameter File will be the same row count (plus one if you include the optional “RISKFREE” row).

Number of Groups=3

Number of Factor Groups in the model. The Fundamental Model has three Groups. This must match the number of rows in the Group Description File.
Default Working Directory=c:\NisPerf32\fund

If a file parameter is not explicitly given a full file path, then the default path will be used

Start Date =2001/07/31

The Start Date must be specified in YYYY/MM/DD format. Dates are injected into file names in place of the %DATE% string (with the ‘/’s removed).

The Start Date parameter value is set for the end of the month (June in this case), even though the example problem is considering attribution for the whole month of July. By convention NIS monthly model data is collected at the end of the month, and the naming convention is implicit that the data is from the end of month. The system will look at the factor model data set for the prior month, and is assumed that the end of the prior month will be an accurate reflection of the beginning of the month.

End Date=2001/09/03

The End Date is specified strictly in YYYY/MM/DD format. Dates are injected into file names in place of the %DATE% string (with the ‘/’s removed).

Period =Monthly

The periodicity of the problem. For this example end-of-month ‘snapshots’ of model data are used. This is signified by the keyword ‘MONTHLY’ (strictly all-caps). Other periods are also allowed.

Portfolio File=port%DATE%.hld

The naming convention of the user’s portfolio. You are specifying three distinct strings: the portfolio code, date string and the file extension. The portfolio code can be as long as you want (subject to how your OS limits the length of a filename). The “%DATE%” string indicates where the date string is placed in the file name. The extension can be whatever you want (though keep in mind that every input data file must be in CSV format).

If you do not place the “%DATE%” string into the parameter then the system will assume that you are pointing to a static portfolio throughout the problem.

All portfolio files are in CSV format. There are two columns: Symbol (as in ticker,sedol,cusip, etc. and Weight (some appropriate numerical value). If there are quotes or spaces around the symbol then they will be ignored. The symbol for the asset must match exactly (including case) to the symbol in the Model Database File or it will be considered an exception.
**Portfolio Weight=Equal?Percent**

The weighting scheme for the user’s portfolio. In this case the portfolio is equal weighted and uses the ‘EQUAL’ keyword: All keywords are strictly uppercase. All weighting schemes ignore portfolio exceptions. The following valid keywords are:

- **PERCENT**: reported weight represents the percentage of the asset in the portfolio. The total value of the weights should equal 100. If they do not the system will rescale the weights to 100.

- **PERCENT_O**: reported weight represents the percentage of the asset in the portfolio. The total value of the weights should equal 100. If they do not the system will place the unaccounted value into cash.

- **EQUAL**: each asset receives an equal percentage (reported weights are ignored).

- **SHARES**: reported value represents the number of shares of the asset in the portfolio. For each asset, the number of shares is multiplied by the asset price reported in the model database file, and are then scaled to percentage values.

- **MKTCAP or MRKTCAP**: reported weights are ignored. For each asset the reported shares outstanding and the reported asset price in the model database file are multiplied. These values are scaled to percentage values.

The weighting scheme influences what assets are considered exceptions (see **When is an Asset an Exception?** below).

**Benchmark File=bench%DATE%.hld**

The naming convention of the desired benchmark portfolio. See Portfolio File entry above. If you are using a long-short portfolio the keyword “LONGSHORT” is used here. See the **LONG/SHORT** section of the Appendix for more details.

**Benchmark Weight=MKTCAP**

The weighting scheme for the benchmark portfolio. See Portfolio Weight descriptions above.
Group Description File=fund_grp.csv
The file that outlines the nature of the Groups. This file is in CSV format. Each row of the file represents a single Group. The columns are:

1) Factor key: a case-sensitive string key that is used to match factors to a Group in the Factor Description File.

2) Report label: string used to label the Group in the output reports.

3) Factor type: a string representing the type of factor for the Group (keyword CONTINUOUS, DISCRETE, HYBRID, or FRACTIONAL).

4) Number of sectors: whole number value indicating the number of sectors for the Group. If the factor type is DISCRETE or HYBRID then you can assign sectors. Enter ‘0’ in this field if you want to skip sectoring.

[5 + number of sectors – 1]) Sector labels: a series of string entries. Each entry is used as the label for the sectors in the output reports. Total columns should equal the number of sectors in column four.

Factor Description File=fundamental.csv
Describes the fixed characteristics of the factor model. Each row represents a factor. The order of the factor-rows must match the column order of the factors in the Model Database File. The columns are:

1) Report label: a string used to label the Factors in the output reports.

2) Group key: a string that indicates which Group the Factor belongs. This string should match one of the entries in the first column of the Group Description File.

3) Factor type: a string representing the Factor type of Factor (keyword CONTINUOUS, DISCRETE, HYBRID, or FRACTIONAL).

4) Transform code: a string indicating what kind of transformation is required to the factor values in the Model Database File. If none is required then enter a ‘0’ in this field. The legal keywords are:

a) ANTILOG: The factor is stored as the logarithm (base 10) of the actual factor value. The antilog will be taken as the proper value ($10^{\text{reported factor value}}$).
b) INVERT: If the factor value is a numerical representation of a ratio then you may invert the ratio if you wish. For example if the Factor is stored as “Revenue/Price” you might want to consider “Price/Revenue” instead.

5) Sector key: a nonzero positive integer that represents which sector the factor belongs. The number corresponds to the relative order the sector label appears in the Group row in the Group Description File.

[6 + -1]) Stratification buckets: Applies only to Factors of type Continuous. A series of column values descending from left to right. Represents the cut-off points for membership in each stratum of the Stratification reports.

Distributional Parameter File=f_dpf%DATE%.csv

Describes the characteristics of the factor model that change over the life of the problem. The file is in CSV format. Each row represents values for a single Factor. The order of the factor rows must match the column order of the factors in the Model Database File.

There are four fixed columns:

1) Factor name: a string representing the factor name.

2) Factor mean: a numerical value for the mean value of the factor. Required if the factor values are standardized (i.e. The Continuous Factors in the NIS Fundamental Model). Place a ‘0’ here if the factor values do not require de-standardization.

3) Factor standard deviation: a numerical value for the standard deviation of the factor. Required if the factor values are standardized (i.e. The Continuous Factors in the NIS Fundamental Model). Place a ‘1’ here if the factor values do not require de-standardization.

4) Factor return: a numerical value representing the model return for the Factor.

If you know the risk-free rate for the period then you report it in this file using the keyword ‘RISKFREE’ in the Factor Name field and placing the numerical return value in the Factor Return field (place a ‘0’ in the other two fields).

Model Database File=f%DATE%.csv

This file conforms to the standard Model Database File (‘.csv’) format used in the NIS Open Optimizer. If there are quotes around the asset symbol then they will be ignored.
Model Variance File=f%DATE%.mdl
This file conforms to the standard Model Variance File (‘.mdl’) format used in the NIS Open Optimizer. The order of the factor rows in this file must match the column order of the factors in the Model Database File.

Correlation File=f%DATE%.cor
This file conforms to the standard Correlation File (‘.csv’) format used in the NIS Open Optimizer. Row and column order must conform to the column factor order in the Model Database File. If you do not place the “%DATE%” string into the parameter then the system will assume that you are pointing to a static '.cor' file throughout the problem.

Return File=tret%DATE%.csv
A file for each period that contains the asset returns. In csv format with two columns:

1) Asset symbol: If there are quotes around the asset symbol then they will be ignored. The symbol must match an asset in the Model Database File.

2) Asset return: numerical value representing the asset return.

Output File Per Period=out_%DATE%.csv
Specifies the naming convention for the per-period output files.

Aggregate Output File=outrevt.csv
Specifies the filename for the aggregate report.

Log=log.txt
Specifies the filename for the log. If no filename is given then the system defaults to ‘DEFAULT_ERROR.csv’. This is especially useful if the system reports any errors before the desired log filename is determined.
The Return Decomposition Report provides a grand summary of all sources of excess return. It consists of various types of returns, i.e. portfolio, benchmark and factor returns. For each return type, a value is displayed labeled “Model SD”. This is short for “Model Standard Deviation of Return.” The purpose of the following is to explain the motivation for the Model SD values and how it is calculated by return type.

For the sake of discussion, let us assume that the only information at one’s disposal is a series of monthly portfolio returns. If we wanted to measure the risk associated with the portfolio then we would calculate the standard deviation of the return series. In this way we get a measurement of how volatile the portfolio has been over the time period analyzed. In an Open Performance Aggregate Report file, the standard deviation of the portfolio is reported in the “Real SD” column. It is “Real” in the sense that the standard deviation is calculated in the normal ‘text-book’ fashion: it is a function of the return series available. This holds true for the other return types in the Return Decomposition Report.

However, how can we calculate a standard deviation for the portfolio in a Single Period Report? The ‘text-book’ method cannot apply, since we only have one data point at our disposal. Here is where Model SD comes into the picture. It is a “Model” standard deviation in the sense that we have to introduce information about our factor model in order to calculate an estimation of risk at that point in time. How this volatility is estimated varies for each return type in the Return Decomposition Report.

Calculating Model SD

In a Single Period Return Decomposition report, the Model SD per return is derived from an estimated variance. In general:

\[
Model \ SD_i = \frac{\sqrt{Est\ Var_i}}{\sqrt{A}}
\]

where:

\[
Est\ Var_i = \text{Estimated Variance of Return type } i
\]

A = Annual Term. This is the number of data points used by the factor model per year. For most Northfield models, A equals 12.

What follows is an accounting of how the estimated variance is calculated per return type.
The portfolio, benchmark and active return estimated variance calculations all follow a similar pattern. They are all calculated in two parts: the factor risk portion and the stock specific portion. In this way systematic and idiosyncratic risk, respectively, are taken into account.

For each return type, the estimated variance is a function of the stocks held and their weights in the portfolio, benchmark, or both (in the case of the active return). Information specific to the factors (independent of assets held) is used as well. Before we detail the actual calculations, let us give a verbal description of the pieces of information that will be used.

**Factor Exposure**

For each factor in the model, the factor exposure is a summation a collection of securities:

\[ \sum_i w_i e_i \]

where:

- \( w_i \) = weight of security \( i \)
- \( e_i \) = factor loading of security \( i \) to the factor

In a Northfield Model, the factor loadings of a security are listed in the Optimizer asset database file (\*.csv file).

This formula applies to the portfolio, benchmark and active exposure. The weight refers to either the portfolio, benchmark or active weight (portfolio – benchmark weight).

**Reported Factor Variance**

The annualized variance of the return to the factor as predicted by the risk model. In a Northfield Model, this is provided in the Optimizer Model file (\*.mdl file).

**Factor Correlation Coefficient**

The correlation coefficient between two factors. In a Northfield Model the correlation matrix is provided in the \*.cor file.

**Residual Risk of a Stock**

The stock specific risk predicted by the risk model. This is typically listed in column five of a Northfield asset database file (\*.csv file).
Annual Term
The annualization term is the number of data points used by the factor model per year. For most Northfield models, A equals 12.

Portfolio Est. Var. = Portfolio Factor Risk + Portfolio Stock Specific Risk

$$\text{estVar}_{\text{port}} = \text{FR}_{\text{port}} + \text{SR}_{\text{port}}$$

Where:

$$\text{FR}_{\text{port}} = \sum_{i} \sum_{j} e_i e_j \sigma_i \sigma_j \rho_{ij}$$

where:

- $e_i$ = portfolio factor exposure to factor $i$
- $e_j$ = portfolio factor exposure to factor $j$
- $\sigma_i$ = $\sqrt{\text{Report Variance of Factor } i}$
- $\sigma_j$ = $\sqrt{\text{Report Variance of Factor } j}$
- $\rho_{ij}$ = correlation between factor $i$ and factor $j$

$$\text{SR}_{\text{port}} = \sum_{i} (w_i)^2 k_i$$

where:

- $w_i$ = portfolio weight of stock $i$
- $k_i$ = (Residual Risk of stock $i$)$^2 * $ (Annual Term)
Open Performance Attribution

Benchmark Return Estimated Variance

\[ \text{Benchmark Est. Var.} = \text{Benchmark Factor Risk} + \text{Benchmark Stock Specific Risk} \]

\[ \text{EstVar}_{\text{bench}} = \text{FR}_{\text{bench}} + \text{SR}_{\text{bench}} \]

Where:

\[ \text{FR}_{\text{bench}} = \sum_i \sum_j e_i e_j \sigma_i \sigma_j \rho_{ij} \]

where:

- \( e_i \) = benchmark factor exposure to factor \( i \)
- \( e_j \) = benchmark actor exposure to factor \( j \)
- \( \sigma_i \) = \( \sqrt{\text{Report Variance of Factor } i} \)
- \( \sigma_j \) = \( \sqrt{\text{Report Variance of Factor } j} \)
- \( \rho_{ij} \) = correlation between factor \( i \) and factor \( j \)

\[ \text{SR}_{\text{bench}} = \sum_i (w_i)^2 k_i \]

where:

- \( w_i \) = benchmark weight of stock \( i \)
- \( k_i \) = \((\text{Residual Risk of stock } i)^2 \times \text{Annual Term}\)
Open Performance Attribution

Active Return Estimated Variance

\[ \text{Active Est. Var.} = \text{Active Factor Risk} + \text{Active Stock Specific Risk} \]

\[ EstVar_{active} = FR_{active} + SR_{active} \]

Where:

\[ FR_{active} = \sum_i \sum_j e_i e_j \sigma_i \sigma_j \rho_{ij} \]

where:

- \( e_i \) = active factor exposure to factor \( i \)
- \( e_j \) = active factor exposure to factor \( j \)
- \( \sigma_i \) = \( \sqrt{\text{Report Variance of Factor } i} \)
- \( \sigma_j \) = \( \sqrt{\text{Report Variance of Factor } j} \)
- \( \rho_{ij} \) = correlation between factor \( i \) and factor \( j \)

\[ SR_{active} = \sum_i (w_i)^2 k_i \]

where:

- summing over the union of securities in both the benchmark and the portfolio
- \( w_i \) = active weight of the stock \( i \)
- \( k_i \) = \( (\text{Residual Risk of stock } i)^2 \) * (Annual Term)

Factor Model Est. Var. = \( FR_{active} \)

Stock Specific Est. Var. = \( SR_{active} \)
Open Performance Attribution

Factor Return by Group Estimated Variance

Each factor in the model has its own contribution to the active factor risk \( \text{FR}_{\text{active}} \). This is the contributed variance of the factor.

\[
\text{VarContribution}_f = \sum_i e_f e_i \sigma_f \sigma_i \rho_{fi}
\]

where:
- \( e_f \) = active factor exposure to factor \( f \)
- \( e_i \) = active factor exposure to factor \( i \)
- \( \sigma_f \) = \( \sqrt{\text{Report Variance of Factor } f} \)
- \( \sigma_i \) = \( \sqrt{\text{Report Variance of Factor } i} \)
- \( \rho_{fi} \) = correlation between factor \( f \) and factor \( i \)

The Factor Return by Group Estimated Variance equals the sum of the contributed variance of each factor in the group

\[
\text{EstVar}_{g} = \sum \text{VarContribution}_f
\]

where \( f \) is a factor in group \( g \).

In a well constructed model, active factor risk will always be positive. However, this does not guarantee that the total contributed variance of a subset of factors will also be positive. In such cases the Model SD value for a Group is undefined and reported as N/A.

Risk Decomposition

The values in the Single Period Risk Decomposition Report are all defined in the discussion above.

- Active Exposure Column = active factor exposure to factor \( f \)
- Risk Column = \( \text{VarContribution}_f \)
- Factor Tracking Variance = \( \text{FR}_{\text{active}} \)
- Stock Specific Tracking Variance = \( \text{SR}_{\text{active}} \)
- Total Tracking Variance = \( \text{EstVar}_{\text{active}} \)
- Tracking Error = \( \sqrt{\text{EstVar}_{\text{active}}} \)
- Total Risk of Port = \( \sqrt{\text{EstVar}_{\text{port}}} \)
- Total Risk of Bench = \( \sqrt{\text{EstVar}_{\text{bench}}} \)
The Northfield Fundamental U.S. Equity Factor Model is the most widely used Northfield model, designed to explain the covariance among US stock returns. It is assumed that CAPM beta can explain some but not all of the structure of the covariances. For a detailed derivation, see Rosenberg and Guy (Financial Analyst Journal, 1976). There are sixty-seven factors (items of commonality). The sixty-seven factors consist of beta, eleven fundamental company characteristics, and fifty-five industry groups. The model can be written as:

\[ R_{it} = R_{ft} + \beta_{it} \times (R_{mt} - R_{ft}) + \sum_{k=1}^{67} E_{ikt} \times a_{kt} + \epsilon_{it} \]  

(1)

- **R**<sub>it</sub> = return on stock i during period t
- **β**<sub>it</sub> = estimated beta of stock at time t
- **R**<sub>mt</sub> = return on the market (our reference universe) during period t
- **R**<sub>ft</sub> = risk free rate of return during period t (three month Treasury bill)
- **E**<sub>ikt</sub> = exposure of stock i to factor k at time t
  - exposures are standardized values of continuous variables such as yield
  - dummy variables for industry membership
- **a**<sub>kt</sub> = Jensen’s alpha associated with factor k during period t
- **ε**<sub>it</sub> = error term associated with stock i during period t

Essentially, it is nothing more than a standard CAPM with an effort made to sub-divide the alpha term into 66 components. To the extent we can associate portions of alpha to common factors we increase the ability of the model to explain covariance, unlike the simple CAPM, which assumes that beta alone explains all covariance among securities.

The model is estimated each month in two steps. In the first step, we get preliminary estimates for the beta values (β<sub>it</sub>) for each stock. To get the β<sub>it</sub> values, we first run a traditional CAPM time series (60 months) regression of stock i’s return against the market to get **B**<sub>i</sub>.

\[ R_{it} = R_{ft} + \beta_{it} \times (R_{mt} - R_{ft}) + \epsilon_{it} \]  

(2)

\[ \beta_{ti} = \text{preliminary estimate of beta on stock i} \]
\[ \epsilon_{it} = \text{error term for stock i during period t under traditional CAPM assumptions} \]

To account for the tendency of beta to shift toward one over time, we adopt the method of Blume (Journal of Finance, June 1975).

\[ \beta_{i}^* = K + \beta_{i} \times C \]  

(2a)

K, C = constants
To improve the quality of fit of the model \( (e_{it} < e_{it}) \), we can allow the beta values for each stock to vary over time. For example, it can be observed that highly levered companies (high debt to equity ratios) have higher beta values. We could then imagine that a company that has just taken on a great deal of debt to finance an acquisition would have its beta increase. To capture the changes in beta values over time for a given company, we start by using a cross-sectional regression to estimate the relationships between beta values and company characteristics across the universe.

\[
\beta_{kt}^* = \sum_{k=1}^{66} E_{ik,t} \times b_{kt} + \zeta_{it} \quad (3)
\]

- \( \beta_{kt} \) = sensitivity of beta values with respect to differences from stock to stock in exposure to fundamental characteristic \( k \) at time \( t \)
- \( \zeta_{it} \) = error term for the beta of stock \( i \) at time \( t \)

We assume then that the \( \beta_{kt} \) values that are derived from an analysis across the universe of companies can then be applied to a single company as its characteristics change through time. Once we have the \( \beta_{kt} \) values, we estimate the contemporaneous value for \( \beta_{it} \).

\[
\beta_{it} = \sum_{k=1}^{66} E_{ik,t} \times \beta_{kt}^* \quad (4)
\]

Incidentally, this rather complicated procedure for getting a beta has one additional benefit. We can get a reasonable estimate of beta for a stock with no return history, such as an initial public offering. Even though it has no return history, fundamental characteristics such as P/E, yield, and industry are immediately observable and equation (4) can still be used.

Once the beta values are estimated, we can substitute the \( \beta_{it} \) values into the equation (1) above and run a cross-sectional regression to estimate the \( \alpha_{kt} \) values. The observations in all cross-sectional regressions are weighted by square root of market capitalization which compensates for the skewness in the distribution of market capitalization. If the observations are equally weighted, the analysis is biased toward small capitalization names that are far more numerous. If the observations are purely capitalization weighted, the effective number of observations gets far too small for the large number of independent variables. This procedure provides essentially the same result as generalized-least squared methods that weight observations by inverse error terms (see Grinold and Kahn, Active Portfolio Management, 1st Edition, footnote #9, pg. 59). Each coefficient represents the amount of return associated with a unit exposure to a factor.\(^1\)

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\(^1\) For a more in-depth discussion of the logic underlying this estimation process, refer to chapter 3 of Rudd, Andrew and Henry K. Clasing, Jr., Modern Portfolio Theory: The Principles of Investment Management, Orinda, CA, Andrew Rudd, 1988.
As previously noted, the estimation of factor returns \((\alpha_{kt})\) is done on a reference universe of all US common stocks with more than $250 million market capitalization. The return on the market portfolio \((R_{mt})\) is the return on this subset portfolio. This return computation is weighted by square root of market capitalization of the members.

Once we have the periodic returns to a factor, we can string them together into time series. It is these time series, with their fluctuations, that represent the actual risk factors. One can use these series to observe trends in returns to a factor, for example, and adjust one’s portfolio's exposure accordingly. The covariance of the returns among the time series form the factor covariance matrix. The factor return covariances are based on the last sixty monthly observations (weighted equally).

An aspect of our current research is the impact of using conditional estimates of variance rather than simple sample variance. This technique is meant to compensate for situations where strong serial correlation effects are evident in factor returns through time. For more information, see diBartolomeo (Journal of Index Issues in Investment, March 2000)

For the purpose of historic performance attribution, the usage of the model is simple. Since the factor exposures of each stock in portfolio sum to the factor exposures of the portfolio, equation (1) also holds for portfolios. Once all items in equation (1) have been estimated at the stock level we can calculate the beta and factor exposures for a given portfolio and immediately observe which “bets” paid off and which did not during a particular period. The stock specific risk is initially estimated for this type of model as the time series standard deviation (60 months in this case) of the Equation (1) error term \((\epsilon_{it})\). However, traditional portfolio theory assumes that these error terms are uncorrelated through time. Since empirical evidence exists that shows this assumption to be weak, we make an upward adjustment to the magnitude of the stock specific risk estimate to compensate for this effect (see Parkinson, Journal of Business, 1980).